

BIJU PATNAIK INSTITUTE OF INFORMATION TECHNOLOGY & MANAGEMENT STUDIES, BHUBANESWAR

QUANTITATIVE TECHNIQUE

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Module - I (Quantitative Techniques) (1)

There are 2 types of data

1) Primary data - The person is directly collected the data.
2) Secondary data - The data is already collected by someone and we are following it.

examples of primary data :- Interview, mail, survey
" of secondary data :- Journal, Newspaper, Internet, social media, Govt notification.

Data Classification

There are 4 types of classification

1) Qualitative
2) Quantitative
3) Geographical
4) Chronological

- There are two different variables used in statistics for representation of data.
  1) Discrete Variables (It is easily countable and finite in nature)
  2) Continuous Variables (It is not easily " and infinite in nature)

- There are three different series in statistic to represent the data.
  1) Individual Series - It has no repetation
  2) Discrete series - It has repetation with whole no. and frequency
  3) Continuous Series - Class boundaries (0-10, 10-20, 9-19..)

→ Continuous series are of 2 types.

1) Exclusive type
2) Inclusive type

In exclusive class, lower limit is included and upper limit is excluded.

For ex:-

C.I	f
0-10	3
10-20	4
20-30	5

Similarly in Inclusive types both lower limit and upper limit are included & through this

ex:-

C.I	f
0-9	3
10-19	4
20-29	2
29-39	4

- **Measure of Central Tendency**

There are 5 different of measures of central tendency.

→ Arithmetic Mean
→ Median
→ Mode
→ Geometric mean
→ Harmonic mean

⇔ Arithmetic Mean

- **Mean of Individual Series :-**

$$M = \frac{\Sigma x}{n}$$

- **Mean of Discrete Series :-**

$$M = \frac{\Sigma fx}{N}, \text{ where } N = \Sigma f$$

ex-

x	f	fx
20	2	40
10	3	30
30	4	120
40	6	240
	$N=15$	$\Sigma fx = 430$

$$M = \frac{\Sigma fx}{N} = \frac{430}{15} = 28.6$$

- **Mean of Continuous Series :-**

$$M = \frac{\Sigma f(m)}{N}, \text{ where } N = \Sigma f$$

[Midvalue is sum of upper limit and lower limit, divided by 2]

ex:-

C.I	f	midvalue	f(m)
0-10	2	5	10
10-20	3	15	45
20-30	4	25	100
	$N=9$		$\Sigma fm = 155$

So here $M = \frac{\Sigma fm}{N} = \frac{155}{9} = 17.2$

H.W

Q.1

C.I	f	(m) Midvalue	Fm
0-5	7	2.5	17.5
5-15	3	10	30
15-45	8	30	240
45-55	2	50	100
55-95	5	75	375
	$N=25$		$\Sigma f(m) = 762.5$

So $M = \frac{\Sigma fm}{N} = \frac{762.5}{25} = 30.5$

Q.2

C.I	f	Fx
5	1	5
10	7	70
13	2	26
20	3	60
25	5	125
30	4	120
35	8	280
	$N=30$	$\Sigma fx = 690$

So $M = \frac{\Sigma fx}{N} = \frac{690}{30} = 23$

Q.3

Class	f	midvalue	fm
-80-(-70)	1	-75	-75
-70-(-60)	2	-65	-130
-60-(-50)	3	-55	-165
-50-(-0)	4	-25	-100
	10		-470

So $M = \frac{\Sigma fm}{N} = \frac{-470}{10} = -47$

Q. The Nos 3.2, 5.8, 7.9 & 4.5 have frequencies x, $x+2$, $x-3$ & $x+6$ respectively. (3) whose arithmetic mean is 4.876. Find the value of x.

X	F	F(x)
3.2	x	$3.2x$
5.8	$x+2$	$5.8x+11.6$
7.9	$x-3$	$7.9x-23.7$
4.5	$x+6$	$4.5x+27$
	$N=4x+5$	$\Sigma f(x) = 21.4x+14.9$

So $M = \frac{\Sigma f(x)}{N}$

$\Rightarrow 4.876 = \frac{21.4x+14.9}{4x+5}$

$\Rightarrow 19.504x + 24.38 = 21.4x + 14.9$

$\Rightarrow 1.896x = 9.48$

$\Rightarrow x = \frac{9.48}{1.896} = 5.01 \simeq 5$

Q. Find out the missing frequency from the following distribution. Given $N=200$ & mean $= 1.46$

→

C.I.	F	F(x)
0	46	0
1	f_1	f_1
2	f_2	$2f_2$
3	25	75
4	10	40
5	5	25
	$N=200$	$\Sigma fx = 140 + f_1 + 2f_2$

→ $M = \frac{\Sigma f(x)}{N}$

~~$\Rightarrow 1.46 =$~~

As $N = 200$

$\Rightarrow 86 + f_1 + f_2 = 200$

$\Rightarrow f_1 + f_2 = 114$ —①

Now $M = \frac{\Sigma fx}{N}$

$\Rightarrow 1.46 = \frac{f_1 + 2f_2 + 140}{200}$

$\Rightarrow 292 = f_1 + 2f_2 + 140 = 0$

$\Rightarrow f_1 + 2f_2 = 152$ —②

Now calculating eq^n(1) & (2), we get

$f_1 + f_2 = 114$

$f_1 + 2f_2 = 152$

Subtracting eq^n(1) from eq^n(2)

$\Rightarrow f_1 + f_2 = 114$

$\underline{f_1 + 2f_2 = 152}$

$-f_2 = -38$

$\Rightarrow f_2 = 38$

Putting f_2 value on eq^n(1), we get -

$f_1 + 38 = 114$

$\Rightarrow f_1 = 76$

$\therefore f_1 = 76, f_2 = 38$

Continous : Combined Mean :-

Let $x_1, x_2 \ldots\ldots x_k$ are K no. of series with $n_1, n_2 \ldots\ldots n_k$ frequencies.

Combined Mean is denoted by $\boxed{\bar{X} = \dfrac{n_1\bar{x}_1 + n_2\bar{x}_2 + \ldots\ldots + n_k\bar{x}_k}{n_1 + n_2 + \ldots\ldots + n_k}}$

Q. There are 120 students in a classroom, where 70 are boys and 50 students are girls. So in a mathematics examination, the average mark of boys and girls are 55 & 65 respectively. So find out the average marks of both boys and girls in maths.

→ Lets take boys $\bar{x}_1$ & girls $\bar{x}_2$

Here $N = 120$, $n_1 = 70$ & $n_2 = 50$

$\bar{x}_1 = 55$, $\bar{x}_2 = 65$

So $\bar{X} = \dfrac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$

$= \dfrac{70 \times 55 + 50 \times 65}{120}$

$= \dfrac{3850 + 3250}{120}$

$= \dfrac{7100}{120}$

$= 59.16$

Q. The mean marks of statistic in a class was 72, where the mean marks of boys are 75 and there are 70 boys in the class room and 30 girls in the class room. Find the mean marks of girls.

→ Lets take Boys as $\bar{x}_1$ & Girls as $\bar{x}_2$

So $\bar{x}_1 = 75$, $\bar{x}_2 = ?$

Given $\bar{X} = 72$, $n_1 = 70$ & $n_2 = 30$

So $\bar{X} = \dfrac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$

$72 = \dfrac{75 \times 70 + x_2 \times 30}{100}$

$7200 = 5250 + 30x_2$

$\Rightarrow 30x_2 = 7200 - 5250 = 1950$

$\Rightarrow x_2 = 1950/30 = 65$

Q. The mean mark of 75 students in a class room is 65, later it was found that one observation wrongly taken as 45 instead of 54. Find out the combined mean. (5)

→ Here $\bar{x} = 65$ & $N = 75$

$$\bar{x} = \frac{\Sigma x}{n} \Rightarrow \Sigma x = n \cdot \bar{x}$$

Correct sum = (Wrong sum − Wrongly taken + correct)

$$\bar{x} = \frac{\Sigma x}{n} \Rightarrow 65 = \frac{\Sigma x}{75}$$

$\Rightarrow \Sigma x = 65 \times 75$

$= 4875$ (this is wrong sum)

Now correct sum $= 4875 - 45 + 54$

$= 4884$

Now mean $= \frac{\Sigma x}{n} = \frac{4884}{75} = 65.12$

Q. The avg. salary of 20 employees in an organization is 10,000 later it was found that 2 employee salary 3,000 & 4000 wrongly taken as 9000 & 7000. Find out the correct sum.

$\Rightarrow \bar{x} = 10,000$

$N = 20$

Wrong sum $= n \cdot \bar{x}$

$= 20 \times 10,000$

$= 200,000$

Correct sum = Wrong sum − (9000 + 7000) + 3000 + 4000

$= 200,000 - 9000$

$= 191,000$

Mean $= \frac{\text{Correct sum}}{N}$

$= \frac{191,000}{20}$

$= 9550$

Assignments

(6)

Q.1 Write 3 merits and demerits of arithematic mean.

Q.2 What is exclusive and inclusive type of class with example.

Q.3 What is the difference between Qualitative and Quantitative with examples.

Q.4 " " " " " Primary and Secondary data. 5 different source of Collecting primary and secondary data.

Q.5 What is the difference between discrete and continuous Variables. Explain with examples.

Q.1 Merits : (1) Arithmetic mean is easy to calculate and understand
(2) Arithmetic mean is suitable for various mathematical and ~~statist~~ statistical operations.
(3) It is the most precise estimate of the central value.

Demerits : (1) Arithmetic mean can not be used in qualitative characteristics.
(2) Arithmetic mean can not determine if a single observation is missing or lost.
(3)

Q.2 Exclusive Type :- The exclusive type of class means, in the given data, the lower limit is included and the upper limit is excluded.
ex:- In a exclusive class (20 - 30), all the greater value than 20 are included like 23, 24, 26 and 29 but not 20 or 30.

Inclusive Type :- The inclusive type of class means in the given data both the lower limit and upper limit is included.
ex:- In a inclusive class (20 - 30) all the values greater than or equal to 20 and less than or equal to 30 are included.

Q.3 Qualitative Data :- It is a classification of characteristics or qualities. We can not count Qualitative data.
ex:- She is a beautiful girl.

Quantitative Data :- It is a classification of numbers and values. These data are measurables and countables.
ex:- Her income is 7 lakh per annum.

Q.4 The meaning of primary data is a data which is directly collected by individuals or sources for researches or analysis.
The 5 sources are :- Interviews, Surveys, Experiments, Mail & Observation.

The meaning of secondary data is the data which is already collected by someone for their purposes and we are following it.
The 5 sources are :- Journal, Newspaper, Internet, social media & Govt notification

Q.15 (b)

Size	Frequency	fx
10	3	30
12	7	84
14	?	$14f_1$
16	20	320
18	8	144
20	5	100

→ Let's put the unknown frequency as f_1

Now $\Sigma f = 43 + f_1$ and $\Sigma fx = 678 + 14f_1$

Again, Given $M = 15.38$

So $M = \frac{\Sigma fx}{\Sigma f}$

$\Rightarrow 15.38 = \frac{678 + 14f_1}{43 + f_1}$

$\Rightarrow 15.38 f_1 + 661.34 = 678 + 14f_1$

$\Rightarrow 15.38 f_1 - 14f_1 = 678 - 661.34$

$\Rightarrow 1.38 f_1 = 16.66$

$\Rightarrow f_1 = \frac{16.66}{1.38} = 12.07 \simeq 12$

Q.16

Class	f	M	f(M)
0-20	17	10	170
20-40	f_1=?	30	$30f_1$
40-60	32	50	1600
60-80	f_2=?	70	$70f_2$
80-100	19	90	1710

Now $\Sigma f(M) = 30f_1 + 70f_2 + 3480$

$\Sigma f = f_1 + f_2 + 68$

$\Rightarrow 120 = f_1 + f_2 + 68$

$\Rightarrow f_1 + f_2 = 52$ —(1)

So $M = \frac{\Sigma f(M)}{N}$

$\Rightarrow 50 = \frac{30f_1 + 70f_2 + 3480}{120}$

$\Rightarrow 30f_1 + 70f_2 + 3480 - 6000 = 0$

$\Rightarrow 30f_1 + 70f_2 = 2520$

$\Rightarrow 3f_1 + 7f_2 = 252$ —(2)

Q.16(b)

Marks	No of students	M	FM
0-10	3	5	15
10-20	8	15	120
20-30	11	25	275
30-40	15	35	525
40-50	f_1	45	$45f_1$
50-60	25	55	1375
60-70	40	65	2600
70-80	f_2	75	$75f_2$
80-90	2	85	170
90-100	1	95	95
	$f_1 + f_2 + 105$		$45f_1 + 75f_2 + 5175$

Given $M = 65$ & $N = 150$

As $N = 150 \Rightarrow f_1 + f_2 + 105 = 150 \Rightarrow f_1 + f_2 = 45$ —(1)

Again $M = \frac{\Sigma fM}{N}$

$\Rightarrow 65 = \frac{45f_1 + 75f_2 + 5175}{150}$

$\Rightarrow 9750 = 45f_1 + 75f_2 + 5175$

$\Rightarrow 45f_1 + 75f_2 = 4575$

$\Rightarrow 3f_1 + 5f_2 = 305$ —(2)

Multiplying 3 on eq^n(1) & and substracting with eq^n(2), we get -

$3f_1 + 3f_2 = 135$

$3f_1 + 5f_2 = 305$

$-2f_2 = -170$

$\Rightarrow f_2 = 85$

Putting f_2 value on eq^n(1), we get

$f_1 + 85 = 45$

$\Rightarrow f_1 = -45$

$\therefore (f_1, f_2) = (-45, 85)$

multiplying 3 on eq^n(1) & substracting on eq^n(2)

3 × eq^n(1) $= 3f_1 + 3f_2 = 156$

$3f_1 + 7f_2 = 252$

$-4f_2 = -96$

$\Rightarrow f_2 = \frac{96}{4} = 24$

As $f_1 + f_2 = 52$

$f_1 + 24 = 52$

$\Rightarrow f_1 = 28$

$\therefore f_1 = 28, f_2 = 24$

17(a) The average salary of 49 out of 50 employees in a firm is Rs. 100. The salary of the 50th employee is Rs. 97.50 more than the average salary of all the 50 workers. Find the mean salary of all the employees in the firm. (8)

→ There are 50 employees in the firm.

49 employees mean salary is 100

Let mean salary 50 employees is x

The salary of 50th employer is $x + 97.50$

Total salary of 49 employee is

$$\frac{\Sigma x}{49} = 100 \Rightarrow \Sigma x = 4900.$$

$$x = \frac{\text{Total salary of all 50 employee}}{50}$$

$$= \frac{4900 + x + 97.50}{50}$$

$$\Rightarrow 50x - x = 4997.50 \Rightarrow 49x = 4997.50 \Rightarrow x = \frac{4997.5}{49} = 101.98$$

- **Step Deviation Method**

Assume mean method along with step deviation method.

$$\bar{X} = A + \frac{\Sigma f(d)}{N}, \text{ where } d = X - A$$

$$\bar{X} = A + \frac{\Sigma f(d)}{N} \cdot h \text{ where } d = \frac{X-A}{h}$$

where h = length

A = Assumed mean

x = Mid value

Q.

C.I	f	x	$d = \frac{x-A}{h}$	f(d)
0-20	17	10	-2	-34
20-40	24	30	-1	-24
40-60	32	50	0	0
60-80	28	70	1	28
80-100	19	90	2	38
	N=120			$\Sigma f(d) = 8$

$A = 50$

$h = 20$

$A + \frac{\Sigma f(d)}{N} \cdot h$

→ $\bar{X} = A + \frac{\Sigma f(d)}{N}$

$= 50 + \frac{8}{120}$

$= 50 + 1.3$

$= 51.3$

• Median (Positional Approach)

It is defined as the mid value of perticular series. Median divides the whole series in two equal parts, where one part is greater than the median value and another part is less than the median.

→ Median for Individual Series

(i) Arrange the series either in ascending order or descending order

(ii) Check wheather it is odd or even.

(iii) If it is odd, the median is $(\frac{n+1}{2})$ th term, if it is even, the median is $\frac{(\frac{n}{2})^{th} + (\frac{n}{2}+1)^{th}}{2}$ term

where n is no. of observation present in the series.

Q.

x	f
6	19
7	5
13	4
14	3
15	2
	1

→ Median for x is $(\frac{n+1}{2})$ th term

$(\frac{5+1}{2})$ term

3rd term i.e. 13

Median for f is $\frac{\frac{n}{2} + (\frac{n}{2}+1)}{2}$ term

$= \frac{6/2 + (6/2+1)}{2}$

$= \frac{3+4}{2} = \frac{7}{2} = 3.5$ median

H.W

Q.

- **Median of Discrete Series**

X	f	cf (successive addition of frequency)
20	2	2
30	3	5
31	4	9
33	9	18
39	6	24
65	7	31
66	8	39
67	4	43
	N=43	

39 is the median

In discrete series, first arrange the series on ascending or descending order, then find out the cumilative frequency column. Calculate the $\left(\frac{N+1}{2}\right)$th term, where n is sum of F and the value of $\left(\frac{N+1}{2}\right)$ in CF column, corcresponding to the observation is known as Median.

How to find the median of continuous series

C.I.	f	CF
0-10	4	4
10-20	6	10
20-30	8	18
30-40	9	27
40-50	10	37
50-60	12	49
	N=49	

Here $L_1 = 30$, $L_2 = 40$
$f = 9$, $M = 24.5$
$c = 18$

$\therefore$ Median class is $\frac{N}{2} = \frac{49}{2} = 24.5$

and Median $= L_1 + \frac{L_2 - L_1}{f_1}(M - c)$

where L_1 is lower limit of median
L_2 is upper " " "
f is the frequency of "
M is $\frac{N}{2}$
c is the preceeding CF of median class
Preceeding - Previous
Successive - After

$\rightarrow$ So $Md = L_1 + \frac{L_2 - L_1}{f}(M - c)$

$= 30 + \frac{40-30}{9}(24.5 - 18)$

$= 30 + \frac{10}{9} \times 6.5$

$= 30 + 7.5$

$= 37.5$

Q. Find the missing frequency from the following data.

Q. Find the missing frequency from the following data. Given N = 100 & M = 50

C.I	f	CF
0-20	?	14
20-40	f_1	$14+f_1$
40-60	27	$41+f_1+f_2$
60-80	f_2	$41+f_1+f_2$
80-100	15	$56+f_1+f_2$

→ Let f_1 & f_2 for missing frequency

As we know,

$$Md = \frac{N}{2} = \frac{100}{2} = 50$$

Now $N = 56+f_1+f_2$

$\Rightarrow f_1+f_2+56 = 100$

$\Rightarrow f_1+f_2 = 44$ —①

Again, $Md = L_1 + \frac{L_2-L_1}{f}(M-C)$

$$= 40 + \frac{60-40}{27}(50-14-f_1)$$

$$\Rightarrow 50 = 40 + \frac{20}{27}(36-f_1)$$

$$\Rightarrow 50 = 40 + \frac{720-20f_1}{27}$$

$\Rightarrow 1350 = 1080 + 720 - 20f_1$

$\Rightarrow 1350 = 1800 - 20f_1$

$\Rightarrow 450 - 20f_1 = 0$

$\Rightarrow f_1 = \frac{450}{20} = 22.5$

As, from eq(1), we know

$f_1+f_2 = 44$

So $22.5 + f_2 = 44$

$\Rightarrow f_2 = 44 - 22.5$

$\Rightarrow f_2 = 21.5$

Assignments

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Weight	No. of Boys	CF
10-19	8	8
20-29	15	23
30-39	21	44
40-49	34	78
50-59	45	123
60-69	26	149
70-79	20	169
80-89	15	184
90-99	10	194
100-109	6	200
	N=200	

→ Median class, $M = \frac{N}{2} = \frac{200}{2} = 100$

Here $l_1 = 50, l_2 = 59, f = 45, C = 78$

Now $Md = L_1 + \frac{L_2-L_1}{f_1}(M-C)$

$$= 50 + \frac{59-50}{45}(100-78)$$

$$= 50 + \frac{9}{45} \times 22$$

$$= 50 + 44$$

$$= 54.4$$

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Marcks	No. of students	C.F
0-10	2	2
10-20	5	7
20-30	8	15
30-40	10	25
40-50	25	50
50-60	16	66
60-70	12	78
70-80	8	86
80-90	4	90
90-100	5	95
	N=95	

→ Median class, $M = \frac{N}{2} = \frac{95}{2} = 47.5$

So $Md = L_1 + \frac{L_2-L_1}{f_1}(M-C)$

$$= 40 + \frac{50-40}{25}(47.5-25)$$

$$= 40 + \frac{10}{25} \times 22.5$$

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Midvalue	f	C.I.	CF
5	1	2.5-7.5	1
10	2	7.5-12.5	3
15	3	12.5-17.5	6
20	4	17.5-22.5	10
25	5	22.5-27.5	15
30	6	27.5-32.5	21
35	7	32.5-37.5	28
40	8	37.5-42.5	36
45	9	42.5-47.5	45
	N=45		

Median class, $m = \frac{N}{2}, \frac{45}{2} = 22.5$

$L_1 = 32.5, L_2 = 37.5, f_1 = 7,$

$c = 21$

So $Md = L_1 + \frac{L_2 - L_1}{f_1}(m - c)$

$= 32.5 + \frac{37.5 - 32.5}{7}(22.5 - 21)$

$= 32.5 + \frac{5}{7} \times 1.5$

$= 32.5 + \frac{7.5}{7}$

$= 32.5 + 1.07$

$= 33.57$

45

C.I	F	CF
30-32	3	3
32-34	8	11
34-36	24	35
36-38	31	66
38-40	50	116
40-42	61	177
42-44	38	215
44-46	21	236
46-48	12	248
48-50	2	250
	N=250	

Here, $M = \frac{N}{2} = \frac{250}{2} = 125$

So $L_1 = 40, L_2 = 42, f_1 = 61, c = 116$

So $Md = L_1 + \frac{L_2 - L_1}{f_1}(m - c)$

$= 40 + \frac{42 - 40}{61}(125 - 116)$

$= 40 + \frac{2}{61} \times 9 = 40 + \frac{18}{61} = 40 + 0.295$

$= 40.295$

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C.I.	F	CF
3-5	6	6
5-7	f_1	$6 + f_1$
7-9	18	$24 + f_1$
9-11	20	$44 + f_1$
11-13	f_2	$44 + f_1 + f_2$
13-15	10	$54 + f_1 + f_2$
	$f_1 + f_2 + 54$	

→ Given $N = 80, Md = 9.20$

Here $f_1 + f_2 + 54 = 80 \Rightarrow f_1 + f_2 = 26$ —(1)

So Mode class $= \frac{N}{2} = \frac{80}{2} = 40$

Here $l_1 = 9, l_2 = 11, f_1 = 20$ & $c = 24 + f_1$

Thus $Md = L_1 + \frac{L_2 - L_1}{f_1}(m - c)$

$\Rightarrow 9.20 = 9 + \frac{11 - 9}{20} \times (40 - 24 - f_1)$

$\Rightarrow 9.20 = 9 + \frac{2}{20} \times 16 - f_1 = 9 + \frac{16 - f_1}{10}$

$\Rightarrow 9.20 = 9 + \frac{32 - f_1}{20} = \frac{90 + 16 - f_1}{10}$

~~$\Rightarrow 184 = 180 + 32 - f_1$~~ $\Rightarrow 92 = 90 + 16 - f_1$

~~$\Rightarrow -f_1 = 184 - 180 - 32$~~ $\Rightarrow f_1 = 106 - 92$

~~$\Rightarrow -f_1 = -28$~~ $\Rightarrow f_1 = 14$

~~$\Rightarrow f_1 = 28$~~

Putting f_1 value on eq(1), we get -

$14 + f_2 = 26 \Rightarrow f_2 = 26 - 14 = 12$

$\therefore f_1 = 14, f_2 = 12$

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x	F	CF
10	6	6
11	9	15
12	25	40
13	15	55
14	20	75
15	16	91
16	11	102
17	10	112
18	8	120
	N=120	

So Median $= \frac{N+1}{2} = \frac{120+1}{2} = \frac{121}{2} = 60.5$

Mode

It is defined as most frequently observed value in a perticular series.

There are 3 different types of mode.

1) Unimodal (1, 2, 2, 3, 4, 5)
2) Bimodal (1, 2, 2, 3, 3, 4, 5)
3) Multimodal (1, 2, 2, 3, 3, 4, 4, 5)

A series having single mode is known as unimodal series.

A series having two mode then it is bimodal series.

A series having more than two mode is multimodal series.

There is no chance of mode in Individual series.

In a discrete series mode is the observation which have highest frequency.

Continuous series

C.I.	f
0-10	2
10-20	5
20-30	8
30-40	10
40-50	25
50-60	16
60-70	12

$$\text{Mode} = 3\,\text{median} - 2\,\text{mean}$$

Empirical relation between mean, median and mode

[If it is unable to find mode of a continuous series with the help of usual technique, then in that case we need to follow the impercical relation]

In a continuous series first find out the modal class, modal class is defined as the class having highest frequencies.

Here modal class is 40 - 50 16

$$M_0 = L_1 + \frac{f_1 - f_0}{2f_1 - f_2 - f_0}(L_2 - L_1)$$

$L_1 = 40$, $L_2 = 50$

f_0 = preceeding frequency of modal class.

f_1 = corrcesponding " " " "

f_2 = succeeding " " " " .

$$M_0 = 40 + \frac{25 - 10}{50 - 16 - 10}(50 - 40)$$

$$= 40 + \frac{15}{24} \times 10$$

$$= 40 + \frac{150}{24}$$

$$= 40 + 6.25$$

$$= 46.25$$

Assignments

① Write 3 merits and demerits of ~~median~~ mode.

→ Merits :- (i) It is easy to calculate

(ii) It is very much useful for both

Q.2

X	f	C.F
0-10	4	4
10-20	16	20
20-30	f_1	f_1+20
30-40	f_2	f_1+f_2+20
40-50	f_3	$f_1+f_2+f_3+20$
50-60	6	$f_1+f_2+f_3+26$
60-70	4	$f_1+f_2+f_3+30$

Given $N = 230$, $M_0 = 34$, $M_d = 33.5$

As $M_0 = 34$, it lies in class (30-40) and having frequency f_2.

So here $l_1 = 30$, $l_2 = 40$, $f_0 = f_1$, $f_1 = f_2$

& $f_2 = f_3$

$$\Rightarrow M_0 = l_1 + \frac{f_1 - f_0}{2f_1 - f_2 - f_0}(l_2 - l_1)$$

$$\Rightarrow 34 = 30 + \frac{f_2 - f_1}{2f_2 - f_3 - f_1}(40-30)$$

$$\Rightarrow 34 = \frac{60f_2 - 30f_3 - 30f_1 + 10f_2 - 10f_1}{2f_2 - f_3 - f_1}$$

$$\Rightarrow 68f_2 - 34f_3 - 34f_1 - 60f_2 + 30f_3 + 30f_1 - 10f_2 + 10f_1 = 0$$

$$\Rightarrow -2f_2 - 4f_3 + 6f_1 = 0$$

$$\Rightarrow 3f_1 - f_2 - 2f_3 = 0 \quad \text{—— ③}$$

Now Given $N = 230$

$$\Rightarrow f_1 + f_2 + f_3 + 30 = 230$$

$$\Rightarrow f_1 + f_2 + f_3 = 200 \quad \text{—— ②}$$

Again, Median = 33.5

this implies the median class is (30-40) and having frequency f_2.

So as we know median $= l_1 + \frac{l_2 - l_1}{f_1}(m - c)$

(Here $m = \frac{N}{2} = \frac{230}{2} = 115$)

$$\Rightarrow 33.5 = 30 + \frac{40-30}{f_1}(115 - f_1 - 20)$$

$$\Rightarrow 33.5 = 30 + \frac{10}{f_2} \times 95 - f_1$$

$$\Rightarrow 33.5 = \frac{30f_2 + 950 - 10f_1}{f_2}$$

$$\Rightarrow 33.5f_2 + 10f_1 = 950 \quad \text{—— (3)}$$

$$\Rightarrow 3.5f_2 + 10f_1 = 950 \quad \text{—— (3)}$$

Now Solving 3 eq'n using Substitution method

$$f_1 + f_2 + f_3 = 200 \quad \text{—— (2)}$$

$$3f_1 - f_2 - 2f_3 = 0 \quad \text{—— (1)}$$

$$3f_1 - f_2 - 2f_3 = 0 \quad \text{—— (1)}$$

$$f_1 + f_2 + f_3 = 200 \quad \text{—— (2)}$$

$$10f_1 + 3.5f_2 = 950 \quad \text{—— (3)}$$

Now multiplying eq 2 in eq'n (2) & adding with eq'n (1), we get -

$$3f_1 - f_2 - 2f_3 = 0$$

$$2 \times \text{eq(2)} \quad 2f_1 + 2f_2 + 2f_3 = 400$$

$$5f_1 + f_2 = 400 \quad \text{—— (4)}$$

Now, multiplying 2 in eq'n (4) & substracting with eq'n (3)

$$2 \times \text{eq'n (4)} = 10f_1 + 2f_2 = 800$$

$$10f_1 + 3.5f_2 = 950$$

$$-1.5f_2 = -150$$

$$\Rightarrow f_2 = \frac{150}{1.5} = 100$$

Putting f_2 value in eq'n (4), we get -

$$5f_1 + 100 = 400 \Rightarrow f_1 = \frac{300}{5} = 60$$

Now putting f_1 & f_2 value in eq'n (2), we get -

$$60 + 100 + f_3 = 200$$

$$\Rightarrow f_3 = 40$$

$\therefore (f_1, f_2, f_3) = (60, 100, 40)$

Q.

C.I (X)	F	C.F	M	$d = M - A$	$d' = \frac{d}{c}$	fd'
100 – 110	6	6	105	-10	-1	-6
110 – 120	4	10	115	0	0	0
120 – 130	20	30	125	10	1	20
130 – 140	32	62	135	20	2	64
140 – 150	39	101	145	30	3	117
150 – 160	17	118	155	40	4	68
160 – 170	8	126	165	50	5	40
170 – 180	2	128	175	60	6	12
	N=128					$\Sigma fd' = 315$

$\rightarrow \bar{X} = A + \frac{\Sigma fd'}{N} \times c$

$= 115 + \frac{315}{128} \times 10$

$= 115 + \frac{3150}{128}$

$= 115 + 24.60$

$= 139.60$

$M = (\frac{N}{2})^{th}$ term $= \frac{128}{2} = 64^{th}$

Median class in between (140 – 150)

$M = L_1 + \frac{L_2 - L_1}{f}(m - c)$

$= 140 + \frac{150 - 140}{39}(64 - 62)$

$= 140 + \frac{10}{39} \times 2$

$= 140 + \frac{20}{39}$

$= 140 + 0.51$

$= 140.51$

Now Mode $= L_1 + \frac{f_1 - f_0}{2f_1 - f_0 - f_2}(L_2 - L_1)$

$= 140 + \frac{39 - 32}{2 \times 39 - 32 - 17}(150 - 140)$

$= 140 + \frac{7}{78 - 32 - 17} \times 10$

$= 140 + \frac{70}{29}$

$= 140 + 2.41$

$= 142.41$

- **Geometric Mean** (For Individual Series)

If $x_1, x_2, x_3 \ldots\ldots x_n$ are 'n' observation in a series, then the Geometric mean is defined as n^{th} root of product of all observation.

$$G = \sqrt[n]{x_1 \cdot x_2 \cdot \ldots \cdot x_n} \quad \text{or} \quad G.M = (x_1 \cdot x_2 \cdots x_n)^{1/n}$$

ex:- G.M of 5 & 6

$$G.M = (x_1 \cdot x_2)^{1/2}$$
$$= (5 \times 6)^{1/2}$$
$$= (30)^{1/2}$$
$$= 5.47$$

Q. Mean of 2 observation is 25 and GM is 20, then find out both the observation

→ Here $G.M = 20$

Lets take 2 observation i.e. x & y

$$M = \frac{x+y}{2} = 25 \quad \text{—(1)}$$

$$G.M = \sqrt{xy} = 20 \quad \text{—(2)}$$

from eq^n(1), we can get - $x+y = 50$ —(1)

" eq^n(2), " " " - $\sqrt{(xy)^2} = 400$

$\Rightarrow xy = 400$

Now we know $(x-y)^2 = (x+y)^2 - 4xy$

$\Rightarrow (x-y)^2 = (50)^2 - 4 \times 400$

$= 2500 - 1600$

$= 900$

$\Rightarrow (x-y)^2 = 900$

$\Rightarrow (x-y) = 30$

$\Rightarrow x-y = 30 \quad \text{—(2)}$

Adding eq^n(1) & (2), we get -

$$x+y = 50$$
$$x-y = 30$$
$$\overline{2x = 80}$$
$$x = 40.$$

Putting x value in eq^n(1), we get

$\rightarrow 40+y = 50$

$\Rightarrow y = 10$

→ The application of Geometric mean is helpful in calculating the annual percentage growth of any activities, mostly applied.

→ When the frequency of any observation is very large at the same time the observation is small and vice versa. In that case GM is mostly applicable than AM.

ex:-

X	f
2	8
4	10
30	2
40	4

→

$$G.M = (X_1 \cdot X_2 \cdots X_n)^{1/n}$$

$$\log G.M = \log (X_1 \cdot X_2 \cdots X_n)^{1/n}$$

$$= \frac{1}{n} \log(X_1 \cdot \cdots X_n)$$

$$= \frac{1}{n} (\log X_1 + \log X_2 + \cdots \log X_n)$$

$$= \frac{1}{n} \left(\sum_{i=1}^{n} \log X_i\right)$$

$$\log GM = \frac{\sum_{i=1}^{n} \log X_i}{n}$$

$$GM = AL\left(\frac{\sum_{i=1}^{n} \log X_i}{n}\right)$$

$$\left[\begin{array}{l} \log m^n = n \log m \\ \log(mn) = \log m + \log n \end{array}\right]$$

Assignments

Q. Find G.M of (2, 3, 7, 9)

→

X	log X
2	0.30
3	0.47
7	0.84
9	0.95
	$\Sigma \log X = 2.56$

$$G.M = AL\left(\frac{\Sigma \log x_i}{n}\right)$$

$$= AL\left(\frac{2.56}{4}\right)$$

$$= AL(0.64)$$

$$= 4.40$$

Q. Find GM of (7, 7, 3, 3, 3, 2, 2, 4, 4, 4)

X	f	log X	f log X
7	2	0.84	1.69
3	3	0.47	1.41
2	2	0.30	0.60
4	3	0.60	2.4
	N=11		$\Sigma = 6.13$

$$G.M = AL\left(\frac{\Sigma f \log x}{n}\right)$$

$$= AL\left(\frac{6.13}{11}\right)$$

$$= AL(0.55)$$

$$= 3.54$$

Q Find out AM & GM of following series.

X	f	log x	f log x	f(x)
2	18	0.30	5.4	36
4	10	0.60	6	40
30	2	1.47	2.94	60
40	4	1.60	6.4	160
	N=34		Σ=20.74	Σ=296

$$AM = \frac{\Sigma f(x)}{N} = \frac{296}{34} = 8.70$$

$$GM = AL\left(\frac{\Sigma f \log x}{N}\right) = AL\left(\frac{20.74}{34}\right)$$

$$= AL(0.61) = 4.07$$

For this particular series the arithematic mean is 8.75 and geometric mean is 4.07 and this shows that 4.07 is representing the whole series thats why geometric mean is more applicable, where observation is small corresponding to large frequency and observation is large corresponding to small frequency.

Assignments

If the GM of the following series 70, then find out the missing frequency of the series. where N is 40.

C.I	f	log x	f log x
10	5	1	5
20	7	1.30	9.1
30	a	1.47	1.47a
40	b	1.60	1.60b
50	9	1.69	15.21
	a+b+21		1.47a+1.60b+29.31

↦ Here Given N = 40, G.M = 70

If N=40 ⇒ a+b+21 = 40

⇒ a+b = 19 —(1)

Now $GM = AL\left(\frac{\Sigma f \log x}{N}\right)$

$$\Rightarrow 70 = AL\left(\frac{1.47a + 1.60b + 29.31}{40}\right)$$

$$\Rightarrow \log 70 \times 40 = 1.47a + 1.60b + 29.31$$

$$\Rightarrow 1.84 \times 40 = 1.47a + 1.60b + 29.31$$

$$\Rightarrow 73.8 = 1.47a + 1.60b + 29.31$$

$$\Rightarrow 1.47a + 1.60b = 73.8 - 29.31$$

$$\Rightarrow 1.47a + 1.60b = 44.49 \quad —(2)$$

Now multiplying 1.47 on eq^n(1) & substracting with eq^n(2), we get -

$$1.47 \times \text{eq}^n(1) = 1.47a + 1.47b = 27.93$$

$$1.47a + 1.60b = 44.49$$

$$-0.13b = -16.56$$

$$\Rightarrow b = \frac{16.56}{0.13}$$

$$= 127.38$$

On putting the value of b, on eq^n(1), we get -

$$a+b = 19$$

$$\Rightarrow a + 127.38 = 19$$

$$\Rightarrow a = 19 - 127.38$$

$$\Rightarrow a = -108.38$$

Q. Find the GM of the following series.

X	f	log x	f log x
2	4	0.30	1.2
4	3	0.60	1.8
8	2	0.90	1.8
16	1	1.20	1.2
	10		6

→ $GM = AL\left(\frac{\Sigma f \log x}{N}\right)$

$= AL\left(\frac{6}{10}\right)$

$= AL(0.6)$

$= 3.98 \simeq 4$

Q. Find the GM of the following series (M9)

X	f	M	log M	f log M
0-10	5	5	0.69	3.45
10-20	7	15	1.17	8.19
20-30	15	25	1.39	20.85
30-40	25	35	1.54	38.5
40-50	8	45	1.65	13.2
	60			84.19

$GM = AL\left(\frac{\Sigma f \log x}{N}\right)$

$= AL\left(\frac{84.19}{60}\right)$

$= AL(1.40)$

$= 25.11$

Combined Mean (VVIP 4 Exam)

If the mean series 1 is 30 with observation 9 and the mean of series 2 is 20 with observation 5. Find the combined mean.

→ Here $\bar{X}_1 = 30$, $\bar{X}_2 = 20$

$n_1 = 9$, $n_2 = 5$

$$\text{Combined Mean} = \frac{n_1\bar{X}_1 + n_2\bar{X}_2}{n_1 + n_2}$$

$$= \frac{270 + 100}{14}$$

$$= \frac{370}{14}$$

$$= 26.4$$

(short Q - 2 mark - VIP)

Q. The mean mark of 75 students in a class room is 65, later it was found that one observation wrongly taken as 45 instead of 54. Find the correct mean.

→ Here $\bar{X} = 65$ & $N = 75$

We know $\bar{X} = \frac{\Sigma X}{n}$

$\Rightarrow 65 = \frac{\Sigma X}{75}$

$\Rightarrow \Sigma X = 4875$

Correct sum = wrong sum − wrongly taken ob + right ob

$\Rightarrow$ Correct sum = $4875 - 45 + 54 = 4884$

Now mean = $\frac{\Sigma X}{n} = \frac{4884}{75} = 65.12$

Combined Geometric mean

Let G_1, G_2, G_3 are Geometric mean of 3 different Series with n_1, n_2 & n_3 observation. Then the combined Geometric mean is

$$G = AL\left(\frac{n_1 \log G_1 + n_2 \log G_2 + n_3 \log G_3}{n_1 + n_2 + n_3}\right)$$

→ If $G_1 = 10$, $G_2 = 7$, $G_3 = 8$

$n_1 = 10$, $n_2 = 8$, $n_3 = 6$

So combined Geometric mean $= AL\left(\frac{n_1 \log G_1 + n_2 \log G_2 + n_3 \log G_3}{n_1 + n_2 + n_3}\right)$

$= AL\left(\frac{10\times 10 + 8\times 7 + 6\times 8}{10+8+6}\right)$

$= AL\left(\frac{10\times 1 + 8\times 0.84 + 6\times 0.90}{10+8+6}\right)$

$= AL\left(\frac{10 + 6.72 + 5.4}{24}\right)$

$= AL\left(\frac{22.12}{24}\right) = AL(0.92) = 8.317$

Q. Given $n_1 = 8$, $n_2 = 7$, $n_3 = 5$

$G_1 = 8$, $G_2 = 10.12$ $G_3 = 7.75$

→ $CGM = AL\left(\frac{n_1 \log G_1 + n_2 \log G_2 + n_3 \log G_3}{n_1 + n_2 + n_3}\right)$

$= AL\left(\frac{8\times \log 8 + 7 \log(10.12) + 5 \log(7.75)}{8+7+5}\right)$

$= AL\left(\frac{8\times 0.90 + 7\times 1 + 5\times 0.88}{20}\right)$

$= AL\left(\frac{7.2 + 7 + 4.4}{20}\right)$

$= AL\left(\frac{18.6}{20}\right)$

$= AL(0.93)$

$= 8.51$

Assignments

	A	B	C	Combined
Number	10	8	?	24
Mean	20	?	6	15
GM	10	7	?	8.397

Here Given A=10, B=8 and the combined value of 3 no = 24

So $A+B+C=24$

$\Rightarrow 10+8+C=24$

$\Rightarrow C=24-18=6$

Now, we have Mean of A=20, Mean of C=6 and the combined mean of ABC=15

Thus combined mean of ABC $= \dfrac{\bar{X}_1 A + \bar{X}_2 B + \bar{X}_3 C}{A+B+C}$, where $\bar{X}_1$ = Mean of A

$\bar{X}_2$ = Mean of B

$\bar{X}_3$ = Mean of C

$$\Rightarrow 15 = \frac{20\times 10 + \bar{X}_2 \times 8 + 6\times 6}{10+8+6}$$

$$\Rightarrow 15 = \frac{200 + 8\bar{X}_2 + 36}{24}$$

$$\Rightarrow 360 = 8\bar{X}_2 + 236$$

$$\Rightarrow 8\bar{X}_2 = 124$$

$$\Rightarrow \bar{X}_2 = \frac{124}{8} = 15.5$$

$\therefore$ So Mean of B = 15.5

Again, we have GM of A = 10, GM of B = 7 and the combined GM of ABC = 8.397

Thus combined GM of ABC $= AL\left(\dfrac{A\log G_1 + B\log G_2 + C\log G_3}{A+B+C}\right)$ $\{\because G_1$ = GM of A, G_2 = GM of B, G_3 = GM of C$\}$

$$\Rightarrow 8.397 = AL\left(\frac{10\times\log 10 + 8\log 7 + 6\times\log G_3}{10+8+6}\right)$$

$$\Rightarrow 8.397 = AL\left(\frac{10\times 1 + 8\times 0.84 + 6\log G_3}{24}\right)$$

$$\Rightarrow \log(8.397)\times 24 = 10 + 6.72 + 6\log G_3$$

$$\Rightarrow 0.92\times 24 = 16.72 + 6\log G_3$$

$$\Rightarrow 22.08 = 16.72 + 6\log G_3$$

$$\Rightarrow 6\log G_3 = 22.08 - 16.72 = 5.36$$

$$\Rightarrow \log G_3 = \frac{5.36}{6} = 0.89 \Rightarrow G_3 = AL(0.89) = 7.76$$

• Harmonic Mean

Let $x_1, x_2 \dots x_n$ are n no. of observation, then the harmonic mean is defined as the reciprocal of arithmetic mean of reciprocal of observation.

Individual

	reciprocal
x_1	$\frac{1}{x_1}$
x_2	$\frac{1}{x_2}$
x_3	$\frac{1}{x_3}$
⋮	⋮
x_n	$\frac{1}{x_n}$

So AM = $\frac{\frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3} + \dots + \frac{1}{x_n}}{n}$

Thus Harmonic mean is $\frac{n}{\frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3} + \dots + \frac{1}{x_n}}$

Discrete

$$H.M = \frac{N}{\frac{f_1}{x_1} + \frac{f_2}{x_2} + \dots \frac{f_n}{x_n}} = \sum_{i=1}^{n} \frac{N}{f_i/x_i}, (N = \Sigma n)$$

continuous

$$H.M = \frac{N}{f_1/m_1 + f_2/m_2 + \dots f_n/m_n} = \sum_{i=1}^{n} \frac{N}{f_i/m_i}$$

where $N = \Sigma n$
m = midvalue of class

Q. Find the Harmonic mean of 2, 4, 5

Here $n = 3$

So $H.M = \frac{n}{\frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3}}$

$= \frac{3}{0.5 + 0.25 + 0.2}$

$= \frac{3}{0.95}$

$= 3.15$

Q. Find H.M of 10, 20, 40, 60, 120

So ~~H.M~~, $n = 5$

$H.M = \frac{5}{0.1 + 0.2 + \frac{0.4}{0.25} + 0.6 + 1.2}$ $\frac{5}{0.1 + 0.5 + 0.25 + 0.16 + 0.008}$

$= \frac{5}{2.5}$ $= \frac{5}{1.993}$

$= 2$ $= 0.$

Q. Find the $\frac{N}{\sum f_i/x_i}$

C.I	f
10	5
12	18
14	20
16	10
18	6
20	1
	60

$$H.M = \sum_{i=1}^{n} \frac{N}{\sum f_i/x_i}$$

$$= \frac{60}{0.5+1.5+1.42+0.62+0.33+0.05}$$

$$= \frac{60}{4.42}$$

$$= 13.57$$

Q. From the following, find H.M

X	f
5	5
15	10
25	7
35	3
45	2

$$H.M = \frac{27}{1+0.66+0.28+0.08+0.04}$$

$$= \frac{27}{2.06}$$

$$= 13.10 \simeq 13$$

Q.

class	f	m
0 – 20	11	10
20 – 40	13	30
40 – 60	17	50
60 – 80	21	70
80 – 100	23	90
	85	

$$N = \sum_{i=1}^{n} \frac{N}{f_i/m_i}$$

$$= \frac{85}{1.1+0.65+0.56+0.52+0.46+0.43+0.34+0.3+0.25}$$

$$= \frac{85}{2.42}$$

$$= 25.83$$

Q. Find H.M of

wage in rupees	no of f	m
5 – 10	1	7.5
10 – 15	2	12.5
15 – 20	3	17.5
20 – 25	4	22.5
25 – 30	3	27.5
30 – 35	2	32.5
	15	

$$H.M = \frac{15}{0.13+0.16+0.17+0.17+0.10+0.06}$$

$$= \frac{15}{0.79}$$

$$= 18.57 \simeq 18$$

Relation between AM, GM & HM

Find the AM, GM & HM of 2,4,8 and tell me

$$AM = \frac{\Sigma x}{n} = \frac{14}{3} = 4.66$$

$$GM = \sqrt[3]{2*4*8} = (64)^{1/3} = 3.9 \cong 4$$

$$HM = \frac{n}{\Sigma f} = \frac{3}{\frac{1}{2}+\frac{1}{4}+\frac{1}{8}} = \frac{3}{0.5+0.25+0.125}$$

$$= \frac{3}{0.875} = 3.42$$

So the relation between AM, GM & HM is $\boxed{AM \geq GM \geq HM}$

Assignments

(1) Write 3 advantages and disadvantages of H.M.

(2) Write 3 uses of HM

(3) Find out the missing frequency of the following series where mode is 34 & N is 60 (Ans = 10, 5)

C.I	f	C.F
0-10	8	8
10-20	7	15
20-30	x	x+15
30-40	20	x+35
40-50	y	x+y35
50-60	6	x+y+41
60-70	4	x+y+45
	x+y+45	

Given N = 60

$\Rightarrow x+y+45 = 60$

$\Rightarrow x+y = 15$ —(1)

Now, the mode is 34 that means it lies in class (30-40) having ft frequency 20.

So here $l_1 = 30$, $l_2 = 40$, $f_0 = x$, $f_1 = 20$, $f_2 = y$

Thus $M_0 = l_1 + \frac{f_1 - f_0}{2f_1 - f_2 - f_0}(l_2 - l_1)$

$\Rightarrow 34 = 30 + \frac{20-x}{40-y-x}(40-30)$

$\Rightarrow 34 = \frac{1200-30y-30x+20-x}{40-y-x}(10)$

$\Rightarrow 34(40-y-x) = 1200-30y-30x+200-10x$

$\Rightarrow 1360-34y-30x = 1200-30y-30x+200-10x$

$\Rightarrow 1360-1400-34y-34x = 1400-30y-40x$

$\Rightarrow -40-4y+6x = 0$

$\Rightarrow 6x-4y = 40$ —(2)

Now multiplying 4 on eq'(1) and adding with eq'(2), we get -

4 × eq'(1) $4x+4y = 60$

$6x-4y = 40$

$10x = 100$

$\Rightarrow x = 10$

Now putting x value on eq'(1), we get -

$x+y = 15$

$\Rightarrow 10+y = 15$

$\Rightarrow y = 5$

$\therefore (x, y) = (10, 5)$

• Measures of Dispersion

1. Range
2. Quantile
3. Mean Deviation
4. Standard Deviation
5. Variance.

*Standard Deviation

Standard deviation is the most appropriate measure in measures of dispersion. If the standard deviation is less then there is less dispersion in data. If the S.D is more then there is highest dispersion in the particular series of data.

If the S.D is zero then there is no dispersion i.e. the data means are same.

For Individual Series

$$\sigma x = \sqrt{\frac{(x-\bar{x})^2}{n}}$$

Q. Find the standard deviation of the series

→ The mean of the data

$$\bar{X} = \frac{20}{2}$$

	$(x-\bar{x})$	$(x-\bar{x})^2$
5	−15	225
10	−10	100
15	−5	25
20	0	0
25	5	25
30	10	100
35	15	225
		700

$$\therefore \sigma x = \sqrt{\frac{\Sigma (x-\bar{x})^2}{n}} = \sqrt{\frac{700}{20}}$$

Q. Find the value of x, from the following data where S.D is 3.

C.I
3
5
9
x
10

Q.2

Marcks	No. of students	fx	$(x-\bar{x})$	$(x-\bar{x})^2$	$f(x-\bar{x})^2$
10	8	80	-20.83	433.88	3471.04
20	12	240	-10.83	117.28	1407.36
30	20	600	-0.83	0.68	13.6
40	10	400	9.17	84.08	840.8
50	7	350	19.17	367.48	2572.36
60	3	180	29.17	850.88	2552.64
	N=60	$\Sigma fx = 1850$			10857.8

→ Here Mean $\bar{x} = \frac{\Sigma fx}{N} = \frac{1850}{60} = 30.83$

So $\sigma_x = \sqrt{\frac{\Sigma f(x-\bar{x})^2}{N}} = \sqrt{\frac{10857.8}{60}} = \sqrt{180.96} = 13.45$

Q.4

C.I	f	m	fm	$(m-\bar{x})$	$(m-\bar{x})^2$	$f(m-\bar{x})^2$
10-19	3	14.5	43.5	-27.52	757.35	2272.05
20-29	61	24.5	1494.5	-17.52	306.95	18723.95
30-39	233	34.5	8038.5	-7.52	56.55	13176.15
40-49	137	44.5	6096.5	2.48	6.15	842.55
50-59	53	54.5	2888.5	12.48	155.75	8254.75
60-69	79	64.5	5095.5	22.48	505.35	39922.65
70-79	4	74.5	298	32.48	1054.95	4219.8
	N=570		$\Sigma fm = 23955$			$\Sigma = 87411.9$

→ Here Mean, $\bar{x} = \frac{\Sigma fm}{N} = \frac{23955}{570} = 42.02$

Thus $\sigma_x = \sqrt{\frac{\Sigma f(m-\bar{x})^2}{N}} = \sqrt{\frac{87411.9}{570}} = \sqrt{153.35} = 12.38$

Q.5

Wages	No. of Persons
Less than 10	12
" " 20	30
" " 30	65
" " 40	107
" " 50	157
" " 60	202
" " 70	222
" " 80	230

→ The meaning of this data is 12 no. of persons having wages less than 10, and 30 persons having wages less than 20, 65 persons having wages less than 40 and so on.

Thus, the no. of persons are cumilative frequency.

So we need find out frequency of the data first.

→

C.I	CF	f	m	fm	$(m-\bar{x})$	$(m-\bar{x})^2$	$f(m-\bar{x})^2$
0-10	12	12	5	60	-34.34	1179.23	14150.76
10-20	30	(30-12)=18	15	270	-24.34	592.43	10663.74
20-30	65	(65-30)=35	25	625	-14.34	205.63	7197.05
30-40	107	(107-65)=42	35	1470	-4.34	18.83	790.86
40-50	157	(157-107)=50	45	2250	5.66	32.03	1601.5
50-60	202	(202-157)=45	55	2475	15.66	245.23	11035.35
60-70	222	(222-202)=20	65	1300	25.66	658.43	13168.6
70-80	230	(230-222)=8	75	600	35.66	1271.63	10173.04
		N=230		Σfm=9050			Σ=68780.9

→ Here, Mean, $\bar{x} = \frac{\Sigma fm}{N} = \frac{9050}{230} = 39.34$

Thus $\sigma x = \sqrt{\frac{\Sigma f(m-\bar{x})^2}{N}} = \sqrt{\frac{68780.9}{230}} = \sqrt{299.04} = 17.29$

• Step Deviation Method

$$\sigma x = \sqrt{\frac{\Sigma fd^2}{N} - \left(\frac{\Sigma fd}{N}\right)^2}$$

$$\sigma x = h\sqrt{\frac{\Sigma fd^2}{N} - \left(\frac{\Sigma fd}{N}\right)^2}$$

* $A = 64.5, h = 9$

Class	f	x	$d = \frac{x-A}{h}$	d^2	fd	fd^2
90-99	2	94.5	3.33	11.08	6.66	22.16
80-89	12	84.5	2.22	4.92	26.64	59.04
70-79	22	74.5	1.11	1.23	24.42	27.06
60-69	20	64.5	0	0	0	0
50-59	14	54.5	-1.11	1.23	-15.54	17.22
40-49	4	44.5	-2.22	4.92	-8.88	19.68
30-39	1	34.5	-3.33	11.08	-3.33	11.08
	75				29.97	156.24

$$\sigma_x = h \sqrt{\frac{\Sigma fd^2}{N} - \left(\frac{\Sigma fd}{N}\right)^2}$$

$$= 9 \sqrt{\frac{156.24}{75} - \left(\frac{29.97}{75}\right)^2}$$

$$= 9 \sqrt{2.08 - (0.39)^2}$$

$$= 9 \sqrt{2.08 - 0.15}$$

$$= 9 \sqrt{1.93} = 9 \times 1.38 = 12.50$$

Q.

Class	f	x	$d = \frac{x-A}{h}$	d^2	fd	fd^2
11-12	5	11.5	-4	16	-20	80
13-14	426	13.5	-3	9	-1278	3834
15-16	720	15.5	-2	4	-1440	2880
17-18	741	17.5	-1	1	-741	741
19-20	665	(19.5)	0	0	0	0
21-22	395	21.5	1	1	395	395
23-24	38	23.5	2	4	76	152
25-26	8	25.5	3	9	24	72
27-28	5	27.5	4	16	20	80
29-30	7	29.5	5	25	350	175
	3010				-2929	8409

$$\sigma_x = h \sqrt{\frac{\Sigma fd^2}{N} - \left(\frac{\Sigma fd}{N}\right)^2} = 2 \sqrt{\frac{8409}{3010} - \left(\frac{-2929}{3010}\right)^2} = 2 \sqrt{2.79 - 0.94}$$

$$= 2 \sqrt{1.84}$$

$$= 2 \times 1.356$$

$$= 2.71$$

Q.

Class	f	m	$d=\frac{x-A}{h}$	d^2	fd	fd^2
1-3	9	2	-2	4	-18	36
4-6	6	5	-1	1	-6	6
7-9	2	(8)	0	0	0	0
10-12	2	11	1	1	2	2
13-15	1	14	2	4	4	4
	20				-18	$\Sigma fd^2=48$

$$\rightarrow \sigma_x = h\cdot\sqrt{\frac{\Sigma fd^2}{N}-\left(\frac{\Sigma fd}{N}\right)^2}$$

$$= 3\sqrt{\frac{48}{20}-\left(\frac{-18}{20}\right)^2}$$

$$= 3\sqrt{2.4-0.81}$$

$$= 3\sqrt{1.59}$$

$$= 3.126$$

$$= 3.78$$

H.W

14.(a)

Class	f	x	$d=\frac{x-A}{h}$	d^2	fd^2	fd
0-10	5	5	-3	9	45	-15
10-20	7	15	-2	4	28	-14
20-30	14	25	-1	1	14	-14
30-40	12	(35)	0	0	0	0
40-50	9	45	1	1	9	9
50-60	6	55	2	4	24	12
60-70	2	65	3	9	18	6
	55				138	$\Sigma fd=-16$

$$\rightarrow \sigma_x = h\cdot\sqrt{\frac{\Sigma fd^2}{N}-\left(\frac{\Sigma fd}{N}\right)^2}$$

$$= 10\sqrt{\frac{138}{55}-\left(\frac{-16}{55}\right)^2}$$

$$= 10\sqrt{2.50-0.084}$$

$$= 10\sqrt{2.416}$$

$$= 10\times 1.55$$

$$= 15.54$$

(b)

C.I	f	x	$d=\frac{x-A}{h}$	d^2	fd	fd^2
100-105	200	102.5	-5	25	-1000	5000
105-110	210	107.5	-4	16	-840	3360
110-115	230	112.5	-3	9	-690	2070
115-120	320	117.5	-2	4	-640	1280
120-125	350	122.5	-1	1	-350	350
125-130	520	127.5	0	0	0	0
130-135	410	132.5	1	1	410	410
135-140	320	137.5	2	4	640	1280
140-145	280	142.5	3	9	840	2520
145-150	210	147.5	4	16	840	3360
150-155	160	152.5	5	25	800	4000
155-160	90	157.5	6	36	540	3240
	3360				550	26870

$$\sigma_x = h \cdot \sqrt{\frac{\sum fd^2}{N} - \left(\frac{\sum fd}{N}\right)^2}$$

$$= 5\sqrt{\frac{2680}{3360} - \left(\frac{550}{3360}\right)^2}$$

$$= 5\sqrt{7.99 - 0.026}$$

$$= 5\sqrt{7.96}$$

$$= 5 \times 2.82$$

$$= 14.10$$

Q. 15(a)

C.I	f	x	$d=\frac{x-A}{h}$	d^2	fd	fd^2
20-29	5	24.5	-3.33	11.08	-16.65	55.4
30-39	12	34.5	-2.22	4.92	-26.64	59.04
40-49	15	44.5	-1.11	1.23	-16.65	18.45
50-59	20	(54.5)	0	0	0	0
60-69	18	64.5	1.11	1.23	19.98	22.14
70-79	10	74.5	2.22	4.92	22.2	49.2
80-89	6	84.5	3.33	11.08	19.98	66.48
90-99	4	94.5	4.44	19.71	17.76	78.84
	90				19.98	349.55

$$\sigma_x = 9 \times \sqrt{\frac{349.55}{90} - \left(\frac{19.98}{90}\right)^2} = 9\sqrt{3.88 - 0.049} = 9 \times 1.95 = 17.61$$

$$\text{Mean } \bar{x} = A + \frac{\sum fd}{N} \cdot h = 54.5 + \frac{19.98}{90} \times 9 = 54.5 + 1.998 = 56.49$$

(b)

Age under (Years)	No. of persons dying
10	15
20	30
30	53
40	75
50	100
60	110
70	115
80	125

The data says 15 persons dying under age of 10, 30 persons dying under age of 20 and so on. So the no. of persons dying column is the C.F. Hence in order to find AM and SD, we have to find frequency from CF first.

C.I	f	x	$d = \frac{x-A}{h}$	d^2	fd	fd^2
0-10	15	5	-3	9	-45	135
10-20	15	15	-2	4	-30	60
20-30	23	25	-1	1	-23	23
30-40	22	(35)	0	0	0	0
40-50	25	45	1	1	25	25
50-60	10	55	2	4	20	40
60-70	5	65	3	9	15	45
70-80	10	75	4	16	40	160
	125				2	488

Now Mean, $\bar{x} = A + \frac{\Sigma fd}{N} \cdot h$

$= 35 + \frac{2}{125} \times 10$

$= 35 + 0.16$

$= 35.16$

and $\sigma_x = h \sqrt{\frac{\Sigma fd^2}{N} - \left(\frac{\Sigma fd}{N}\right)^2}$

$= 10 \sqrt{\frac{488}{125} - \left(\frac{2}{125}\right)^2}$

$= 10 \sqrt{3.904 - 0.00256}$

$= 10 \sqrt{3.90}$

$= 10 \times 1.974$

$= 19.74$

Q. Find the standard deviation of n natural numbers.

→ S.D of 1nd $= \sqrt{\frac{\Sigma (x-\bar{x})^2}{n}}$

$= \sqrt{\frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2}$

$\sigma_x = \sqrt{\frac{\Sigma x^2}{n} - (\bar{x})^2}$

$\Sigma x^2 = 1^2 + 2^2 + 3^2 + \cdots + n^2$

$= \frac{n(n+1)(2n+1)}{6}$

$\Rightarrow$ ~~$n(n+1)$~~

Now $\sigma_x = \sqrt{\frac{\Sigma x^2}{n} - (\bar{x})^2}$

$= \sqrt{\frac{\frac{n(n+1)(2n+1)}{6}}{n} - \left(\frac{n+1}{2}\right)^2}$

$= \sqrt{\frac{(n+1)(2n+1)}{6} - \left(\frac{n+1}{4}\right)^2}$

$= \sqrt{\left(\frac{n+1}{2}\right)\left(\frac{2n+1}{3} - \frac{n+1}{2}\right)}$

$= \sqrt{\frac{n+1}{2} \times \frac{4n+2-3n-3}{6}}$

$= \sqrt{\frac{n+1}{2} \times \frac{n-1}{6}}$

$= \sqrt{\frac{n^2-1}{12}}$

$\sigma_x^2 = \frac{\Sigma x^2}{n} - (\bar{x})^2$

$\frac{\Sigma x^2}{n} = \sigma_x^2 + (\bar{x})^2$

$\Rightarrow \Sigma x^2 = n\left(\sigma_x^2 + (\bar{x})^2\right)$

Q. The AM and the standard deviation of 9 items are 43 and 5 respectively. If an item 63 is added to the set then what is the new mean and new S.D.

→ Here $n = 9$, $\bar{x} = 43$ & $\sigma_x = 5$

So $\Sigma x = n \cdot \bar{x}$

$\Rightarrow \Sigma x = 9 \times 43$

$\Rightarrow \Sigma x = 387$

For addition of new item 63,

$\Sigma x = 387 + 63$

$= 450$

Thus $\bar{x} = \frac{\Sigma x}{10}$ ($\because$ 9 items + 63, $n = 10$)

$= \frac{450}{10} = 45$

Now in order to find the ~~SD~~ new S.D, first we need Σx^2

Thus $\Sigma x^2 = n\left(\sigma_x^2 + (\bar{x})^2\right)$

$= 9\left(5^2 + 43^2\right) = 9(25 + 1849)$

$= 16866$

For addition of new item 63,

$\Sigma x^2 = 16866 + 63^2 = 20835$

Hence $\sigma_x = \sqrt{\frac{20835}{10} - (45)^2}$

$= \sqrt{2083.5 - 2025}$

$= \sqrt{58.5} = 7.64$

* Relation between Variance and standard deviation

Variance defined as σ_x^2

Q. The variance of a series 2, 3, 11, x is 12.25. Find the value of x.

$\sigma_x^2 = 12.25$, $n = 4$

The Mean, $\bar{x} = \frac{2+3+11+x}{4} = \frac{16+x}{4}$

x	x^2
2	4
3	9
11	121
x	x^2

So $\Sigma x^2 = 134 + x^2$

$\sigma_x^2 = \frac{\Sigma x^2}{n} - (\bar{x})^2$

$\Rightarrow 12.25 = \frac{134+x^2}{4} - \left(\frac{16+x}{4}\right)^2$

$\Rightarrow 12.25 = \frac{134+x^2}{4} - \frac{256+x^2+32x}{16}$

$\Rightarrow 16(12.25) = 536 + 4x^2 - 256 - x^2 - 32x$

$\Rightarrow 196 = 536\ 4x^2 - x^2 - 32x + 280$

$\Rightarrow 196 - 280 = 4x^2\ 3x^2 - 32x$

$\Rightarrow 3x^2 - 32x = -84$

$\Rightarrow 3x^2 - 32x + 84 = 0$

Now $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$ $(a=3, b=-32, c=84)$

$= \frac{32 \pm \sqrt{32^2 - 4\times3\times84}}{6}$

$= \frac{32 \pm \sqrt{1024-1008}}{6}$

$= \frac{32 \pm \sqrt{16}}{6}$

$= \frac{32 \pm 4}{6}$

$= \frac{32+4}{6}$ or $\frac{32-4}{6}$

$= \frac{36}{6}$ or $\frac{28}{6}$

$= 6$ or 4.6

HW

Q. The mean of 5 observation is 4.4 and variance is 8.24. If 3 observation 1, 2 & 6. Find the other two.

→

C.I	x^2
1	1
2	4
6	36
x	x^2
y	y^2
x+y+9	x^2+y^2+41

Here given AM of 5 observation, $\bar{X} = 4.4$

and $\sigma x^2 = 8.24$

Thus $\sigma x^2 = \frac{\Sigma X^2}{n} - (\bar{X})^2$

$\Rightarrow 8.24 = \frac{x^2+y^2+41}{5} - (4.4)^2$

$\Rightarrow 8.24 = \frac{x^2+y^2+41}{5} - 19.36$

$\Rightarrow 5(8.24 - 19.36) = x^2+y^2+41$

$\Rightarrow 138 = x^2+y^2+41$

$\Rightarrow x^2+y^2 = 97$ ——— (1)

As $\bar{X} = 4.4$

$\Rightarrow \bar{X} = \frac{\Sigma x}{n}$

$\Rightarrow 4.4 = \frac{x+y+9}{5}$

$\Rightarrow 22 = x+y+9$

$\Rightarrow x+y = 13$ ——— (2)

$\Rightarrow y = (13-x)$

Now putting the value of y on eq^n(1), we get -

$x^2+(13-x)^2 = 97$

$\Rightarrow x^2+169+x^2-26x-97 = 0$

$\Rightarrow 2x^2-26x+72 = 0$

$\Rightarrow x^2-13x+36 = 0$

So $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$ $(a=1, b=-13, c=36)$

$= \frac{13 \pm \sqrt{169-4\times1\times36}}{2}$

$= \frac{13 \pm \sqrt{169-144}}{2}$

$= \frac{13 \pm \sqrt{25}}{2}$

$= \frac{13 \pm 5}{2} = \frac{13+5}{2}$ or $\frac{13-5}{2} = (9 \text{ or } 4)$

Putting x value on eq^n(1), we get-

if $x=9$, then $9+y=13$

$\Rightarrow y = 4$

If $x=4$, then $4+y=13$

$\Rightarrow y = 9$

So $(x,y) = (4,9)$ or $(9,4)$

- **Combined Standard Deviation**

Let $\sigma_1, \sigma_2 \cdots \sigma_k$ are standard deviation of K no. of series, and $\bar{x}_1, \bar{x}_2 \cdots \bar{x}_k$ are no. of means of K series. Then the combined standard deviation is defined as

$$\sigma^2 = \frac{n_1(\sigma_1^2 + d_1^2) + n_2(\sigma_2^2 + d_2^2) + \cdots + n_k(\sigma_k^2 + d_k^2)}{n_1 + n_2 + \cdots + n_k}$$

where $d_k = \bar{x}_k - \bar{x}$

and $\bar{x}$ is combined mean.

$x_1 - \bar{x}$

$\sigma_1, n_2, \bar{x}_1, \bar{x}_2$

$d_1 = x_1 - \bar{x}$ or $\bar{x} = \frac{n_1 x_1 + n_2 x_2}{n_1 + n_2}$

The means of two sample size 50 & 100 are 54.1 and 50.3, ~~the mean~~ and standard deviations are 8 & 7. Find

→ Here $n_1 = 50, n_2 = 100$

$\bar{x}_1 = 54.1, \bar{x}_2 = 50.3$

$\sigma_1^2 = 8, \sigma_2^2 = 7$

$d_1 = \bar{x}_1 - \bar{x} =$

$= 54.1 - 51.56$

$= 2.53$

$d_2 = \bar{x}_2 - \bar{x}$

$= 50.3 - 51.56$

$= -1.26$

$\frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$

$\bar{x} = \frac{50 \times 54.1 + 100 \times 50.3}{150}$

$= \frac{2705 + 5030}{150} = \frac{7735}{150} = 51.56$

Now Combined standard deviation

$$\sigma^2 = \frac{n_1(\sigma_1^2 + d_1^2) + n_2(\sigma_2^2 + d_2^2)}{n_1 + n_2}$$

$$= \frac{50(64 + 6.40) + 100(49 + 1.58)}{150}$$

$$= \frac{50 \times 70.4 + 100 \times 50.58}{150}$$

$$\sigma^2 = \frac{3520 + 5058}{150} = \frac{8578}{150} = 57.18$$

$$\sigma = \sqrt{57.18} = 7.56$$

Q. The mean weight of 150 student 60 kg, where mean weight boys 70 kg and mean weight of girls 55 kg. and the S.D are 10 & 15 kg respectively. Find the no. of boys and the combined S.D. boys = 50, girls = 100

→ $\bar{X} = 60$kg, $\bar{x}_1 = 70$, $\bar{x}_2 = 55$

$\sigma_1 = 10$, $\sigma_2 = 15$

$n_1 = 50$, $n_2 = 100$

$d_1 = \bar{x}_1 - \bar{X} = 10$

$d_2 = \bar{x}_2 - \bar{X} = -5$

$\sigma^2 = n_1$

$$\sigma^2 = \frac{n_1(\sigma_1^2 + d_1^2) + n_2(\sigma_2^2 + d_2^2)}{n_1 + n_2}$$

$$= \frac{50(100+100) + 100(225+25)}{150}$$

$$= \frac{10000 + 25000}{150}$$

$$= \frac{35000}{150}$$

$$= 233.33$$

$$\sigma = \sqrt{233.33}$$

$$= 12.90 = 15.27$$

$$\bar{X} = \frac{n_1 + \bar{x}_1 + n_2 + \bar{x}_2}{n_1 + n_2}$$

$$60 = \frac{n_1 70 + 55 n_2}{150}$$

$$\Rightarrow 9000 = 70n_1 + 55n_2$$

$$\Rightarrow 70n_1 + 55n_2 = 9000 \quad (1)$$

$$70 n_1 + 70 n_2 = 10500$$

70 × eq (2),

$$70n_1 + 55n = 9000$$

$$- 70n_1 + 70n_2 = 10500$$

$$-15n_2 = -1500$$

$$\Rightarrow 15n_2 = 1500$$

$$\Rightarrow n_2 = 100$$

$$n_1 + n_2 = 150$$

$$\Rightarrow n_1 + 100 = 150$$

$$\Rightarrow n_1 = 50$$

$(n_1, n_2) = (50, 100)$

Q. Find the missing information from the following

	I	II	III	combined
Number	50	?	90	200
Standard deviation	6	7	?	7.746
Mean	113	?	115	116

→ Given combined of 3 no = 200

$\Rightarrow 50 + x + 90 = 200$

$\Rightarrow x = 200 - 140$

$\Rightarrow x = 60$

Now, combined mean, $\bar{X} = 116$,

$\bar{X}_1 = 113$

$\bar{X}_3 = 115$

we need to find $\bar{X}_2$.

As we know $\bar{X} = \dfrac{n_1\bar{X}_1 + n_2\bar{X}_2 + n_3\bar{X}_3}{n_1 + n_2 + n_3}$

$\Rightarrow 116 = \dfrac{50 \times 113 + 60\bar{X}_2 + 90 \times 115}{200}$

$\Rightarrow 23200 = 5650 + 60\bar{X}_2 + 10350$

$\Rightarrow 23200 = 60\bar{X}_2 + 16000$

$\Rightarrow 60\bar{X}_2 = 23200 - 16000 = 7200$

$\Rightarrow \bar{X}_2 = \dfrac{7200}{60} = 120$

Again $\sigma_1 = 6$, $\sigma_2 = 7$, & $\sigma = 7.746$, we need to find σ_3 value.

Here $n_1 = 50$, $n_2 = 60$, $n_3 = 90$

$\bar{X}_1 = 113$, $\bar{X}_2 = 120$, $\bar{X}_3 = 115$, $\bar{X} = 116$

$d_1 = \bar{X}_1 - \bar{X} = -3$, $d_2 = \bar{X}_2 - \bar{X} = 4$, $d_3 = \bar{X}_3 - X = -1$

$\sigma^2 = \dfrac{n_1(\sigma_1^2 + d_1^2) + n_2(\sigma_2^2 + d_2^2) + n_3(\sigma_3^2 + d_3^2)}{200}$

$$\sigma^2 = \frac{50(36+9)+60(49+16)+90(\sigma_3^2+1)}{200}$$

$$= \frac{50\times45+60\times65+90\sigma_3^2+90}{200}$$

$$= \frac{90\sigma_3^2+6240}{200}$$

$$(7.746)^2 = \frac{90\sigma_3^2+6240}{200}$$

$$\Rightarrow 60 = \frac{90\sigma_3^2+6240}{200}$$

$$\Rightarrow 12000 = 90\sigma_3^2+6240$$

$$\Rightarrow 90\sigma_3^2 = 12000-6240$$

$$\Rightarrow 90\sigma_3^2 = 5760$$

$$\Rightarrow \sigma_3^2 = \frac{5760}{90} = 64$$

$$\Rightarrow \sigma_3 = \sqrt{64} = 8$$

* Assignments

12.(a) From the following information, find the standard deviation of x & y variables

→ ~~Given~~ For Variable x

→ Given $\Sigma x = 235$, $\Sigma x^2 = 6750$, $N = 10$

So $\sigma_x = \sqrt{\frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2} = \sqrt{\frac{6750}{10} - \left(\frac{235}{10}\right)^2} = \sqrt{675 - (23.5)^2} = \sqrt{675 - 552.25}$

$= \sqrt{122.75} = 11.07$

For Variable y

Given $\Sigma y = 250$, $\Sigma y^2 = 6840$, $N = 10$

So $\sigma_y = \sqrt{\frac{\Sigma y^2}{N} - \left(\frac{\Sigma y}{N}\right)^2} = \sqrt{\frac{6840}{10} - \left(\frac{250}{10}\right)^2} = \sqrt{684 - 625} = \sqrt{59} = 7.68$

(b)

13.(a)

→ Let $X = 1, 2, 3, \dots n$

$$\Sigma X = \frac{n+1}{2}$$

So $X^2 = 1^2, 2^2, 3^2, \dots n^2$

$$\Sigma X^2 = 1^2 + 2^2 + 3^2 + \dots + n^2$$

$$= \frac{n(n+1)(2n+1)}{6}$$

Now standard deviation of n natural numbers,

$$\sigma_x = \sqrt{\frac{\Sigma X^2}{n} - \left(\frac{\Sigma X}{n}\right)^2}$$

$$= \sqrt{\frac{n(n+1)(2n+1)}{6} \cdot \frac{1}{n} - \left(\frac{n+1}{2}\right)^2}$$

$$= \frac{(n+1)}{2} \cancel{n}$$

$$= \sqrt{\frac{(n+1)(2n+1)}{6} - \frac{(n+1)^2}{4}}$$

$$= \sqrt{\left(\frac{n+1}{2}\right)\left(\frac{2n+1}{3} - \frac{(n+1)}{2}\right)}$$

$$= \sqrt{\frac{n+1}{2} \times \frac{4n+2-3n-3}{6}}$$

$$= \sqrt{\frac{n+1}{2} \times \frac{n-1}{6}}$$

$$= \sqrt{\frac{n^2-1}{12}}$$

(b) S.D of 1, 2, 3, 4, 5

Here $n = 5$,

So $\sigma_x = \sqrt{\frac{5^2-1}{12}}$

$$= \sqrt{\frac{25-1}{12}}$$

$$= \sqrt{\frac{24}{12}}$$

$$= \sqrt{2}$$

(c) Mean of 10 items is 50 and S.D is 14.

So $n = 10$, $\bar{X} = 50$, $\sigma_x = 14$

Thus $\sigma_x = \sqrt{\frac{\Sigma X^2}{n} - \left(\frac{\Sigma X}{n}\right)^2}$

$$\Rightarrow 14 = \sqrt{\frac{\Sigma X^2}{10} - (50)^2}$$

On squaring both sides, we get -

$$\Rightarrow 196 = \frac{\Sigma X^2}{10} - 2500$$

$$\Rightarrow 196 + 2500 = \frac{\Sigma X^2}{10}$$

$$\Rightarrow 2696 = \frac{\Sigma X^2}{10}$$

$$\Rightarrow \Sigma X^2 = 2696 \times 10 = 26960$$

Q.14

C.I	f	m	fm	$(m-\bar{X})$	$(m-\bar{X})^2$	$f(m-\bar{X})^2$
0-10	5	5	25	-27.09	733.86	3669.3
10-20	7	15	105	-17.09	292.06	2044.42
20-30	14	25	350	-7.09	50.26	703.64
30-40	12	35	420	2.91	8.46	101.52
40-50	9	45	405	12.91	166.66	1499.94
50-60	6	55	330	22.91	524.86	3149.16
60-70	2	65	130	32.91	1083.06	2166.12
						13334.1

Here Mean, $\bar{X} = \frac{\Sigma fm}{N} = \frac{1765}{55} = 32.09$

Thus $\sigma_x = \sqrt{\frac{\Sigma f(m-\bar{X})^2}{N}}$

$$= \sqrt{\frac{13334.1}{55}}$$

$$= \sqrt{242.43}$$

$$= 15.57$$

(b)

C.I	f	x	$d=\frac{x-A}{h}$	d^2	fd	fd^2
100-105	200	102.5	-5	25	-1000	5000
105-110	210	107.5	-4	16	-840	3360
110-115	230	112.5	-3	9	-690	2070
115-120	320	117.5	-2	4	-640	1280
120-125	350	122.5	-1	1	-350	350
125-130	520	(127.5)	0	0	0	0
130-135	410	132.5	1	1	410	410
135-140	320	137.5	2	4	640	1280
140-145	280	142.5	3	9	840	2520
145-150	210	147.5	4	16	840	3360
150-155	160	152.5	5	25	800	4000
155-160	90	157.5	6	36	540	3240
	3360				550	26870

$$\rightarrow \sigma_x = h \cdot \sqrt{\frac{\Sigma fd^2}{N} - \left(\frac{\Sigma fd}{N}\right)^2}$$

$$= 5\sqrt{\frac{26870}{3360} - \left(\frac{550}{3360}\right)^2}$$

$$= 5\sqrt{7.997 - 0.026}$$

$$= 5\sqrt{7.971}$$

$$= 5 \times 2.83$$

$$= 14.15$$

12.(b)

Given $\Sigma x = 240$, $\Sigma x^2 = 6400$

$\Sigma y = 250$, $\Sigma y^2 = 7060$

Also ten items are included in both the survey

So $n = 10$

Now, $\sigma_x = \sqrt{\frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2}$

$$= \sqrt{\frac{6400}{10} - \left(\frac{240}{10}\right)^2}$$

$$= \sqrt{640 - (24)^2}$$

$$= \sqrt{640 - 576}$$

$$= \sqrt{64}$$

$$= 8$$

$\sigma_y = \sqrt{\frac{\Sigma y^2}{n} - \left(\frac{\Sigma y}{n}\right)^2}$

$$= \sqrt{\frac{7060}{10} - \left(\frac{250}{10}\right)^2}$$

$$= \sqrt{706 - (25)^2}$$

$$= \sqrt{706 - 625}$$

$$= \sqrt{81}$$

$$= 9$$

$(\because \sigma_x, \sigma_y = 8, 9)$

Measures of Dispersion

There are two types of measures dispersion.

(I) Absolute measure

(II) Relative measure

~~Range, Quartile deviation, mean deviation and standard deviation~~

(a) Range
(b) Quartile deviation
(c) Mean deviation
(d) standard deviation
(e) Variance

Relative Measure

(a) Co-efficient range
(b) Co-efficient of Quartile deviation
(c) Co-efficient of M.D
(d) Co-efficient of Variation

Difference between absolute measure & Relative measure

In absolute measure, the results are dependent on particular unit. But in case of co-efficient of variation, the results are independent of unit or unit-3

In case of absolute measure, we can only compare with same unit data. But in case of Co-efficient of variation, we can compare any distribution.

The Co-efficient Variation is defined as $CV = \frac{\sigma}{\bar{x}}$, where σ is S.D & $\bar{x}$

$$\boxed{CV = \frac{\sigma}{\bar{x}} \times 100}$$

Q. The A.M of runs scored by 3 batsman in 10 innings are 50, 48, 12 and their standard deviation 15, 12 & 2. Now find out the who is more consistent among the 3 batsman. Calculate CV of all the 3 batsman.

→ Here $\bar{x}_1 = 50$, $\sigma_1 = 15$

$\bar{x}_2 = 48$, $\sigma_2 = 12$

$\bar{x}_3 = 12$, $\sigma_3 = 2$

So $CV_1 = \frac{\sigma_1}{\bar{x}} \times 100$, $CV_2 = \frac{\sigma_2}{\bar{x}_2} \times 100$, $CV_3 = \frac{\sigma_3}{\bar{x}_3} = \frac{2}{12} \times 100$

$= \frac{15}{50} \times 100$ $= \frac{12}{48} \times 100$ $= 16.6$

$= 30$ $= 25$

Here the lowest CV is belongs to batsman 3. So he is more consistent than other two.

Co-efficient of variance of 2 series are 75% & 90% & their S.D are 15 & 18 respectively. Find the mean.

$CV_1 = 75$, $CV_2 = 90$

$\sigma_1 = 15$, $\sigma_2 = 18$ $\frac{75}{100} \times$

So $CV_1 = \frac{\sigma_1}{\bar{x}}$ $CV_1 = \frac{\sigma_1}{\bar{x}_1} \times 100$

$\Rightarrow 75 = \frac{15}{\bar{x}} = \bar{x} = \frac{15}{75} = 0.2$ $\Rightarrow \frac{75}{100} = \frac{15}{\bar{x}_1} \times 100$

$CV_2 = \frac{\sigma_2}{\bar{x}_2} \Rightarrow \bar{x}_2 = \frac{\sigma_2}{CV_2}$ $\Rightarrow 75\bar{x}_1 = 15000$

$= \frac{18}{}$ $\Rightarrow \bar{x}_1 = \frac{15000}{75} = 20$

Now $CV_2 = \frac{\sigma_2}{\bar{x}_2} \times 100$

$\Rightarrow 90 = \frac{18}{x_2} \times 100$

$\Rightarrow x_2 = \frac{18000}{90} = 20$

Q.

→ Here $\bar{X}_1 = 78$, S.D = 8.0

$$CV_1 = \frac{\sigma_1}{\bar{X}_1} \times 100 \qquad CV_2 = \frac{\sigma_2}{\bar{X}_2} \times 100$$

$$= \frac{8.0}{78} \times 100 \qquad = \frac{7.6}{80} \times 100$$

$$= 10.25 \qquad = 9.5$$

c.

Here CV_1 is 10.25 and CV_2 is 9.5, this is clear that CV_1 is greater than CV_2 so the students of 1995 are more consistent in their result in comparing to the students of 1990.

It can be observe that over the time from 1990 to 1995, the avg performance of students has increased and they are more consistent in their performance.

Q. 6

	Company A		Company B	
	Males	Females	Males	Females
Mean	30	20	27	32
Variance	8	3	12	5
No. of emp	40	10	20	30

→ For Company A,

the Combine Mean = $\frac{40 \times 30 + 10 \times 20}{50}$

$= \frac{1200 + 200}{50}$

$= \frac{1400}{50}$

$= 28$

Again Combine variance for Combine S.D

$d_1 = \bar{X}_1 - \bar{X} = 30 - 28 = 2$

$d_2 = \bar{X}_2 - \bar{X} = 20 - 28 = -8$

$$\sigma = \left[\frac{n_1(\sigma_1^2 + d_1^2) + n_2(\sigma_2^2 + d_2^2)}{50}\right]^{1/2}$$

$$= \left(\frac{40(8+4) + 10(3+64)}{50}\right)^{1/2}$$

$$= \left(\frac{480 + 670}{50}\right)^{1/2} = \left(\frac{1150}{50}\right)^{1/2}$$

$= 4.79$

For Company B

the Combine Mean = $\frac{20 \times 27 + 30 \times 32}{50}$

$= \frac{540 + 960}{50}$

$= 30$

$d_1 = \bar{X}_1 - \bar{X} = -3$

$d_2 = \bar{X}_2 - \bar{X} = 2$

$$CV_2 = \left(\frac{20(12+9) + 30(5+4)}{50}\right)^{1/2}$$

$$= \left(\frac{3060 + 870}{50}\right)^{1/2}$$

$= (78.6)^{1/2} = 8.86$

$$CV_A = \frac{4.79}{28} \times 100 = 17.10$$

$$CV_2 = \left(\frac{20(12+9) + 30(5+4)}{50}\right)^{1/2}$$

$$= \left(\frac{420 + 270}{50}\right)^{1/2}$$

$$= \left(\frac{690}{50}\right)^{1/2}$$

$$= (13.5)^{1/2}$$

$$= 3.71$$

$$CV_2 = \frac{\sigma_B}{30} \times 100$$

$$= \frac{3.71}{30} \times 100$$

$$= 12.36$$

Here CV_B is less than CV_A, so productivity in company B is more consistent, whereas highest productivity observe in Comp A.

Assignments

Q.2

Co-efficient of variation of the first n natural numbers.

$n = 1, 2, 3, \ldots n$

As we know the standard deviation of n natural numbers is

$$\sqrt{\frac{n^2-1}{12}}$$

Also the mean of n natural numbers is

$$\left(\frac{n+1}{2}\right)$$

Thus C.V of n natural numbers $= \frac{\sigma_n}{\bar{x}_n} = \frac{\sqrt{\frac{n^2-1}{12}}}{\frac{n+1}{2}}$

$$\Rightarrow C.V_n = \sqrt{\frac{n^2-1}{12}} \times \frac{2}{n+1}$$

On squaring both side, we get -

$$(C.V_n)^2 = \frac{n^2-1}{12} \times \frac{4}{(n+1)^2}$$

$$= \frac{4(n^2-1^2)}{12(n+1)^2}$$

$$= \frac{4(n+1)(n-1)}{3 \cancel{12}(n+1)^2}$$

$$= \frac{(n-1)}{3(n+1)}$$

$$\Rightarrow C.V_n = \sqrt{\frac{n-1}{3(n+1)}}$$

Q.4

→ Here $\bar{x}_1 = 50, \sigma_1 = 15$

$\bar{x}_2 = 48, \sigma_2 = 12$

$\bar{x}_3 = 12, \sigma_3 = 2$

So $CV_1 = \frac{\sigma_1}{\bar{x}_1} \times 100$

$= \frac{15}{50} \times 100$

$= 30$

$CV_2 = \frac{\sigma_2}{\bar{x}_2} \times 100$

$= \frac{12}{48} \times 100$

$= 25$

$CV_3 = \frac{2}{12} \times 100$

$= 16.67$

Q.4 If two samples A & B having same standard deviation. But the mean of A is greater than B, then obviously, the CV of A is will be greater than B. Because if in two cases, the numerators are same, and the denominator of A is greater than the denominator B. Then the ~~data having~~ CV of A is greater than CV of B.

Q.5 (a) Here, $CV = 60\%$, $\sigma = 12$, $\bar{X} = ?$

As we know $CV = \frac{\sigma}{\bar{X}} \times 100$

$\Rightarrow 60 = \frac{12}{\bar{X}} \times 100$

$\Rightarrow \frac{60}{100} = \frac{12}{\bar{X}}$

$\Rightarrow 6\bar{X} = 120$

$\Rightarrow \bar{X} = \frac{120}{6} = 20$

(b) Here $\sigma^2 = 16$, $n = 20$ & $\Sigma n = 160$, $CV = ?$

Thus $\bar{X} = \frac{\Sigma n}{n} = \frac{160}{20} = 8$

As $\sigma^2 = 16$,

$\sigma = \sqrt{16} = 4$

So $CV = \frac{\sigma}{\bar{X}} \times 100$

$= \frac{4}{8} \times 100$

$= 50\%$.

Q.6 (a) Here $CV_1 = 60\%$, $CV_2 = 80\%$.

$\sigma_1 = 20$, $\sigma_2 = 16$

$\bar{X}_1 = ?$ $\bar{X}_2 = ?$

For CV_1,

$CV_1 = \frac{\sigma_1}{\bar{X}_1} \times 100$

$60 = \frac{20}{\bar{X}_1} \times 100$

$\Rightarrow \bar{X}_1 = \frac{2000}{60}$

$= 33.33$

For CV_2

$CV_2 = \frac{\sigma_2}{\bar{X}_2} \times 100$

$\Rightarrow 80 = \frac{16}{\bar{X}_2} \times 100$

$\Rightarrow \bar{X}_2 = \frac{1600}{80}$

$= 20$

(b) Here $CV_1 = 60\%$. $CV_2 = 80\%$.

$\sigma_1 = 24$ $\sigma_2 = 20$

$\bar{X}_1 = ?$ $\bar{X}_2 = ?$

For CV_1,

$CV_1 = \frac{\sigma_1}{\bar{X}_1} \times 100$

$\Rightarrow 60 = \frac{24}{\bar{X}_1} \times 100$

$\Rightarrow \bar{X}_1 = \frac{2400}{60}$

$= 40$

For CV_2,

$CV_2 = \frac{\sigma_2}{\bar{X}_2} \times 100$

$\Rightarrow 80 = \frac{20}{\bar{X}_2} \times 100$

$\Rightarrow \bar{X}_2 = \frac{2000}{80}$

$= 25$

Q.7

After settlement, the mean increased from 800 to 1200 & the standard deviation had increased from 200 to 250.

So $\bar{x}_1 = 800$ & $\bar{x}_2 = 1200$

$\sigma_1 = 200$ & $\sigma_2 = 250$

Thus $CV_1 = \frac{200}{800} \times 100$ & $CV_2 = \frac{250}{1200} \times 100$

$= 25$ $= 20.83$

Now we can say the wages have become ~~higher~~ more uniform after settlement.

Q.8

$\bar{x}_1 = 800$ (weekly avg. wages of workers)

$\bar{x}_2 = 1200$ (increase avg. wages ")

& their $\sigma_1 = 100$ to $\sigma_2 = 500$

Thus $CV_1 = \frac{100}{800} \times 100$ & $CV_2 = \frac{500}{1200} \times 100$

$= 12.5$ $= 41.66$

Hence the wages after increment become less uniform.

Q.9 Students ~~having~~ secured average marks 50 with a standard deviation of 2 in the first year.

So $\bar{x}_1 = 50$, $\sigma_1 = 2$

that implies $CV_1 = \frac{2}{50} \times 100 = 4\%$

After five years same students secured avg. marks 60 with an increased S.D of 3.

So $\bar{x}_2 = 60$, $\sigma_2 = 3$

$\Rightarrow CV_2 = \frac{3}{60} \times 100$

$= 5\%$

Hence we can say, the students showed improved performance.

Q.10

Here $C.V_m = 55\%$, $C.V_f = 70\%$

$\sigma_m = 22$, $\sigma_f = 15.40$

Calculate the combined average wages of all the workers, if 80% of the workers were males.

that means $n_m = 80$ and obviously $n_f = 20$

Now, $C.V_m = \frac{\sigma_m}{\bar{x}_m} \times 100$, and

$\Rightarrow 55 = \frac{22}{\bar{x}_m} \times 100$

$\Rightarrow \bar{x}_m = \frac{2200}{55} = 40$

again $C.V_f = \frac{\sigma_f}{\bar{x}_f} \times 100$

$\Rightarrow 70 = \frac{15.40}{\bar{x}_f} \times 100$

$\Rightarrow \bar{x}_f = \frac{1540}{70} = 22$

So the combined mean, $\bar{x} = \frac{n_m \bar{x}_m + n_f \bar{x}_f}{n_m + n_f}$

$= \frac{80 \times 40 + 20 \times 22}{100}$

$= \frac{3200 + 440}{100}$

$= \frac{3640}{100}$

$\bar{x} = 36.40$

Q.11

Q.11

	Brand I	Brand II
Means	800 hours	770 hours
S.D	100 hours	60 hours

→ Here $\bar{x}_1 = 800$, $\sigma_1 = 100$

$\bar{x}_2 = 770$, $\sigma_2 = 60$

So $CV_1 = \frac{\sigma_1}{\bar{x}_1} \times 100$

$= \frac{100}{800} \times 100$

$= 12.5$

$CV_2 = \frac{60}{770} \times 100$

$= \frac{600}{77}$

$= 7.79$

Brand II is more uniform.

Q.12 (b)

In factory A, Variance, $\sigma^2 = 16 \Rightarrow \sigma = 4$

Mean $\bar{x} = 120$

So $CV_A = \frac{\sigma}{\bar{x}} \times 100$

$= \frac{4}{120} \times 100$

$= 3.3$

In factory B, Variance, $\sigma^2 = 25 \Rightarrow \sigma = 5$

Mean, $\bar{x} = 200$

So $CV_B = \frac{\sigma}{\bar{x}} \times 100$

$= \frac{5}{200} \times 100$

$= 2.5$

Q.13

In factory A, $\sigma^2 = 9$

So $\sigma = 3$

and Mean, $\bar{x} = 120$

So $C.V_A = \frac{\sigma}{\bar{x}} \times 100$

$= \frac{3}{120} \times 100$

$= \frac{30}{12} = 2.5$

In factory B, $\sigma^2 = 16$

$\sigma = 4$

and $\bar{x} = 85$

So $CV_B = \frac{\sigma}{\bar{x}} \times 100$

$= \frac{4}{85} \times 100$

$= 4.70$

Q.14

In 1960, thousand students of B.A. Hons. Economics gave avg. marks secured as 50%. So $\bar{x}_1 = 50$ with a S.D of 3, $\sigma_1 = 3$.

Thus $CV_1 = \frac{\sigma_1}{\bar{x}_1} \times 100$

$= \frac{3}{50} \times 100$

$= 6$

In 1970, $\bar{x}_2 = 55\%$. $\sigma_2 = 5$

So $CV_2 = \frac{\sigma_2}{\bar{x}_2} \times 100$

$= \frac{5}{55} \times 100$

$= 9.09$

Thus the results have improved from 1960 to 1970.

Range

Range is the difference between largest observation and the smallest observation in the data set.

$$R = L - S$$

Absolute Range = Largest value - Smallest value.

Q. Find the range of the following distribution

16 - 20
20 - 24
24 - 28
28 - 32

→ Here Range = 32 - 16 = 16

In case of range of a continious series, if the series is exclusive, then

R = Upper limit of last class - Lower limit of first class.

but if it is a inclusive series, then convert it to exclusive and find range.

x	f
16 - 20	3
21 - 25	9
26 - 30	7

→

x
15.5 - 20.5
20.5 - 25.5
25.5 - 30.5

30.5 - 15.5 = 15

Co-efficient of Range

It is defined as $\frac{L-S}{L+S}$

Q. Find the range & co-efficient of range from the following observation

65
70
82
59
81
59
81
76
57
60
55
50

Range = 82 - 50 = 32

Co-efficient of range = $\frac{L-S}{L+S}$

$= \frac{32}{132} = 0.24$

Q. From the monthy income of 10 families, given below. Calculate No.1 median, GM, Co-Range

Family 1	F.1	log X
145	73	1.86
367	115	2.06
268	145	2.16
73	185	2.26
185	268	2.42
619	280	2.44
280	315	2.49
115	367	2.56
870	619	2.79
315	870	2.93
		23.97

$$\frac{\left(\frac{n}{2}\right)+\left(\frac{n}{2}+1\right)}{2} = \frac{5+6}{2} \quad \frac{11}{2} = 5.5$$

$$\frac{268+280}{2} =$$

$$A\left(\frac{\Sigma \log y}{n}\right)$$

$$\text{Med} = AL\left(\frac{\Sigma \log y}{n}\right)$$

$$= AL\left(\frac{23.97}{10}\right)$$

$$= AL(2.397)$$

$$= 249.45$$

$$\text{Co-range} = \frac{870-73}{870+73} = \frac{797}{943} = 0.84$$

Quartile

Quartile divides the whole data set into 4 equal parts.

Q Find the quartile of following series

94	69	
95	73	
96	79	
93	87	
87	93	
79	94	
73	95	
69	96	
	N=8	

$$\frac{n}{2} + \frac{n}{2}+1 \rightarrow \frac{4^{th}+5^{th}}{2} = \frac{87+93}{2} = \frac{180}{2} = 90$$

$Q_1 = 38$

$Q_2 = 90$

$Q_3 = 47.5$

$Q_2 = 90$

$$Q_1 = \frac{N}{4} = \frac{180}{4} = 45$$

$$Q_3 = \frac{3N}{4} = \frac{135}{4} =$$

$$Q_1 = \frac{\frac{N}{4}\text{th} + \left(\frac{N}{4}+1\right)\text{th}}{4}$$

$$= \frac{2^{th}+3^{rd}}{4}$$

$$= \frac{73+79}{4}$$

$$= 38$$

$$\frac{3N}{4} \quad \frac{57+46}{4} =$$

$$Q_3 = \frac{3N}{4} = \frac{3\frac{N}{4}\text{th} + 3\left(\frac{N}{4}+1\right)\text{th}}{4}$$

$$= \frac{6^{th}+7^{th}}{4}$$

$$= \frac{94+95}{4} = \frac{189}{4} = 47.5$$

Continuous Series

x	f	cf	
10-19	1	1	
20-29	6	7	
30-39	4	11	
40-49	3	14	Q_1
50-59	14	28	Q_2
60-69	19	47	Q_3

N=99
47

→ Median class $= \frac{N}{2} = \frac{47}{2} = 23.5$

$Q_1 \quad \frac{N}{4} = \frac{47}{4} = 11.75$

$Q_3 = \frac{3N}{4} = 3 \cdot \frac{47}{4} = 35.25$

So $Q_1 = L_1 + \frac{L_2 - L_1}{f_1}\left(\frac{N}{4} - c\right)$

$= 40 + \frac{49-40}{3}(11.75 - 11)$

$= 40 + 3(0.75)$

$= 40 + 2.25$

$= 42.25$

& $Q_3 = L_1 + \frac{L_2 - L_1}{f_1}\left(\frac{3N}{4} - c\right)$

$= 60 + \frac{69-60}{19}(35.25 - 28)$

$= 60 + \frac{9}{19} \times 7.25$

$= 60 + 0.47 \times 7.25$

$= 60 + 3.43$

$= 63.43$

Assignments

Find the quartile of ?

Quartile deviation: $\frac{Q_3 - Q_1}{2}$

& co-Q.D $= \frac{Q_3 - Q_1}{Q_3 + Q_1}$

Q.

H.I	f	cf	
50-53	2	2	
53-56	7	9	
56-59	24	33	Q_1
59-62	27	60	Q_2
62-65	13	73	
65-68	3	76	
	76		

Here Q_1 class $= \frac{N}{4} = \frac{76}{4} = 19$

So $Q_1 = L_1 + \frac{L_2 - L_1}{f_1}\left(\frac{N}{4} - c\right)$

$= 56 + \frac{59-56}{24}(19-9)$

$= 56 + \frac{3}{24} \times 10$

$= 56 + 0.125 \times 10$

$= 56 + 1.25$

$= 57.25$

Now Q_2 class $= \frac{N}{2} = \frac{76}{2} = 38$

So $Q_2 = L_1 + \frac{L_2 - L_1}{f_1}\left(\frac{N}{2} - c\right)$

$= 59 + \frac{62-59}{27}(38-33)$

$= 59 + \frac{3}{27} \times 5$

$= 59 + 0.55$

$= 59.55$

Now Q_3 class $= \frac{3N}{4} = \frac{3 \cdot 76}{4} = 57$

So $Q_3 = L_1 + \frac{L_2 - L_1}{f_1}\left(\frac{3N}{4} - c\right)$

$= 59 + \frac{62-59}{27}(57-33)$

$= 59 + \frac{3}{27} \times 24$

$= 59 + 2.66$

$= 61.66$

Quartile deviation $= \frac{Q_3 - Q_1}{2} = \frac{61.66 - 57.25}{2} = \frac{4.41}{2} = 2.205$

Q.2

C.I	f	CF	
20-29	306	306	Q_1
30-39	182	488	Q_3
40-49	144	632	
50-59	96	728	
60-69	42	770	
70-79	34	804	
	804		

Here Q_1 class $= \frac{N}{4} = \frac{804}{4} =$ ~~402~~ 201

So $Q_1 = L_1 + \frac{L_2 - L_1}{f_1}\left(\frac{N}{4} - c\right)$

$= 39 + \frac{39-30}{182}(402-306)$

$= 39 + \frac{9}{182}(96)(-105)$

$= 39 + 0.049 \times 96$

$= 39 + 4.74$

$= 43.74$

Q_2 class $= \frac{N}{2} = \frac{804}{2}$

Here Q_1 class $= \frac{N}{4} = \frac{804}{4} = 201$

So $Q_1 = L_1 + \frac{L_2 - L_1}{f_1}\left(\frac{N}{4} - c\right)$

$= 20 + \frac{29-20}{306}$

→

Midpoint	f	CF
24.5	306	306
34.5	182	488
44.5	144	632
54.5	96	728
64.5	42	770
74.5	34	804

~~$Q_1 = \frac{N}{4} = \frac{804}{4} = 201$~~

~~$Q_3 = \frac{3N}{4} =$~~

~~$Q_1 = \frac{\text{N}^{\text{th}} + \left(\frac{N}{2}+1\right)^{\text{th}}}{4}$~~

$=$

$Q_1 = \frac{N+1}{4} = \frac{804+1}{4} = \frac{805}{4} = 201.25$

C.I	f	cF	
20-29	306	306	Q_1
30-39	182	488	
40-49	144	632	Q_3
50-59	96	728	
60-69	42	770	
70-79	34	804	

Q_1 Class $= \frac{N}{4} = \frac{804}{4} = 201$

So $Q_1 = L_1 + \frac{L_2 - L_1}{f_1}\left(\frac{N}{4} - c\right)$

$= 20 + \frac{29-20}{306}(201 - 0)$

$= 20 + \frac{9}{306} \times 201$

$= 20 + 5.91$

$= 25.91$

$Q_3 = \frac{3N}{4} = \frac{3 \times 804}{4} = 603$

So $Q_3 = L_1 + \frac{L_2 - L_1}{f_1}\left(\frac{3N}{4} - c\right)$

$= 40 + \frac{49-40}{144}(603 - 488)$

$= 40 + \frac{9}{144} \times 115$

$= 40 + 7.18$

$= 47.18$

So Quartile deviation $= \frac{Q_3 - Q_1}{2} = \frac{47.18 - 25.91}{2}$

$= 10.63$

Mean Deviation

Mean deviation of a series is the arithmetic average of the deviation of various items from a measure of central tendency.

→ It is based on all the items of the series.

→ While calculating deviations from the selected average, the signs (+ or −) of deviations are ignored and all the deviations are taken as positive.

Individual Series

~~Mean Deviation from~~

- Calculate the specific average (Mean or Median) from which mean deviation is to be found out.
- Obtain absolute (positive) deviations of each observation from the specific average.
- Absolute deviations are totalled up to find out $\Sigma|D|$
- Apply the formula: Mean Deviation from Mean $(MD_x) = \frac{\Sigma|x-\bar{x}|}{n} = \frac{\Sigma|D|}{n}$
- Co-efficient of Mean Deviation: $\frac{\text{Mean Deviation}}{\text{Mean}}$ (for mean), $\frac{\text{Mean Deviation}}{\text{Mode}}$ (for mode)

Ex Find mean deviation for the data, 1, 2, 3, 4, 5, 6, 7.

→

x	$\lvert x-\bar{x}\rvert$
1	3
2	2
3	1
4	0
5	1
6	2
7	3
28	$\Sigma = 12$

Here Mean $= \frac{\Sigma x}{n} = \frac{28}{7} = 4$

So Mean Deviation $(MD_x) = \frac{\Sigma|x-\bar{x}|}{n} = \frac{12}{7} = 1.7$

& Co-efficient of $MD_x = \frac{MD_x}{\bar{x}} = \frac{1.7}{4} = 0.42$

Again for median,

Median of the data $= \left(\frac{n+1}{2}\right)$ th term

$= \left(\frac{7+1}{2}\right)^{th}$ term

$= 4^{th}$ term

$= 4$

As Median $= 4$, so $|x-\bar{x}|$ column will be same as for mean.

Thus Mean deviation from median $(MD_x) = \frac{\Sigma|x-\bar{x}|}{n} = \frac{12}{7} = 1.7$

and Co-efficient of $MD_x = \frac{MD_x}{\bar{x}} = \frac{1.7}{4} = 0.42$

For Discrete Series

M.D about mean = $\frac{\Sigma f|x-\bar{x}|}{N}$, here $\bar{x}$ is mean

M.D about median = $\frac{\Sigma f|x-\bar{x}|}{N}$, here $\bar{x}$ is median

M.D about mode = $\frac{\Sigma f|x-\bar{x}|}{N}$, here $\bar{x}$ is mode.

ex:- Calculate the mean deviation about arithmetic mean and co-efficient of mean deviation.

x	f	fx	$\lvert x-\bar{x}\rvert$	$f\lvert x-\bar{x}\rvert$
10	3	30	1.86	5.58
11	12	132	0.86	10.32
12	18	216	0.14	2.52
13	12	156	1.14	13.68
	$\Sigma=45$	$\Sigma fx=534$		$\Sigma=32.1$

So Mean = $\frac{\Sigma fx}{N} = \frac{534}{45} = 11.86$

Mean deviation $MD_x = \frac{\Sigma f|x-\bar{x}|}{N} = \frac{32.1}{45} = 0.71$

and co-efficient of $MD_x = \frac{MD}{\bar{x}} = \frac{0.71}{11.86} = 0.06$

For Median

x	f	CF
10	3	3
11	12	15
12	18	33
13	12	45
	N=45	

So Median = $\left(\frac{N+1}{2}\right)^{th}$ term

$= \left(\frac{45+1}{2}\right)^{th}$ term

$= 23$

So

x	f	CF	$\lvert x-\bar{x}\rvert$	$f\lvert x-\bar{x}\rvert$
10	3	3	13	39
11	12	15	12	144
12	18	33	11	198
13	12	45	10	120
	$\Sigma=45$			$\Sigma=501$

So Mean deviation $(MD_x) = \frac{\Sigma f|x-\bar{x}|}{N}$

$= \frac{501}{45} = 11.13$

Co-efficient of $MD_x = \frac{MD}{\bar{x}}$

$= \frac{11.13}{23} = 0.48$

For continuous Series

In case of continuous series, the formula of mean deviation are same as discrete series.

The mid point of class intervals have to found out and they are taken as 'M'

Mean Deviation from Mean $(MD_{\bar{x}}) = \frac{\Sigma f|M-\bar{x}|}{N} = \frac{\Sigma f|D|}{N}$

Mean Deviation from Median $(MD_{Me}) = \frac{\Sigma f|M-Me|}{N} = \frac{\Sigma f|D|}{N}$

where M = mid point, $\bar{x}$ = Arithmetic mean & Me = Median

* Calculate the mean deviation from mean for the following data.

class	f	M	fM	$\lvert M-\bar{x}\rvert$	$f\lvert M-\bar{x}\rvert$
2-4	3	3	9	2.2	6.6
4-6	4	5	20	0.2	0.8
6-8	2	7	14	1.8	3.6
8-10	1	9	9	3.8	3.8
	$\Sigma=10$		$\Sigma=52$		14.8

So Mean $\frac{\Sigma fM}{N} = \frac{52}{10} = 5.2$

Hence the mean deviation, $MD_{\bar{x}} = \frac{\Sigma f|M-\bar{x}|}{N} = \frac{14.8}{10} = 1.48$

Assignments

class	f	M	fM	CF	$\lvert M-\bar{M}e\rvert$	$f\lvert M-\bar{M}e\rvert$
50-100	7	75	525	7	125	875
100-150	18	125	2250	25	75	1350
150-200	25	175	4375	50	25	625
200-250	31	225		81	25	775
250-300	15	275		96	75	1125
300-350	4	325		100	125	500
	100					$\Sigma=5250$

$(M.D_{Me}) = \frac{\Sigma f|M-Me|}{N} = \frac{5250}{100} = 52.50$

Co-efficient of $(MD.Me) = \frac{MD}{Me} = \frac{52.50}{200} = 0.26\frac{25}{}$

Median class, $\frac{N}{2} = \frac{100}{2} = 50$, So it lies in class (150-200) having f & CF are 25, 50.

Median $= L_1 + \frac{L_2-L_1}{f_1}(m-c)$

$= 150 + \frac{200-150}{25}(50-25) = 150 + \frac{50}{25} \times 25 = 200$

Assignment

(a)

Q.1 Calculate the mean deviation from mean

X	f	M	fM	$\lvert M-\bar{X}\rvert$	$f\lvert M-\bar{X}\rvert$
0-10	5	5	25	22	110
10-20	8	15	120	12	96
20-30	15	25	375	2	30
30-40	16	35	560	8	128
40-50	6	45	270	18	108
	$\Sigma = 50$		$\Sigma = 1350$		$\Sigma = 472$

So Mean, $\bar{X} = \frac{\Sigma fM}{N} = \frac{1350}{50} = 27$

Hence Mean Deviation $(MD_X) = \frac{\Sigma f\lvert M-\bar{X}\rvert}{N}$

$= \frac{472}{50} = 9.44$

And, Co-efficient of $MD_X = \frac{MD}{Mean} = \frac{9.44}{27} = 0.35$

(b) calculate the mean deviation from the mean for the following data

X	f	M	fM	$\lvert M-\bar{X}\rvert$	$f\lvert M-\bar{X}\rvert$	
0-10	6	5	30	28.4	152.4	170.4
10-20	5	15	75	18.4	77	92
20-30	8	25	200	8.4	43.2	67.2
30-40	15	35	525	1.6	64	24
40-50	7	45	315	11.6	102.2	81.2
50-60	6	55	330	21.6	147.6	129.6
60-70	3	65	195	31.6	103.8	94.8
	$N = 50$		$\Sigma = 1670$		$\Sigma = 695.2$	$\Sigma = 659.2$

So Mean, $\bar{X} = \frac{\Sigma fM}{N} = \frac{1520}{50} = 30.4$ $\frac{1670}{50} = 33.4$

Thus Mean Deviation $(MD_X) = \frac{\Sigma f\lvert M-\bar{X}\rvert}{N} = \frac{695.2}{50}$ $\frac{659.2}{50} = 13.184$

Q.8(a) Mean deviation may be calculated from the arithmetic mean or the median or the mode. Which of these three measures is the minimum?

→ Mean deviation is minimum, when taken about median.

(b) Find out mean deviation and its co-efficient from median.

x	f	c.f	fx	$\lvert x-\bar{x}\rvert$	$f\lvert x-\bar{x}\rvert$
4	2	2	2	6	12
6	1	3	3	4	4
8	3	6	6	2	6
10	6	12	12	0	0
12	4	16	16	2	8
14	3	19	19	4	12
16	1		20	6	6
	$N=20$				$\Sigma = 42$

$= 48$

So Median $= \left(\frac{N+1}{2}\right)^{th}$ term

$= \left(\frac{19+1}{2}\right)^{th}$ term

$= 10^{th}$ term

$= 10$

$\frac{48}{20} = 2.4$

Hence Mean deviation from Median $= \frac{\Sigma f\lvert x-\bar{x}\rvert}{N} = \frac{42}{19} = 2.21$

and co-efficient of $(MD_{Me}) = \frac{MD}{\text{Median}} = \frac{2.21}{10} = 0.22$

Q.9

x	f	fx	$\lvert x-\bar{x}\rvert$	$f\lvert x-\bar{x}\rvert$
5	8	40	24	192
15	12	180	14	168
25	10	250	4	40
35	8	280	6	48
45	3	135	16	48
55	2	110	26	52
65	7	455	36	252
	50	1450		$\Sigma = 800$

So Mean $= \frac{\Sigma fx}{N} = \frac{1450}{50} = 29$

So Mean deviation $= \frac{\Sigma f\lvert x-\bar{x}\rvert}{N}$

$= \frac{800}{50} = 16$

For median

X	f	CF	|X- Me|	f|X- Me|
5	8	8	20	160
15	12	20	15	180
25	10	30	0	0
35	8	38	10	80
45	3	41	20	60
55	2	43	30	60
65	7	50	40	280
				Σ = 820

Hence MD of median = $\frac{\Sigma f|X-Me|}{N}$

$= \frac{820}{50}$

$= 16.4$

and co-efficient of $(MD_{Me}) = \frac{MD}{Me}$

$= \frac{16.4}{25}$

$= 0.65$

Median = $(\frac{N+1}{2})^{th}$ term

$= (\frac{50+1}{2})^{th}$ term

$= (\frac{51}{2})^{th}$ term

= 25.5 in CF i.e 25

10.

X	f	M	CF	|M- Me|	f|M- Me|
17-19.5	9	18.25	9	~~22.62~~ 20	~~203.58~~ 180
20-25.5	16	~~[illegible]~~ 22.75	25	~~18.12~~ 15.5	~~289.92~~ 248
26-35.5	12	30.75	37	~~10.12~~ 7.5	~~121.44~~ 90
36-40.5	26	38.25	63	~~2.62~~ 0	~~68.12~~ 0
41-50.5	14	45.75	77	~~4.88~~ 7.5	~~68.32~~ 105
51-55.5	12	53.25	89	~~12.38~~ 15	~~148.56~~ 180
56-60.5	6	58.25	95	~~17.38~~ 20	~~104.28~~ 120
61-70.5	5	65.75	100	~~24.88~~ 27.5	~~124.4~~ 137.5
					Σ = ~~1128.62~~ = 1060.5

Here Median class, $\frac{N}{2} = \frac{100}{2} = 50$, So it lies in class (51-55.5) having frequency ~~20~~ 12 and CF ~~89~~ 63

~~Median = l1 + (l2-l1)/f1 (m-c)~~

$= 51 + \frac{55.5 - 51}{12}(50 - 63)$

$= 51 + \frac{4.5}{12}(-27)$

$= 51 - \frac{121.5}{12}$

$= 51 - 10.125$

$= 40.875$

Thus Mean deviation from median, $(MD_{Me}) = \frac{\Sigma f|M-Me|}{N}$

$Me = 36 + \frac{50-36}{26}(50-37)$

$= 36 + \frac{4.5}{26} \times 13$

$= 36 + \frac{58.5}{26}$

$= 36 + 2.25$

$= 38.25$

$= \frac{1060.5}{100}$

$= 10.605$

11.

X	f	M	cf	$\lvert M-Me \rvert$	$f\lvert M-Me \rvert$
20-25	6	22.5	6	20	120
25-30	12	27.5	18	15	180
30-40	17	35	35	7.5	127.5
40-45	30	42.5	65	0	0
45-50	10	47.5	75	5	50
50-55	10	52.5	85	10	100
55-60	8	57.5	93	15	120
60-70	5	65	98	22.5	112.5
70-80	2	75	100	32.5	65
					$\Sigma = 875$

Median class $\frac{N}{2} = 50^{th}$, (40-45)

Median, $l + \frac{\frac{N}{2} - cf}{f_1}(m-c)$

$= 40 + \frac{5}{30}(50-35)$

$= 40 + 2.5$

$= 42.5$

$MD_{me} = \frac{\Sigma f|M-Me|}{N}$

$= \frac{875}{100}$

$= 8.75$

Coefficient of $MD_{me} = \frac{8.75}{42.5} = 0.205$

Co-relation

In a bivariant distribution if there is any relationship between the two variables, then we can study the co-relation between the variables.

For ex:- ① Height, weight

② Family income and expenditure of luxury items

③ Price & supply of a commodity

④ Price & demand " "

⑤ sales of woolen items & the season

There are two types of co-relation

Positive correlation

Negative correlation

The correlation coefficient is defined as 'r' and the range of lies in between $-1 \leq r \leq 1$

when $r = 1$, then it is positively correlated

when $r = -1$ then it is negatively correlated

(if both the variables going in same direction then it means +ve correlation)

where $r = 0$, then there is correlation.

Positive Correlation

when both the variable increased or decreased in same direction, then we can say that there is a positive correlation

Negative correlation

When both the variable increased or decreased in different direction, then there is negative correlation.

Formula for 'r'

$$r = \frac{\Sigma dx\,dy}{\sqrt{\Sigma dx^2 \cdot \Sigma dy^2}}$$

where $dx = x - \bar{x}$

$dy = y - \bar{y}$

This r is known as Karl Pearson's co-efficient of correlation.

Q. calculate the 'r' between expenditure on advertisement and sales from the following data (6)

$r = \frac{\Sigma dxdy}{\sqrt{\Sigma dx^2 \Sigma dy^2}}$ $dx = x - \bar{x}$ $dy = y - \bar{y}$

expense on adv.	sales	dx $x - \bar{x}$	dy $y - \bar{y}$	$dxdy$	dx^2	dy^2
39	47	-26	-19	494	676	361
65	53	0	-13	0	0	169
62	58	-3	-8	24	9	64
90	86	25	20	500	625	400
82	62	17	-4	-68	289	16
75	68	10	2	20	100	4
25	60	-40	-6	240	1600	36
98	91	33	25	825	1089	625
36	51	-29	-15	435	841	225
78	84	13	18	234	169	324
		$\Sigma = 0$	$\Sigma = 0$	2704	5398	2224

Here Mean of $\bar{x} = \frac{650}{10} = 65$

Mean of $\bar{y} = \frac{660}{10} = 66$

$$r = \frac{\Sigma dxdy}{\sqrt{\Sigma dx^2 . \Sigma dy^2}} = \frac{2704}{\sqrt{5398 \times 2224}} = \frac{2704}{\sqrt{12005152}}$$

$$= \frac{2704}{3464.84}$$

$$r = 0.78$$

So there is a fairly +ve correlation between expenditure on adv. and sales.

Therefore sales increased with an increase in the advertisement expenses

Q: Calculate the ~~coef~~ 'r' for ages of husbands and wives

Husband X	Wives	$x-\bar{x}$	$y-\bar{y}$	$dxdy$	dx^2	dy^2
23	18	-8.1	-7.7	62.37	65.61	59.29
27	22	-4.1	-3.7	15.17	16.81	13.69
28	23	-3.1	-2.7	8.37	9.61	7.29
29	24	-2.1	-1.7	3.57	4.41	2.89
30	25	-1.1	-0.7	0.77	1.21	0.49
31	26	-0.1	0.3	0.03	0.01	0.09
33	28	1.9	2.3	4.37	3.61	5.29
35	29	3.9	3.3	12.87	15.21	10.89
36	30	4.9	4.3	21.07	24.01	18.49
39	32	7.9	6.3	49.77	62.41	39.69
				178.59	202.89	168.1

$$\bar{X} = \frac{311}{10} = 31.1$$

$$\bar{Y} = \frac{257}{10} = 25.7$$

$$r = \frac{\Sigma dxdy}{\sqrt{\Sigma dx^2 \cdot \Sigma dy^2}}$$

$$= \frac{178.59}{\sqrt{32076.0}}$$

$$= \frac{178.59}{179.10}$$

$$= 0.99$$

Here $r = 0.9$, so there is a strongly positive correlation between the ages of husbands and wives.

Find the co-relation coefficient

x	y	dx $x-\bar{x}$	dy $y-\bar{y}$	$dxdy$	dx^2	dy^2
45	25	-20	-19	380	400	361
48	32	-17	-12	204	289	144
50	35	-15	-9	135	225	81
55	30	-10	-14	140	100	196
65	42	0	-2	0	0	4
70	50	5	6	30	25	36
75	45	10	1	10	100	1
72	55	7	11	77	49	121
80	60	15	16	240	125	256
85	65	20	21	420	400	441
				1636	1813	1641

$\bar{x} = \frac{645}{10} = 64.5 = 65$

$\bar{y} = \frac{439}{10} = 43.9 = 44$

$r = \frac{\Sigma dxdy}{\sqrt{\Sigma dx^2 \cdot \Sigma dy^2}}$

$= \frac{1636}{\sqrt{2646933}}$

$= \frac{1636}{1613.4}$

$= 1.01$

$\frac{1636}{\sqrt{2975133}}$

2975133

1724.8

$\frac{1636}{\sqrt{2646933}}$

$\frac{1636}{1626.9}$

$= 0.94$

$= \frac{1636}{1724.8}$

$= 0.93$

Here $r = 0.94$

Q. 13

x	y	dx	dy	dxdy	dx^2	dy^2
6	10	-12	-9	108	144	81
8	12	-10	-7	70	100	49
12	15	-6	-4	24	36	16
15	15	-3	-4	12	9	16
18	18	0	1	0	0	1
20	25	2	6	12	4	36
24	22	6	3	18	36	9
28	26	10	7	70	100	49
31	28	13	9	117	169	81
168	171			431	598	338

$\bar{X} = \frac{162}{9} = 18$

$\bar{y} = \frac{171}{9} = 19$

So $r = \frac{\Sigma dx.dy}{\sqrt{\Sigma dx^2 . \Sigma dy^2}}$

$= \frac{431}{\sqrt{202124}} = \frac{431}{449.5} = 0.95$

Q.14

x	y	dx	dy	dxdy	dx^2	dy^2
80	45	17	-13	-221	289	169
60	71	-3	13	-39	9	169
51	60	-12	2	-24	144	4
69	57	6	-1	-6	36	1
58	62	-5	4	-20	25	16
62	58	-1	0	0	1	0
64	48	1	-10	-10	1	100
72	50	9	-8	-72	81	64
56	62	-7	4	-28	49	16
58	69	-5	11	-55	25	121
630	582			-475	660	660

Mean, $\bar{X} = \frac{630}{10} = 63$

Mean, $\bar{y} = \frac{582}{10} = 58.2 \simeq 58$

So $r = \frac{\Sigma dxdy}{\sqrt{\Sigma dx^2 . \Sigma dy^2}} = \frac{-475}{\sqrt{435600}}$

$= \frac{-475}{660}$

$= -0.65$

Q.3

X_1	X_2	dx	dy	$dxdy$	dx^2	dy^2
10	9	0	-1	0	0	4
6	4	-4	-7	28	16	49
9	6	-1	-5	5	1	25
10	9	0	-2	0	0	4
12	11	2	0	0	4	0
13	13	3	2	6	9	4
11	8	1	-3	-3	1	9
9	4	-1	-7	7	1	49
				$\Sigma = 43$	$\Sigma = 32$	$\Sigma = 144$

Here $\bar{X}_1 = \frac{80}{8} = 10$

$\bar{X}_2 = \frac{64}{8} = 8$

Thus $r = \frac{\Sigma dx\,dy}{\sqrt{\Sigma dx^2 . \Sigma dy^2}}$

$= \frac{43}{\sqrt{4608}}$

$= \frac{43}{67.88}$

$=$

X	Y
10	
9	

Q_1 and Q_3 of an individual series is calculated as $\frac{N+1}{4}$ and $3\left(\frac{N+1}{4}\right)^{th}$ term of the series.

X
2
4
7
8
9
10
N=6

$Q_1 = \left(\frac{N+1}{4}\right)^{th}$

$= \left(\frac{6+1}{4}\right)$

$= \left(\frac{7}{4}\right)^{th}$ term

$= 1.7^{th}$ term

= 1st term + .75(2nd − 1st)

= 2 + .75(4−2)

= 2 + 1.5

= 3.5

$Q_3 = 3\left(\frac{N+1}{4}\right)$

$= 3 \times 1.75$

$= 5.25$

* Relation between standard deviation, MD, QD and range.

$\Rightarrow$ 4SD = 5MD = 6QD

and SD = ~~$\frac{1}{4}$ range~~ $\frac{1}{6}$ range

Q. Find Mean of data is 25.6 & coefficient of variance 18.75. Find the SD

$\rightarrow \bar{X} = 25.6$, CV = 18.75

$CV = \frac{\sigma}{X} \times 100$

$\Rightarrow 18.75 = \frac{\sigma}{25.6} \times 100$

$\Rightarrow 480 = 100\sigma$

$\Rightarrow \sigma = \frac{480}{100} = 4.8$

Q Quartile of distribution 47.3 & 52.7

MD and range

$Q_1 = 47.3$, $Q_3 = 52.7$

$QD = \frac{52.7 - 47.3}{2} = \frac{5.4}{2} = 2.7$

$6QD = 5MD$

$MD = \frac{6QD}{5} = \frac{16.2}{5} = 3.24$

Now

$4SD = 5MD$

$SD = \frac{5MD}{4} = \frac{16.2}{4} = 4.05$

So range $= 6SD$

$= 6 \times 4.05$

$= 24.3$

X	Y	dx	dy	dxdy	dx^2	dy^2
10	9	0	-1	0	0	1
6	4	-4	-4	16	16	16
9	6	-1	-2	2	1	4
10	9	0	1	0	0	1
12	11	2	3	6	4	9
13	13	3	5	15	9	25
11	8	1	0	0	1	0
9	4	-1	-4	4	1	16
80	64			43	32	72

$\rightarrow \bar{X} = \frac{80}{8} = 10$

$\bar{Y} = \frac{64}{8} = 8$

$r = \frac{43}{\sqrt{32 \times 72}} = \frac{43}{\sqrt{2304}} = \frac{43}{48}$

$= 0.89$

Q Find r' using Karl where $\bar{x}=6, \bar{y}=8$

x	y	dx	dy	dxdy	dx^2	dy^2
6	9	0	1	0	0	1
2	11	-4	3	-12	16	9
10	?5	4	a-8(-3)	4a-32	16	$(a-8)^2 9$
4	8	-2	0	0	4	0
8	7	2	-1	-2	4	1
35+x				4a-56	40	$a^2+64+16+1$
				-26		20

Here $\bar{x}=\frac{30}{5}=6$

$\bar{y}=8$

$dx=x-\bar{x}$

$dy=y-\bar{y}$ So $r=\frac{4a-56}{40\times(a^2+81)}$

$\frac{35+x}{5}=8$

$\Rightarrow 35+x=40$

$\Rightarrow x=5$

$= \frac{4a-56}{\sqrt{40a^2+3240}}$

$= \frac{20-56}{\sqrt{40\times25+3240}}$

$= \frac{-26}{\sqrt{1000+3240}} = \frac{-26}{\sqrt{800}}$

$= \frac{-26}{\sqrt{4240}} = \frac{-26}{\sqrt{28.28}}$

$= \frac{-26}{65.11} = -0.91$

Q.

Karl Pearson co-efficient also defined as $\frac{\text{co-variance of } xy}{\sigma x \cdot \sigma y}$

$$r = \frac{cov(x,y)}{\sigma x \cdot \sigma y}$$

$$= \frac{\Sigma dx.dy}{\sqrt{\Sigma dx^2 \cdot \Sigma dy^2}}$$

of x, y

Co-variance is defined as the relationship between two variable x and y and mathematically

$$cov(x,y) = \frac{\Sigma(x-\bar{x})\,\Sigma(y-\bar{y})}{n}$$

$$\sigma x = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n}}$$

$$r = \frac{cov(x,y)}{\sigma x \cdot \sigma y} = \frac{\Sigma(x-\bar{x})(y-\bar{y})}{n} \times \sqrt{\frac{n}{\Sigma(x-\bar{x})^2}} \sqrt{\frac{n}{\Sigma(y-\bar{y})^2}}$$

$$= \frac{\Sigma dx \cdot \Sigma dy}{n} \times \frac{n^2}{\sqrt{\Sigma(x-\bar{x})^2 \cdot \Sigma(y-\bar{y})^2}}$$

$$= \frac{\Sigma dx.dy}{\sqrt{\Sigma dx^2 \cdot \Sigma dy^2}}$$

Calculate r from the following series, in x, n = 15

(X)

	X	Y
no. of obs.	15	15
AM	25	18
S.D	3.01	3.03
$\Sigma(x-\bar{x})^2$ / $\Sigma(y-\bar{y})^2$	136	138
$\Sigma(x-\bar{x})(y-\bar{y})$	122	

$$r = \frac{\Sigma dx.dy}{\sqrt{\Sigma dx^2 . \Sigma dy^2}}$$

$$= \frac{122}{\sqrt{136 \times 138}}$$

$$= \frac{122}{\sqrt{18768}}$$

$$= \frac{122}{136.99}$$

$$r = 0.89$$

Here $\frac{\Sigma dx.dy}{\Sigma (x-\bar{x})^2 . \sigma x}$

$$Cov(x,y) = \frac{\Sigma (x-\bar{x})(y-\bar{y})}{n}$$

$$= \frac{122}{15} = 8.13$$

$$\frac{Cov(x,y)}{\sigma x . \sigma y} = \frac{122}{15}$$

$$= \frac{0.89}{3.01 \times \sigma y} = \frac{122}{15}$$

$$\Rightarrow (0.89) \times 15 = 122 (0.31 \times \sigma y)$$

$$\Rightarrow 13.35 = 367.22 \sigma y$$

$$\Rightarrow \sigma y = \frac{1335}{367.22}$$

$$\frac{Cov(x,y)}{\sigma x . \sigma y} = \frac{\Sigma (x-\bar{x})(y-\bar{y})}{\Sigma (x-\bar{x})^2 . \Sigma (y-\bar{y})^2}$$

$$r = \frac{Cov(x,y)}{\sigma x . \sigma y}$$

$$\& \; Cov(x,y) = \frac{\Sigma (x-\bar{x})(y-\bar{y})}{n} = \frac{122}{15} = 8.13$$

$$=$$

Now $r = \frac{8.13}{\sigma x . \sigma y}$

$$\Rightarrow 0.89 = \frac{8.13}{3.01 \sigma y}$$

$$\Rightarrow (0.89)(3.01)\sigma y = 8.13$$

~~Cov(x,y)~~ $\sigma^2(x,y)$ $\sigma x = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n}}$

$r = 0.48$ $\sigma^2 = c'$ σx^2

$Cov(x,y) = 36.$

$\sigma x^2 = 16 \Rightarrow \sigma x = 4.$

$\sigma x =$

$$\frac{Cov(x,y)}{\sigma x \cdot \sigma y} = \frac{36}{4 \times \sigma y} = 0.48$$

$$\Rightarrow 36 = 0.48 \times 4\sigma y$$

$$\Rightarrow 1.92\sigma y = 36$$

$$\Rightarrow \sigma y = \frac{36}{1.92} = 18.75$$

Q.

$r = 0.8$

$\Sigma xy = 16$

$\sigma y = 2.5$

$\Sigma x^2 = 90$

where x and y are the deviation from their respective mean, find n,

Q. ~~Cov beta~~ $Cov(x,y) = 96$

$\sigma^2 x : \sigma^2 y = 4 : 9$

Find σx & σy

Q. $Cov(x,y) = 144$

Q.1

$r = 0.8$

$\Sigma xy = \Sigma(X-\bar{X})(Y-\bar{Y}) = 16$

$\sigma_y = 2.5$

$\Sigma x^2 = \Sigma(X-\bar{X})^2 = 90$

Thus $r = \dfrac{\Sigma(X-\bar{X})(Y-\bar{Y})}{}$

As $\Sigma x^2 = 90$

$\Rightarrow \Sigma(X-\bar{X})^2$

Correlation coefficient also defined as $r = \frac{n\Sigma xy - \Sigma x \cdot \Sigma y}{\sqrt{\{n\Sigma x^2 - (\Sigma x)^2\}}}$

$$r = \frac{n\Sigma xy - \Sigma x \cdot \Sigma y}{\sqrt{n\cdot\Sigma x^2 - (\Sigma x)^2 (n\Sigma y^2 - (\Sigma y)^2)}}$$

The merit of this formula is if $\bar{x}$ & $\bar{y}$ both will come in the form of decimal, then we can use this formula.

	x	y	xy	x^2	y^2
	6	9	54	36	81
	2	11	22	4	121
	10	5	50	100	25
	4	8	32	16	64
	8	7	56	64	49
			214	220	340

$\bar{x} = \frac{30}{5} = 6$

$\bar{y} = \frac{40}{5} = 8$

$$r = \frac{n\cdot\Sigma xy - \Sigma x\cdot\Sigma y}{\sqrt{(5\times220 - 30^2)(5\times340 - 40^2)}}$$

$$= \frac{5\cdot214 - 30\times40}{\sqrt{(5\times220 - 30^2)(5\times340 - 40^2)}}$$

$$= \frac{1070 - 1200}{\sqrt{(1100 - 900)(1700 - 1600)}}$$

$$= \frac{-130}{\sqrt{200\times100}}$$

$$= \frac{-130}{\sqrt{20,000}} = \frac{-130}{141} = -0.92$$

Q

x	y	xy	x^2	y^2
1	1	1	1	1
2	4	8	4	16
3	9	27	9	81
4	16	64	16	256
10	30	100	30	354

$$r = \frac{4 \times 100 - 300}{\sqrt{(4 \times 30 - 100)(4 \times 354 - 900)}}$$

$$= \frac{400 - 300}{\sqrt{20 \times 1416 - 900}}$$

$$= \frac{100}{\sqrt{26520}} \quad \frac{100}{10320}$$

$$= \frac{100}{162.84} \quad \frac{100}{101.58}$$

$$= 0.98$$

$$r = \frac{n\Sigma uv - \Sigma u . \Sigma v}{\sqrt{n.\Sigma u^2 - (\Sigma u)^2 (n\Sigma v^2 - (\Sigma v)^2}}$$

$$u = \frac{x - A}{h}$$

$$v = \frac{y - A}{h}$$

Continuous Series

x	y		v	uv	u^2	v^2
17.5	75	-2	5	-10	4	25
22.5	60	-1	2	-2	1	4
27.5	50	0	0	0	0	0
32.5	50	1	0	0	1	0
37.5	45	2	-1	-2	4	1
42.5	40	3	-2	-6	9	4
		3		-20	19	34

$$r = \frac{6 \times (-20) - 3 \times 4}{\sqrt{(6 \times 19 - 9)(6 \times 34 - 16)}} = \frac{-120 - 12}{\sqrt{(114 - 9)(204 - 16)}} = \frac{-132}{\sqrt{105 \times 188}}$$

$$= \frac{-132}{\sqrt{19740}} = \frac{-132}{140.4} = -0.94$$

X	Y	XY	X^2	Y^2
2	18	36	4	324
4	12	48	16	144
5	10	50	25	100
6	8	48	36	64
8	7	56	64	49
11	5	55	121	25
36	60	293	266	706

$r = \frac{6 \times 293 - 36 \times 60}{\sqrt{(6 \times 266 - (36)^2)(6 \times 706 - 60^2)}}$

$= \frac{1758 - 2160}{\sqrt{(1596 - 1225) \times (4236 - 3600)}}$

$= \frac{-402}{\sqrt{371 \times 636}}$

$= \frac{-342}{\sqrt{235,956}}$

$= \frac{-342}{485.75}$

$\frac{-402}{\sqrt{190,800}} = \frac{-402}{436.8}$

$= -0.92$

2. calculate 'r' from the following data

$n = 10$, $\Sigma x = 140$, $\Sigma y = 150$, $\Sigma (x-10)^2 = 180$ & $\Sigma (y-15)^2 = 215$

$\Sigma (x-10)(y-15) = 60$

Here $r = \frac{cov(x,y)}{\sigma x \cdot \sigma y} = \frac{6}{4.24 \times 4.63} = 0.30$

$cov(xy) = \frac{\Sigma (x-10)(y-15)}{10} = \frac{60}{10} = 6$

$\sigma x = \sqrt{\frac{\Sigma (x-10)^2}{10}} = \sqrt{\frac{180}{10}} = \sqrt{18} = 4.24$

$\sigma y = \sqrt{\frac{\Sigma (y-15)^2}{n}} = \sqrt{\frac{215}{10}} = \sqrt{21.5} = 4.63$

Q. calculate r, use uv method.

X	Y	$U = X - A$	$V = Y - A$	U^2	V^2	UV
23	18	-7	-7	49	49	49
27	22	-3	-3	9	9	9
28	23	-2	-2	4	4	4
29	24	-1	-1	1	1	1
(30)	(25)	0	0	0	0	0
31		1	0	1		1
33	26	3	1	9	1	9
35	28	5	3	25	9	
36	29	6	4	36	16	20
39	30	9	5	81	25	30
311	32	11	7	215	163	63
	257		7			186

$$r = \frac{n \cdot \Sigma UV - \Sigma U \cdot \Sigma V}{\sqrt{(n\Sigma U^2 - (\Sigma U)^2)\,[n\Sigma V^2 - (\Sigma V)^2]}}$$

$$= \frac{10 \times 186 - 11 \times 7}{\sqrt{(10 \times 215 - 121)\,(10 \times 163 - 49)}}$$

$$= \frac{1860 - 77}{\sqrt{(2150 - 121)\,(1630 - 49)}}$$

$$= \frac{1783}{\sqrt{2029 \times 1581}}$$

$$= \frac{1783}{\sqrt{3207849}}$$

$$= \frac{1783}{1791.04}$$

$$\approx 0.99$$

What do you mean by correlation? What are the diff types of co-relation. (8)
write do two uses of correlation related to biz using suitable examples.

Q.2

X	Y	XY	U	V	UV	U^2	V^2
50	11	550	-15	-5	75	225	25
50	13	650	-15	-3	45	225	9
55	14	770	-10	-2	20	100	4
60	16	960	-5	0	0	25	0
(65)	(16)	1040	0	0	0	0	0
65	15		0	-1	0	0	1
65	15		0	-1	0	0	1
60	14		-5	-2	10	25	4
60	13		-5	-3	15	25	9
50	13		-15	-3	45	225	9
			-70	-20	210	850	62

$$r = \frac{n\Sigma uv - \Sigma u \cdot \Sigma v}{\sqrt{n\Sigma u^2 - (\Sigma u)^2 \times n\Sigma v^2 - (\Sigma v)^2}}$$

$$= \frac{10 \times 210 - (-70 \times -20)}{\sqrt{(10 \times 850 - 4900) \times (10 \times 62 - 400)}}$$

$$= \frac{2100 - 1400}{\sqrt{(8500 - 4900)(620 - 400)}}$$

$$= \frac{700}{\sqrt{3600 \times 220}}$$

$$= \frac{700}{\sqrt{792,000}}$$

$$= \frac{700}{889.94}$$

$$= 0.78$$

Q.7

x	y	d(x-y)	d^2
1	6	-5	25
6	8	-2	4
3	3	0	0
9	7	2	4
5	2	3	9
2	1	1	1
7	5	2	4
10	9	1	1
8	4	4	16
4	10	-6	36
			$\Sigma = 100$

Thus $\rho = 1 - \frac{6\Sigma d^2}{n(n^2-1)}$

$= 1 - \frac{6 \times 100}{10 \times 99}$ (10, 33)

$= 1 - \frac{20}{33}$

$= 1 - 0.6060$

$= 0.394$

Q.8

x	y	d	d^2
1	2	-1	1
2	10	-8	64
3	3	0	0
4	4	0	0
5	5	0	0
6	7	-1	1
7	2	5	25
8	6	2	4
9	8	1	1
10	11	-1	1
11	15	-4	16
12	9	3	9
13	14	-1	1
14	12	2	4
15	16	-1	1
16	13	3	9
			$\Sigma = 136$

Thus $\rho = 1 - \frac{6\Sigma d^2}{n(n^2-1)}$

$= 1 - \frac{6 \times 136}{16 \times 255}$

$= 1 - \frac{816}{4080}$

$= 1 - 0.2$

$= 0.8$

Q.9 (18)

x	y	d	d^2
1	12	-11	121
2	9	-7	49
3	6	-3	9
4	10	-6	36
5	3	2	4
6	5	1	1
7	4	-3	9
8	7	1	1
9	8	1	1
10	2	8	64
11	11	0	0
12	1	11	121
			$\Sigma = 416$

Thus $\rho = 1 - \frac{6\Sigma d^2}{n(n^2-1)}$

$= 1 - \frac{6 \times 416}{12 \times 143}$

$= 1 - \frac{2496}{1716}$

$= 1 - 1.454$

$= -0.454$

∴ As $\rho = -0.454$, then this shows a moderate negative correlation between two judges rankings. Means one judge's rankings increase, the other's decrease and ~~vice-v~~ Vice-Versa.

Q.10

R_1	R_2	R_3	d_{12}	d_{12}^2	d_{23}	d_{23}^2	d_{13}	d_{13}^2
1	4	6	-3	9	-2	4	-5	25
5	8	7	-3	9	1	1	-2	4
4	7	8	-3	9	-1	1	-4	16
8	6	1	2	4	5	25	7	49
9	5	5	4	16	0	0	4	16
6	9	10	-3	9	-1	1	-4	16
10	10	9	0	0	1	1	1	1
7	3	2	4	16	1	1	5	25
3	2	3	1	1	-1	1	0	0
2	1	4	1	1	-3	9	-2	4
				$\Sigma = 74$		$\Sigma = 44$		$\Sigma = 156$

Hence $\rho_{12} = 1 - \frac{6\Sigma d_{12}^2}{n(n^2-1)} = 1 - \frac{6 \times 74}{10 \times 99} = 1 - \frac{444}{990} = 1 - 0.448 = 0.5515$

$\rho_{23} = 1 - \frac{6\Sigma d_{23}^2}{n(n^2-1)} = 1 - \frac{6 \times 44}{10 \times 99} = 1 - \frac{4}{15} = 1 - 0.26 = 0.73$

$\rho_{13} = 1 - \frac{6\Sigma d_{13}^2}{n(n^2-1)} = 1 - \frac{6 \times 156}{10 \times 99} = 1 - \frac{936}{990} = 1 - 0.945 = 0.054$

∴ 1st judge and 2nd judge has the nearest approach to beauty.

Q.11

Judge 'A'	Judge 'B'	Judge 'C'	d_{AB}	d_{AB}^2	d_{BC}	d_{BC}^2	d_{AC}	d_{AC}^2
6	5	4	1	1	1	1	2	4
5	8	9	-3	9	-1	1	-4	16
3	4	8	-1	1	-4	16	-5	25
10	7	1	3	9	6	36	9	81
2	10	2	-8	64	8	64	0	0
4	2	3	2	4	-1	1	1	1
9	1	10	8	64	-9	81	-1	1
7	6	5	1	1	1	1	2	4
8	9	7	-1	1	2	4	1	1
1	3	6	-2	4	-3	9	-5	25
				$\Sigma = 158$		$\Sigma = 214$		$\Sigma = 158$

So $\rho_{AB} = 1 - \frac{6\Sigma d_{AB}^2}{n(n^2-1)} = 1 - \frac{6 \times 158}{10 \times 99} = 1 - \frac{948}{990} = 1 - 0.957 = 0.042$

$\rho_{BC} = 1 - \frac{6\Sigma d_{BC}^2}{n(n^2-1)} = 1 - \frac{6 \times 214}{10 \times 99} = 1 - \frac{1284}{990} = 1 - 1.296 = -0.296$

$\rho_{AC} = 1 - \frac{6\Sigma d_{AC}^2}{n(n^2-1)} = 1 - \frac{6 \times 158}{10 \times 99} = 1 - \frac{948}{990} = 1 - 0.957 = 0.042$

∴ The judges (A & B) and (A & C) have the nearest approach to common tastes in beauty.

Q.12

Equity shares X	Preference shares Y	Rank of X	Rank of Y	d	d^2
97.5	75.1	4	6	-2	4
99.4	75.9	1	5	-4	16
98.6	77.1	2	3	-1	1
96.2	78.2	6	2	4	16
95.1	79	7	1	6	36
98.4	74.8	3	7	-4	16
97.1	76.2	5	4	1	1
					$\Sigma = 90$

Thus $\rho = 1 - \frac{6\Sigma d^2}{n(n^2-1)}$

$= 1 - \frac{6 \times 90}{7 \times 48} = 1 - \frac{540}{336} = 1 - 1.607 = -0.607$

Q.16

X	Y	U	V	UV	U^2	V^2
46	12	-1	-12	12	1	144
33	13	-12	-9	108	144	81
41	(24)	-4	0	0	16	0
38	16	-7	-8	56	49	64
36	15	-9	-9	81	81	81
(45)	14	0	-10	0	0	100
34	21	-11	-3	33	121	9
37	17	-8	-7	56	64	49
50	19	5	-5	-25	25	25
40	19	-5	-5	25	25	25
		-52	-68	346	526	578

$$r = \frac{n\Sigma UV - \Sigma U \cdot \Sigma V}{\sqrt{n\Sigma U^2 - (\Sigma U)^2} \times \sqrt{n\Sigma V^2 - (\Sigma V)^2}}$$

$$= \frac{10 \times 346 - (-52)\times(-68)}{\sqrt{(10 \times 526 - 2704) \times (10 \times 578 - 4624)}}$$

$$= \frac{3460 - 3536}{\sqrt{(5260 - 2704)(5780 - 4624)}}$$

$$= \frac{-76}{\sqrt{2556 \times 1156}}$$

$$= \frac{-76}{\sqrt{2954736}}$$

$$= \frac{-76}{1718.93}$$

$$= -0.0442$$

Q.17 → Given $\bar{X} = 20, \bar{Y} = 70$

X	Y	U	V	UV	U^2	V^2
14	84	-6	14	-84	36	196
16	78	-4	8	-32	16	64
17	70	-3	0	0	9	0
18	75	-2	5	-10	4	25
19	66	-1	-4	4	1	16
20	67	0	-3	0	0	9
21	62	1	-8	-8	1	64
22	58	2	-12	-24	4	144
23	60	3	-10	-30	9	100
		-10	-10	-184	80	618

$$r = \frac{n\Sigma UV - \Sigma U \cdot \Sigma V}{\sqrt{n\Sigma U^2 - (\Sigma U)^2 \times n\Sigma V^2 - (\Sigma V)^2}}$$

$$= \frac{9 \times (-184) - (-10)(-10)}{\sqrt{(9 \times 80 - 100)(9 \times 618 - 100)}}$$

$$= \frac{-1656 - 100}{\sqrt{(720 - 100)(5562 - 100)}}$$

$$= \frac{-1756}{\sqrt{620 \times 5462}}$$

$$= \frac{-1756}{\sqrt{3386440}}$$

$$= \frac{-1756}{1840.2}$$

$$= -0.954$$

Q.18

X	Y	U	V	UV	U^2	V^2
75	62	-1	2	-2	1	4
81	68	5	8	40	25	64
70	65	-6	5	-30	36	25
(76)	(60)	0	0	0	0	0
77	69	1	9	9	1	81
81	72	5	12	60	25	144
84	76	8	16	128	64	256
75	72	-1	12	-12	1	144
		11	64	193	153	718

$$r = \frac{n\Sigma UV - \Sigma U \cdot \Sigma V}{\sqrt{n\Sigma U^2 - (\Sigma U)^2} \times \sqrt{n\Sigma V^2 - (\Sigma V)^2}}$$

$$= \frac{8 \times 193 - 11 \times 64}{\sqrt{(8 \times 153 - 121) \times (8 \times 718 - 4096)}}$$

$$= \frac{1544 - 704}{\sqrt{(1224 - 121)(5744 - 4096)}}$$

$$= \frac{840}{\sqrt{1103 \times 1648}}$$

$$= \frac{840}{\sqrt{1817744}}$$

$$= \frac{840}{1348.23}$$

$$= 0.623$$

Q.19

X	Y	U	V	UV	U^2	V^2
100	98	1	6	6	1	36
101	99	2	7	14	4	49
103	99	4	7	28	16	49
102	97	3	5	15	9	25
100	95	1	3	3	1	9
(99)	(92)	0	0	0	0	0
97	95	-2	3	-6	4	9
98	94	-1	2	-2	1	4
96	90	-3	-2	6	9	4
95	91	-4	-1	4	16	1
		1	30	68	51	186
					61	

$$\Rightarrow r = \frac{n\Sigma UV - \Sigma U \cdot \Sigma V}{\sqrt{n\Sigma U^2 - (\Sigma U)^2} \times \sqrt{n\Sigma V^2 - (\Sigma V)^2}}$$

$$= \frac{10 \times 68 - 30}{\sqrt{(10 \times 51 - 1)(10 \times 186 - 900)}}$$

$$= \frac{680 - 30}{\sqrt{509 \times 960}}$$

$$= \frac{650}{\sqrt{488640}}$$

$$= \frac{650}{221.05} =$$

$$\frac{10 \times 68 - 30}{\sqrt{(609 \times 10 \times 186 - 900)}} = \frac{650}{}$$

$$= \frac{650}{\sqrt{609 \times 960}} = \frac{650}{\sqrt{584640}}$$

$$= \frac{650}{764.61}$$

$$= 0.850$$

Now, $N = 10$ & $\Sigma X = 991$, $\Sigma Y = 950$

So $\bar{X} = \frac{991}{10} = 99.1$

$\bar{y} = \frac{950}{10} = 95$

Thus,

dx	dy	dxdy	dx^2	dy^2
0.9	3	2.7	0.81	9
1.9	4	7.6	3.61	16
3.9	4	5.6	15.21	16
2.9	2	5.8	8.41	4
0.9	0	0	0.81	0
-0.1	-3	0.3	0.01	9
-2.1	0	0	4.41	0
-1.1	-1	1.1	1.21	1
-3.1	-5	15.5	9.61	25
-4.1	-4	16.6	16.81	16
		$\Sigma = 65.2$	$\Sigma = 60.9$	$\Sigma = 96$

Thus $r = \frac{\Sigma dxdy}{\sqrt{\Sigma dx^2 \times \Sigma dy^2}}$

$$= \frac{65.2}{\sqrt{60.9 \times 96}}$$

$$= \frac{65.2}{\sqrt{5846.4}}$$

$$= \frac{65.2}{76.46} = 0.852$$

Q.20

X	Y	$U = \frac{X-120}{10}$	$V = Y-19$	UV	U^2	V^2
80	15	-4	-4	16	16	16
90	15	-3	-4	12	9	16
100	16	-2	-3	6	4	9
110	19	-1	0	0	1	0
120	17	0	-2	0	0	4
130	18	1	-1	1	1	1
140	16	2	-3	-6	4	9
150	18	3	-1	-3	9	1
160	19	4	0	0	16	0
		0	-18	26	60	56

$$r = \frac{n\Sigma UV - \Sigma U \cdot \Sigma V}{\sqrt{n\Sigma U^2 - (\Sigma U)^2 \times n\Sigma V^2 - (\Sigma V)^2}}$$

$$= \frac{9 \times 26 - 0}{\sqrt{(9 \times 60 - 0)(9 \times 56 - 324)}}$$

$$= \frac{234}{\sqrt{540 \times 180}}$$

$$= \frac{234}{\sqrt{97200}} = \frac{234}{311.77} = 0.750$$

Q.21

x	y	dx	dy	dxdy	dx^2	dy^2
13	54	4	-8	-32	16	64
12	53	3	-9	-27	9	81
9	57	0	-5	0	0	25
9	61	0	-1	0	0	1
8	67	-1	5	-5	1	25
6	72	-3	10	-30	9	100
6	70	-3	8	-24	9	64
		0	0	-118	44	360

Here $N = 7$, $\Sigma X = 63$, $\Sigma Y = 434$

So $\bar{x} = 9$, $\bar{y} = \frac{434}{7} = 62$

$$\gamma = \frac{\Sigma dx\,dy}{\sqrt{\Sigma dx^2 \Sigma dy^2}}$$

Q.21

X	Y	U	V	UV	U^2	V^2
13	54	4	-3	-12	16	9
12	53	3	-4	-12	9	16
(9)	(57)	0	0	0	0	0
9	61	0	4	0	0	16
8	67	-1	10	-10	1	100
6	72	-3	15	-45	9	225
6	70	-3	13	-39	9	169
		0	35	-118	44	535

$$r = \frac{n\Sigma UV - \Sigma U \cdot \Sigma V}{\sqrt{n\Sigma U^2 - (\Sigma U)^2 \times n\Sigma V^2 - (\Sigma V)^2}}$$

$$= \frac{7 \times (-118) - 0}{\sqrt{(7 \times 44 - 0)(7 \times 535 - 1225)}} = \frac{-826}{\sqrt{308 \times 2520}} = \frac{-826}{880.9} = -0.937$$

Q.22

X	Y	U	V	UV	U^2	V^2
23	18	-5	-9	45	25	81
27	10	-1	-17	17	1	289
28	22	0	-5	0	0	25
(28)	(27)	0	0	0	0	0
28	21	0	-6	0	0	36
30	29	2	2	4	4	4
30	27	2	0	0	4	0
33	29	5	2	10	25	4
35	28	7	1	7	49	1
38	29	10	2	20	100	4
		20	-30	103	208	444

$$r = \frac{n\Sigma UV - \Sigma U \cdot \Sigma V}{\sqrt{n\Sigma U^2 - (\Sigma U)^2 \times n\Sigma V^2 - (\Sigma V)^2}}$$

$$= \frac{10 \times 103 + 600}{\sqrt{(10 \times 208 - 400)(10 \times 444 - 900)}}$$

$$= \frac{1030 + 600}{\sqrt{1680 \times 3540}}$$

$$= \frac{1630}{\sqrt{5947200}}$$

$$= \frac{1630}{2438.6} = 0.66$$

Q.22

X	Y	$x-\bar{x}$	$y-\bar{y}$	$dx\,dy$	$(x-\bar{x})^2$	$(y-\bar{y})^2$
23	18	-7.22	-7.44	53.71	52.12	55.35
27	20	-3.22	-5.44	17.51	10.36	29.59
28	22	-2.22	-3.44	7.63	4.92	11.83
29	27	-1.22	1.56	-1.90	1.48	2.43
30	29	-0.22	3.56	-0.78	0.04	12.67
31	27	0.78	1.56	1.21	0.60	2.43
33	29	2.78	3.56	9.89	7.72	12.67
35	28	4.78	2.56	12.23	22.84	6.55
36	29	5.78	3.56	20.57	33.40	12.67
272	229			120.07	133.48	$\Sigma = 146.19$

Here $N = 9$, $\Sigma X = 272$, $\Sigma Y = 229$

Then $\bar{X} = \frac{272}{9} = 30.22$

$\bar{Y} = \frac{229}{9} = 25.44$

Now $r = \frac{\Sigma dx.dy}{\sqrt{\Sigma dx^2 . \Sigma dy^2}} = \frac{120.17}{\sqrt{133.48 \times 146.19}} = \frac{120.17}{\sqrt{19513.44}} = \frac{120.17}{139.69} = 0.860$

Q.23 → Given $\bar{x} = 44$, $\bar{y} = 26$

X	Y	$U = X-44$
43	29	
44	31	
46	19	
40	18	
44	19	
42	27	
45	27	
42	29	
38	41	
40	30	
42	26	
57	10	

Q. Coefficient of range correlation bet^n 2 variables is 0.8 and if the sum of the Squares of difference is 33. The find n

$r = 0.8$

$d^2 = 33$

$n = ?$

$$r = 1 - \frac{6\Sigma d^2}{n(n^2-1)}$$

$$\Rightarrow 0.8 = 1 - \frac{6 \times 33}{n^3 - n} \qquad 1 - \frac{6 \times 33}{n(n^2-1)}$$

$$\Rightarrow (0.8)(n(n^2-1)) = -1$$

$$\Rightarrow 0.8 = 1 - \frac{198}{n(n^2-1)}$$

$$\Rightarrow 0.8 \times n(n^2-1) = n(n^2-1) - 198$$

$$\Rightarrow 0.8 \times n(n^2-1) - n(n^2-1) = 0 - 198$$

$$\Rightarrow n(n^2-1)[0.8 - 1) = -198$$

$$\Rightarrow n(n^2-1) = \frac{-198}{-0.2}$$

$$\Rightarrow n(n^2-1) = 990$$

$$\Rightarrow n(n+1)(n-1) = 990$$

$$\Rightarrow (n+1)(n^2 - n + 1) = 990$$

$$\Rightarrow n^3 - n - 990 = 0$$

$$\Rightarrow n^3 - 100n + 99n - 990 = 0$$

$$\Rightarrow n(n^2 - 100) + 99(n - 10) = 0$$

$$\Rightarrow n(n-10)(n+10) + 99(n-10) = 0$$

$$\Rightarrow (n-10)[n(n+10) + 99] = 0$$

$$\Rightarrow n - 10 = 0 \text{ or } n(n+10) + 99 = 0$$

$$\Rightarrow n = 10 \text{ or } n^2 + 10n + 99 = 0$$

$$n = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-10 \pm \sqrt{100 - 396}}{2}$$

$$= \frac{-10 \pm \sqrt{-296}}{2}$$

Q. $r = 0.143$, $\sum d^2 = 48$

$$r = 1 - \frac{6\sum d^2}{n(n^2-1)}$$

$$\Rightarrow 0.143 = 1 - \frac{6 \times 48}{n(n^2-1)}$$

$$\Rightarrow 0.143 = 1 - \frac{288}{n^3-n}$$

$$\Rightarrow 0.143(n^3-n) = (n^3-n) - 288$$

$$\Rightarrow 0.143(n^3-n) - (n^3-n) + 288 = 0$$

$$\Rightarrow n^3 - n(0.143 - 1) = -288$$

$$\Rightarrow n^3 - n = \frac{-288}{-0.857} = 336.05$$

$$\Rightarrow n(n^2-1) = 336.$$

$$\Rightarrow n^3 - n - 336 = 0$$

$$\Rightarrow n^3 - 49n + 48n - 336 = 0$$

$$\Rightarrow n(n^2-49) + 48(n-7) = 0$$

$$\Rightarrow n(n+7)(n-7) + 48(n-7) = 0$$

$$\Rightarrow (n-7)\left[n(n+7) + 48\right] = 0$$

$$\Rightarrow (n-7) = 0 \quad \text{or} \quad n(n+7) + 48 = 0$$

$$\Rightarrow n = 7$$

$$\Rightarrow n^2 + 7n + 48 = 0$$

$$\Rightarrow n = \frac{-7 \pm \sqrt{49 - 192}}{2}$$

$$= \frac{-7 \pm 11.95}{2}$$

$$= 7$$

As $n = 5, n^3 = 125$

$n = 6, n^3 = 216$

$n = 7, n^3 = 343$

Probable Error

Since we are finding the correlation between two variable based on sample of a population / data. So this shows that we are not taking the whole population / data. so there must be some error in our calculation and by the help of we can define a limit where the correlation coefficient lies, whatever sample will dreaw from the population, and that error is known as probable error.

$$P.E = \frac{2}{3}\left(\frac{1-r^2}{\sqrt{n}}\right) = 0.6745\left(\frac{1-r^2}{\sqrt{n}}\right)$$

and limit is defined as $r \pm P.E$

If $r = 0.36$ and $n = 10$, find out P.E & the range of the Correlation Coeffice

$$P.E = \frac{2}{3}\frac{1-r^2}{\sqrt{n}}$$

$$= \frac{2}{3}\frac{1-(0.36)^2}{\sqrt{10}}$$

$$= \frac{2}{3}\frac{1-0.1296}{3.16}$$

$$= \frac{2}{3} \times \frac{0.87}{3.16}$$

$$= 0.66 \times \frac{0.87}{3.16}$$

$$= \frac{0.5742}{3.16} = 0.18$$

limit $= r \pm PE$

$= 0.36 + 0.18$ and $0.36 - 0.18$

$= 0.54$ and 0.18

This is called range of the population correlation coefficient.

Standard Error

$$\frac{1-r^2}{\sqrt{n}}$$

The value of r is significant or reliable if $r > 6 \times P.E$ and insignificant $r < P.E$

r is significant means the relationship bet^n both the variables are reliable or realistic or significant.

And the value of r is insignificant means, the relnship bet^n both the variable are not reliable.

Assignment

Q.31 (a)

Given $\Sigma X = 125$, $\Sigma Y = 100$

$\Sigma X^2 = 650$, $\Sigma Y^2 = 436$

$\Sigma XY = 520$, $n = 25$

Thus $r(X,Y) = \dfrac{n\Sigma XY - \Sigma X \cdot \Sigma Y}{\sqrt{n\Sigma X^2 - (\Sigma X)^2 (n\Sigma Y^2 - (\Sigma Y)^2}}$

$= \dfrac{25 \times 520 - 125 \times 100}{\sqrt{25 \times 650 - (125)^2 \times (25 \times 436 - (100)^2)}}$

$= \dfrac{13000 - 12500}{\sqrt{(16250 - 15625)(10900 - 10000)}}$

$= \dfrac{500}{\sqrt{625 \times 900}} = \dfrac{500}{25 \times 30} = \dfrac{2}{3} = 0.67$

(b) Given $n = 9$, $\Sigma X = 45$, $\Sigma X^2 = 285$, $\Sigma Y = 108$, $\Sigma Y^2 = 1356$, $\Sigma XY = 597$

Hence $r(X,Y) = \dfrac{n\Sigma XY - \Sigma X \cdot \Sigma Y}{\sqrt{[n\Sigma X^2 - (\Sigma X)^2][n\Sigma Y^2 - (\Sigma Y)^2]}}$

$= \dfrac{9 \times 597 - 45 \times 108}{\sqrt{(9 \times 285 - 2025)(9 \times 1356 - 11664)}}$

$= \dfrac{5373 - 4860}{\sqrt{(2565 - 2025)(12204 - 11664)}}$

$= \dfrac{513}{\sqrt{540 \times 540}}$

$= \dfrac{513}{540}$

$= 0.95$

Q.32

Given $\bar{x} = 5.5$ $\bar{y} = 4$

$\Sigma x^2 = 385$, $\Sigma y^2 = 192$

$\Sigma (x+y)^2 = 947$, $n = 10$

→ As $\Sigma (x+y)^2 = 947$

$\Rightarrow \Sigma x^2 + \Sigma y^2 + 2\Sigma xy = 947$

$\Rightarrow 385 + 192 + 2\Sigma xy = 947$

$\Rightarrow 577 + 2\Sigma xy = 947$

$\Rightarrow 2\Sigma xy = 947 - 577$

$\Rightarrow \Sigma xy = \frac{370}{2} = 185$

~~Now $r(x,y) =$~~

Again, $n = 10$, $\bar{x} = 5.5$

as we know $\bar{x} = \frac{\Sigma x}{n}$

$\Rightarrow \Sigma x = n\bar{x}$

$= 10 \times 5.5$

$= 55$

Similarly, $\Sigma y = n\bar{y}$

$= 10 \times 4$

$= 40$

Hence $r(x,y) = \frac{n\Sigma xy - \Sigma x \Sigma y}{\sqrt{n\Sigma x^2 - (\Sigma x)^2}\ \sqrt{n\Sigma y^2 - (\Sigma y)^2}}$

$= \frac{10 \times 185 - 40 \times 55}{\sqrt{(10 \times 385 - 3025)(10 \times 192 - 1600)}}$

$= \frac{1850 - 2200}{\sqrt{(3850 - 3025)(1920 - 1600)}}$

$= \frac{-350}{\sqrt{825 \times 320}}$

$= \frac{-350}{\sqrt{264000}}$

$= \frac{-350}{513.8}$

$= -0.681$

Q.33

→ Given $N=30$, $\Sigma X=120$, $\Sigma X^2=600$, $\Sigma Y=90$, $\Sigma Y^2=250$, $\Sigma XY=356$

Two pairs of observation, which is (8,10) & (12,7) wrongly taken, while the ~~right~~ correct pair is (8,12) & (10,8).

→ As we know Correct sum = Wrong Sum − Wrongly taken Obs. + Correct Obs.

Thus we need to find out correct sum for all the data given.

Now

Correct $\Sigma X = 120 - 8 - 12 + 8 + 10 = 118$

Correct $\Sigma Y = 90 - 10 - 7 + 12 + 8 = 93$

Correct $\Sigma X^2 = 600 - 64 - 144 + 64 + 100 = 556$

Correct $\Sigma Y^2 = 250 - 100 - 49 + 144 + 64 = 309$

Correct $\Sigma XY = 356 - (8\times10) - (12\times7) + (8\times12) + (10\times8)$

$= 356 - 80 - 84 + 96 + 80$

$= 368$

Thus correct $r(X,Y) = \dfrac{n\Sigma XY - \Sigma X \Sigma Y}{\sqrt{n\Sigma X^2 - (\Sigma X)^2} \times n\Sigma Y^2 - (\Sigma Y)^2}$

$= \dfrac{30\times 368 - 118\times 93}{\left[(30\times 556 - (118)^2)(30\times 309 - (93)^2)\right]^{1/2}}$

$= \dfrac{11040 - 10974}{\left[(16680 - 13924)(9270 - 8649)\right]^{1/2}}$

$= \dfrac{66}{(2756 \times 621)^{1/2}}$

$= \dfrac{66}{(1711476)^{1/2}}$

$= \dfrac{66}{1308.2}$

$= 0.0504$

Q.34

Given $n = 20$, $r_{xy} = 0.3$

$\bar{X} = 15$, $\bar{Y} = 20$

$\sigma_x = 4$, $\sigma_y = 5$

At the time of calculations one pair $(x=27, y=30)$ was wrongly taken as $(x=17, y=35)$

Now $\Sigma X = n \cdot \bar{X} = 20 \times 15 = 300$

$\Sigma Y = n \cdot \bar{Y} = 20 \times 20 = 400$

Again, we know ~~$\sigma_x = \frac{\Sigma X - \bar{X}}{n}$~~

$\sigma_x^2 = \frac{\Sigma X^2}{n} - (\bar{X})^2$

$\Rightarrow 16 = \frac{\Sigma X^2}{20} - (15)^2$

$\Rightarrow \frac{\Sigma X^2}{20} = 16 + 225 = 241$

$\Rightarrow \Sigma X^2 = 4820$

~~$\sigma_x^2 = \frac{\Sigma X^2}{n} - (\bar{X})^2$~~

~~$\Rightarrow n\sigma_x^2 = \Sigma X^2 - (\bar{X})^2$~~

~~$\Rightarrow \Sigma X^2 = n\sigma_x^2 - (\bar{X})^2$~~

~~$\Rightarrow \Sigma X^2 = 20 \times 16 - 225$~~

~~$= 320 - 225$~~

~~$= 95$~~

Similarly $\Sigma Y^2 = n[(\bar{Y})^2 + \sigma_y^2]$

$= 20[25 + 400]$

$= 20 \times 425$

$= 8500$

~~Similarly $\Sigma Y^2 = n\sigma_y^2 - (\bar{Y})^2$~~

~~$= 20 \times 25 - 400$~~

~~$= 500 - 400$~~

~~$= 100$~~

~~$\Sigma(X-\bar{X})(Y-\bar{Y})$~~

~~n~~ $\Sigma XY =$

~~$\frac{\Sigma XY}{n} =$~~

Now we need to find ~~all~~ correct sum for all the data

Correct $\Sigma X = 300 - 17 + 27 =$ ~~290~~ 310

Correct $\Sigma Y = 400 - 35 + 30 =$ ~~405~~ 395

Correct $\Sigma X^2 =$ ~~485~~ $4820 - 17^2 + 27^2$

$= 4820 - 289 + 729 = 5260$

Correct $\Sigma Y^2 = 8500 - 35^2 + 30^2 = 8500 - 1225 + 900 = 8175$

Correct $\Sigma XY = 6120 - 17 \times 35 + 27 \times 30 = 120 - 595 + 810 = 6335$

Again $r = \frac{cov(x,y)}{\sigma_x \cdot \sigma_y}$

$\Rightarrow 0.3 = \frac{cov(x,y)}{4 \times 5} = \frac{cov\,xy}{20}$

$\Rightarrow cov(x,y) = 6$

$\Rightarrow \frac{\Sigma XY}{n} = 6120$

~~$\Rightarrow \Sigma XY = 6 \times 20 = 120$~~

~~$\Sigma(XY - \bar{X}Y - \bar{Y}X + \bar{X}\bar{Y})$~~

~~$\bar{X}\bar{Y}$~~

Hence $r(x,y) = \frac{n\Sigma XY - \Sigma X \Sigma Y}{\left[n\Sigma X^2 - (\Sigma X)^2 \times n\Sigma Y^2 - (\Sigma Y)^2\right]^{1/2}}$

$= \frac{20 \times 6335 - 310 \times 395}{\left\{[20 \times 5260 - (310)^2][20 \times 8175 - (395)^2]\right\}^{1/2}}$

$= \frac{126700 - 122450}{\left[(105200 - 96100)(163500 - 156025)\right]^{1/2}}$

$= \frac{4250}{(9100 \times 7475)^{1/2}}$

$= \frac{4250}{(68022500)^{1/2}}$

$= \frac{4250}{8247.57}$

$= 0.515$

Q.34

Given $n = 20$, $r(x,y) = 0.3$, $\bar{x} = 15$, $\bar{y} = 20$, $\sigma_x = 4$ & $\sigma_y = 5$

At the time of calculations one pair $(x=27, y=30)$ was wrongly taken as $(x=17, y=35)$

Now $\Sigma X = n\bar{x} = 20 \times 15 = 300$

$\Sigma Y = n \cdot \bar{y} = 20 \times 20 = 400$

Again, we know $\sigma_x^2 = \frac{\Sigma x^2}{n} - (\bar{x})^2$

$\Rightarrow 16 = \frac{\Sigma x^2}{20} - (15)^2$

$\Rightarrow 16 = \frac{\Sigma x^2}{20} - 225$

$\Rightarrow \Sigma x^2 = 20(16+225)$

$= 4820$

and $\sigma_y^2 = \frac{\Sigma y^2}{n} - (\bar{y})^2$

$\Rightarrow 25 = \frac{\Sigma y^2}{20} - 400$

$\Rightarrow \Sigma y^2 = 20(25+400)$

$= 8500$

And $r = 0.3$ (given)

$\Rightarrow \frac{cov(x,y)}{\sigma_x \sigma_y} = 0.3$

$\Rightarrow \frac{cov(x,y)}{4 \times 5} = 0.3$

$\Rightarrow cov(x,y) = 0.3 \times 20 = 6$

Again $cov(x,y) = 6$

$\Rightarrow \frac{\Sigma(x-\bar{x})(y-\bar{y})}{n} = 6$

$\Rightarrow \Sigma(xy - x\bar{y} - \bar{x}y + \bar{x}\bar{y}) = 120$

$\Rightarrow \Sigma xy - \bar{y}\Sigma x - \bar{x}\Sigma y + n\bar{x}\bar{y} = 120$

$\Rightarrow \Sigma xy = 20 \times 300 - 15 \times 400 + 20 \times 300 = 120$

$\Rightarrow \Sigma xy - 6000 - 6000 + 6000 = 120$

$\Rightarrow \Sigma xy = 6120$

Therefore the correct $r(x,y)$

$$= \frac{n\Sigma xy - \Sigma x \Sigma y}{\left[n\Sigma x^2 - (\Sigma x)^2 \times n\Sigma y^2 - (\Sigma y)^2\right]^{1/2}}$$

$$= \frac{20 \times 6335 - 310 \times 395}{\left\{\left[20 \times 5260 - (310)^2\right]\left[20 \times 8175 - (395)^2\right]\right\}^{1/2}}$$

$$= \frac{126700 - 122450}{\left[(105200 - 96100)(163500 - 156025)\right]^{1/2}}$$

$$= \frac{4250}{(9100 \times 7475)^{1/2}}$$

$$= \frac{4250}{(68022500)^{1/2}}$$

$$= \frac{4250}{8247.57} = 0.515$$

Now we need to find all the correct sum of the data.

Correct $\Sigma X = 300 - 17 + 27 = 310$

Correct $\Sigma Y = 400 - 35 + 30 = 395$

Correct $\Sigma x^2 = 4820 - 17^2 + 27^2 = 4820 - 289 + 729 = 5260$

Correct $\Sigma y^2 = 8500 - 35^2 + 30^2 = 8500 - 1225 + 900 = 8175$

Correct $\Sigma xy = 6120 - 17 \times 35 + 27 \times 30 = 6120 - 595 + 810 = 6335$

Q.35

Given $n = 12$, $\Sigma XY = 334$

$\Sigma X = 30$, $\Sigma Y = 5$

$\Sigma X^2 = 670$, $\Sigma Y^2 = 285$

A pair $(X=11, Y=4)$ was copied wrongly, correct pair is $(X=10, Y=14)$

→ We need to find out correct sum of all the data first.

Correct $\Sigma X = 30 - 11 + 10 = 29$

Correct $\Sigma Y = 5 - 4 + 14 = 15$

Correct $\Sigma X^2 = 670 - 121 + 100 = 649$

Correct $\Sigma Y^2 = 285 - 16 + 196 = 465$

Correct $\Sigma XY = 334 - 44 + 140 = 430$

Thus Correct $r(X,Y) = \dfrac{n\Sigma XY - \Sigma X \Sigma Y}{\left[n\Sigma X^2 - (\Sigma X)^2 \times n\Sigma Y^2 - (\Sigma Y)^2\right]^{1/2}}$

$= \dfrac{12 \times 430 - 15 \times 29}{\left[(12 \times 649 - 29^2) \times (12 \times 465 - 15^2)\right]^{1/2}}$

$= \dfrac{5160 - 435}{\left[(7788 - 841)(5580 - 225)\right]^{1/2}}$

$= \dfrac{4715}{(6947 \times 5355)^{1/2}}$

$= \dfrac{4715}{(37201185)^{1/2}}$

$= \dfrac{4715}{6099.27}$

$= 0.77$

Q.37

X	Y	U	V	UV	U^2	V^2
7	18	2	6	12	4	36
6	16	1	4	4	1	16
(5)	14	0	2	0	0	4
4	(12)	-1	0	0	1	0
3	10	-2	-2	4	4	4
2	6	-3	-6	18	9	36
1	8	-4	-4	16	16	16
		-7	0	54	35	112

Now $r(X,Y) = \dfrac{n\Sigma UV - \Sigma U.\Sigma V}{\left\{[n\Sigma U^2-(\Sigma U)^2][n\Sigma V^2-(\Sigma V)^2]\right\}^{1/2}}$

$= \dfrac{7\times 54 - 0}{[(7\times(-7)-49)(7\times 112-0)]^{1/2}}$

$= \dfrac{378}{(-98\times 784)^{1/2}}$

$=$

Q.37

X	Y	$x-\bar{x}$	$y-\bar{y}$	dxdy	$(x-\bar{x})^2$	$(y-\bar{y})^2$
7	18	3	6	18	9	36
6	16	2	4	8	4	16
5	14	1	2	2	1	4
4	12	0	0	0	0	0
3	10	-1	-2	2	1	4
2	6	-2	-6	12	4	36
1	8	-3	-4	12	9	16
				54	28	112

$\bar{X} = \dfrac{\Sigma X}{n} = \dfrac{28}{7} = 4$

$\bar{Y} = \dfrac{\Sigma Y}{n} = \dfrac{84}{7} = 12$

Thus $r = \dfrac{\Sigma dx.dy}{(\Sigma dx^2.\Sigma dy^2)^{1/2}}$

$= \dfrac{54}{(28\times 112)^{1/2}}$

$= \dfrac{54}{(3136)^{1/2}}$

$= \dfrac{54}{56}$

$= 0.964$

And P.E $= 0.6745\left(\dfrac{1-r^2}{\sqrt{n}}\right)$

$= 0.6745\times\dfrac{1-.93}{\sqrt{7}}$

$= \dfrac{0.6745\times 0.07}{2.645}$

$= \dfrac{0.047}{2.645} = 0.0178$

Q.38

X	Y	$x-\bar{x}$	$y-\bar{y}$	$dxdy$	$(x-\bar{x})^2$	$(y-\bar{y})^2$	R_x	R_y	$D=R_x-R_y$	D^2
15	$\frac{200}{250}\times100=80$	-2	40	-80	4	1600	6	1	5	25
16	$\frac{150}{200}\times100=75$	-1	35	-35	1	1225	5	2	3	9
(17)	$\frac{90}{150}\times100=60$	0	20	0	0	400	4	3	1	1
18	$\frac{48}{120}\times100=$ (40)	1	0	0	1	0	3	4	-1	1
19	$\frac{30}{100}\times100=30$	2	-10	-20	4	100	2	5	-3	9
20	$\frac{12}{80}\times100=15$	3	-25	-75	9	625	1	6	-5	25
		3	-60	-210	19	3950				$\Sigma D^2=70$

$$r=\frac{\Sigma dxdy}{\sqrt{\Sigma dx^2.\Sigma dy^2}}=\frac{n\Sigma UV-\Sigma U.\Sigma V}{\left[n\Sigma U^2-(\Sigma U)^2 \times n\Sigma V^2-(\Sigma V)^2\right]^{1/2}}$$

$$=\frac{-210}{\sqrt{19\times3950}}=\frac{6\times(-210)-180}{\left[(6\times19-9)(6\times3950-3600)\right]^{1/2}}$$

$$=\frac{-210}{\sqrt{75050}}=\frac{-1260-180}{(105-20100)^{1/2}}=\frac{-1440}{}$$

$$=\frac{-210}{273.95}$$

$$=$$

$$R_{xy}=1-\frac{6\Sigma D^2}{6(6^2-1)}$$

$$=1-\frac{420}{210}$$

$$=1-2=(-1)$$

$$P.E=0.6745\left(\frac{1+1}{\sqrt{n}}\right)$$

$$=0.6745\times\frac{2}{}$$

Q.39

X	Y	U	V	UV	U^2	V^2
50	45	3	5	15	9	25
60	52	13	12	156	169	144
75	50	28	10	280	784	100
84	65	37	25	925	1369	625
(47)	(40)	0	0	0	0	0
52	65	5	25	125	25	625
59	50	12	10	120	144	100
44	60	-3	20	-60	9	400
33	32	-14	-8	112	196	64
46	51	-1	11	-11	1	121
		70	110	1662	2706	2204

$$r=\frac{n\Sigma UV-\Sigma U.\Sigma V}{\left[(n\Sigma U^2-(\Sigma U)^2)(n\Sigma V^2-(\Sigma V)^2)\right]^{1/2}}$$

$$=\frac{10\times1662-70\times110}{\left[(10\times2706-4900)(10\times2204-12100)\right]^{1/2}}$$

$$=\frac{16620-7700}{\left[(27060-4900)(22040-12100)\right]^{1/2}}$$

$$=\frac{8920}{(22160\times9940)^{1/2}}$$

Repeated Range correlation Coefficient

$$r = 1 - \frac{6\left(\Sigma d^2 + \Sigma \frac{m_i(m_i^2-1)}{12}\right)}{n(n^2-1)}$$, where m_i is the repeation of variable

X	Y	Rx	Ry	Rx-Ry	D^2
24	37	6	1	5	25
29	35	3	2	1	1
19	16	8.5	9.5	-1	1
14	26	10	4	6	36
30	23	1.5	5	-3.5	12.25
19	27	8.5	3	5.5	30.25
27	19	5	8	-3	9
30	20	1.5	7	5.5	30.25
20	16	7	9.5	-2.5	6.25
28	11	4	11	-7	49
11	21	11	6	5	25
					$\Sigma = 225$

$m_1 = 2$

$m_2 = 2$

$m_3 = 2$

$$r = 1 - \frac{6\left(225 + \frac{2(2^2-1)}{12} + \frac{2(2^2-1)}{12} + \frac{2(2^2-1)}{12}\right)}{11(11^2-1)}$$

$$= 1 - \frac{6(225 + 0.5 + 0.5 + 0.5)}{11(120)}$$

$$= 1 - \frac{6(225 + 1.5)}{1320}$$

$$= 1 - \frac{6 \times 226.5}{1320}$$

$$= 1 - \frac{1359}{1320}$$

$$= 1 - 1.029$$

$$= -0.029$$

Q.

X	Y	Rx	Ry	Rx-y	d^2
48	13	3	5.5	-2.5	6.25
33	13	5	5.5	-0.5	0.25
40	24	4	1	3	9
9	6	10	8.5	1.5	2.25
16	15	8	4	4	16
16	4	8	10	-2	4
65	20	1	2	-1	1
24	9	6	7	-1	1
16	6	8	8.5	-0.5	0.25
57	19	2	3	-1	1
					41

$m_1 = 3, m_2 = 2, m_3 = 2$

$$r = 1 - \frac{6\left(41 + \frac{3(3^2-1)}{12} + \frac{2(2^2-1)}{12} + \frac{2(2^2-1)}{12}\right)}{10(100-1)}$$

$$= 1 - \frac{6(41 + 2 + 0.5 + 0.5)}{990}$$

$$= 1 - \frac{6(44)}{990}$$

$$= 1 - \frac{264}{990}$$

$$= 1 - 0.266$$

$$= 0.734$$

P.E < 1

P.E > 6P.E

P.E < 1

$r = -1 \quad 0$

$P.E = .6745$

$r \pm P.I$

$-1 + 0$

Q.8 (i)

	R1	R2	R3	
	A	B	C	Supply
C1	1	2 [5]	7	4	5 → 0
C2	2	3 [2]	3 [8]	1	8 → 6 → 0
C3	3	5	4 [3]	7 [4]	7 → 4 → 0
C4	4	1	6	2 [14]	14
Demand	7 → 2 → 0	9 → 3	18 → 14		34

⇒

	S1	S2	S3	Supply
D1	2 [5]	7	4	5
D2	3 [2]	3 [6]	1	8
D3	5	4 [3]	7 [4]	7
D4	1	6	2 [14]	14
Demand	7	9	18	34

→ It is a balanced T.P as row sum = column sum = 34.

The given T.P has 12 cells

According to NWCM, 1st allocation made at the cell (1,1), with the allocation min (7,5) = 5. The 2nd allocation made at the cell (2,1), with the allocation min (2,8) = 2. We continue this process again and again until all transportation cost has been used. Finally the number of occupied cell is $m+n-1 = 4+3-1 = 6$.

The transportation cost according to above route

$= 2\times5 + 3\times2 + 3\times6 + 4\times3 + 7\times4 + 2\times14$

$= 102.$

(ii)

	S1	S2	S3	Supply
D1	2 [5]	7	4	5 → 0
D2	3	3	1 [8]	8 → 0
D3	5	4	7	7
D4	1	6	2 [10]	14 → 4
Demand	7 → 2	9	18 → 10 → 0	34

(ii)

	S1	S2	S3	Supply
D1	2	7 [2]	4 [3]	5 → 2
D2	3	3	1 [8]	8 → 0
D3	5	4 [7]	7	7 → 0
D4	1 [7]	6	2 [7]	14 → 7 → 0
Demand	7 → 0	9 → 2	18 → 10 → 3	34

⇒

	S1	S2	S3	Supply
D1	2	7 [2]	4 [3]	5
D2	3	3	1 [8]	8
D3	5	4 [7]	7	7
D4	1 [7]	6	2 [7]	14
Demand	7	9	18	34

→ According to LCM, we search for an element having the least cost. Here we observe the cell (2,3) having the least cost, so we allocate their min (18,8) = 8 and cross the 2nd column. Then the next allocation is at the cell (4,1) with allocation min (7,14) = 7 and cross the 1st row. We continue this process until all transportation has been used. Finally the no. of occupied cell is m+n-1 = 6

The T.P cost according to this route = $7\times2 + 4\times3 + 1\times8 + 4\times7 + 1\times7 + 2\times7$

$= 83.$

(iii)

→ The largest penalty associated with 1st and 2nd row. So we choose 1st row and allocate min(7,5) = 5 to the cell(1,1) having the minimum cost and cross-up the 1st row.

The new revise T.P. as follows.

		S_1	S_2	S_3	Supply
R_1	D_1	2 [5]	7	4	~~7~~ 2 (2) ←
R_2	D_2	3	3	1	8 (2)
R_3	D_3	5	4	7	7 (1)
R_4	D_4	1	6	2	14 (1)
Demand		~~7~~ 2	9	18	
		(1)	(1)	(1)	

The largest penalty associated with 1st ~~and~~ row and 1st column, So we choose 1st row and allocate min(~~8~~,18) = ~~8~~ to the cell (1,3) having the minimum cost and cross up the 1st row.

		S_1	S_2	S_3	Supply
R_1	D_2	3	3	1 [18]	~~8~~ 0 (2) ←
R_2	D_3	5	4	7	7 (1)
R_3	D_4	1	6	2	14 (1)
Demand		2	9	~~18~~ 10	
		(2)	(1)	(1)	

The new revise T.P. as follows.

The largest penalty (5) associated with 3rd column. So we allocate min(10,4) = 4 to the cell (2,3) having the minimum cost and cross up the 2nd row.

		S_1	S_2	S_3	Supply
R_1	D_3	5	4	7 [10]	7 (1)
R_2	D_4	1	6	2	~~14~~ 10 (1)
Demand		2	9	~~10~~ 6	
		(4)	(2)	(5) ↑	

The new revise T.P as follows.

The largest penalty (7) associated with 3rd column. So we allocate min (6,7) = 6 to the cell and cross up the 3rd column.

		S_1	S_2	S_3	Supply
R_1	D_1	5	4	~~7~~ 1 [6]	7 (1)
Demand		~~2~~ 5	9	~~6~~ 0	
				7 ↑	

5	4		7 (1)
1 [2]	6		~~14~~ 12 (5) ←
2 0	9		
(4)	(2)		

The new revise T.P. as follows

The largest penalty (5) associated with 1st column. So we allocate min (2,7) = 2 to the cell and cross up the 1st column

		S_1	S_2	Supply
R_1	D_1	5	4	7 (1)
Demand		~~2~~ 5 ↑	9	

Find out the penalty of each row and each column i.e. the difference between smallest cost and next to smallest cost

identify the highest penalty among the penalties and start allocation in the corresponding smallest cost of respective rows or column and continue this process up to demand fullfilled and supply satisfaction

Q.8 Assignments

	D_1	D_2	D_3	Supply
O_1	2 [5]	7	4	~~5~~ 0
O_2	3	3 [2]	1 [6]	~~8~~ 2
O_3	5 [7]	4	7	7
O_4	1 [2]	6	2 [12]	~~14~~ ~~12~~ 0
Demand	~~7~~ ~~2~~ 0	9	~~18~~ ~~6~~ 0	

	P_1	P_2	P_3	P_4
~~5~~ 0	(2)	–	–	–
~~8~~ 2	(2)	(2)	(2)	(2)
7	(1)	(1)	(3)	(3)
~~14~~ ~~12~~ 0	(1)	(1)	(4) ←	–
P_1	(1)	(1)	(1)	
P_2	(2)↑	(1)	(1)	
P_3	–	(1)	(1)	
P_4	–	(1)	(6)↑	

The T.P cost = $2\times5 + 3\times2 + 1\times6 + 4\times7 + 1\times2 + 2\times12$

= 76

Q.9

	D_1	D_2	D_3	D_4	Sup
O_1	5 [10]	4	2 [10]	6	~~20~~ ~~10~~ 0
O_2	8	8 [30]	5	7	~~30~~ 0
O_3	5	9 [10]	4 [10]	6 [30]	~~50~~ 40
Demand	~~10~~ 0	~~40~~ ~~10~~ 0	~~20~~ ~~10~~ 0	30	

	P_1	P_2	P_3	P_4
~~20~~ ~~10~~ 0	(2)	(2)	–	–
~~30~~ 0	(2)	(2)	(2)	–
~~50~~ 40	(1)	(2)	(2)	(2)
P_1	(3)↑	(1)	(2)	(1)
P_2	–	(1)	(2)↑	(1)
P_3	–	(6)↑	(1)	(1)
P_4	–	(9)↑	(4)	(6)

∴ The T.P. cost = $5\times10 + 2\times10 + 3\times30 + 9\times10 + 4\times10 + 6\times30$

= 470

Q.10

	D_1	D_2	D_3	P_1	P_2
O_1	2 [10]	2	3	10	(1) (0)
O_2	4	1 [15]	2	~~15~~ 0	(1) (3) ←
O_3	1 [10]	3	1 [30]	~~40~~ 10	(2) (2) ←
	20	~~15~~ 0	~~30~~ 0		
P_1	(1)	(1)	(1)		
P_2	(1)	(1)	—		

The T.P cost = $2\times10 + 1\times15 + 1\times10 + 1\times30$

$= 20 + 15 + 10 + 30$

$= 75$

	D_1	D_2	D_3	D_4	P_1	P_2	P_3
O_1	5 [10]	4	2 [10]	6	~~10~~ ~~20~~ 0	(2)	(2) —
O_2	8	3 [30]	5	7	~~30~~ 0	(2)	(2) (2)
O_3	5	9 [10]	4 [10]	6 [30]	50	(2)	(2) (2)
	~~10~~ 0	~~40~~ 10	~~20~~ 10	30			
P_1	(3)↑	(1)	(2)	(1)			
P_2	—	(1)	(2)	(1)			
P_3	—	(6)↑	(1)	(1)			

T.P. Cost = $5\times10 + 2\times10 + 3\times30 + 9\times10 + 4\times10 + 6\times30$

$= 50 + 20 + 90 + 90 + 40 + 180$

$=$

Q.10

	D_1	D_2	D_3	P_1	P_2	P_3
O_1	2	2 [10]	3	~~10~~ 0	(1)	(1) (1) ←
O_2	4	1 [5]	2 [10]	15	(1)	(1) (1)
O_3	1 [20]	3	1 [20]	~~40~~ ~~20~~ 0	(2) ←	(2) ←
	~~20~~ 0	~~15~~ 5	30 10			
P_1	(1)	(1)	(1)			
P_2	—	(1)	(1)			
P_3	—	(1)	(1)			

The T.P cost = $2\times10 + 1\times5 + 2\times10 + 1\times20 + 1\times20$

$= 20 + 5 + 20 + 20 + 20$

$= 85$

	D_1	D_2	D_3	D_4	P_1	P_2	P_3	P_4
O_1	5	4 [10]	2 [10]	6	~~20~~ ~~10~~ 0	(2)	(2)	(2) (4) ←
O_2	8	3 [30]	5	7	~~30~~ 0	(2)	(2) ←	— —
O_3	5 [10]	9	4 [10]	6 [30]	~~40~~ ~~50~~	(1)	(2)	(2) (2)
	~~10~~ 0	~~40~~ ~~10~~ 0	~~20~~ 10	30				
P_1	(3)↑	(1)	(2)	(1)				
P_2	—	(1)	(2)	(1)				
P_3	—	(5)↑	(2)	(0)				
P_4	—	—	(2)	(0)				

	D_1	D_2	D_3	D_4	P_1	P_2	P_3
O_1	5	4	2 [20]	6	~~20~~ 0	(2)	(2) —
O_2	8	3 [30]	5	7	~~30~~ 0	(2)	(2) (4) ←
O_3	5 [10]	9 [10]	4	6 [30]	~~50~~ ~~40~~	(2)	(2) (3)
	~~10~~ 0	~~40~~ 10	~~20~~ 0	30			
P_1	(3)↑	(1)	(2)	(1)			
P_2	—	(1)	(2)↑	(1)			
P_3	—	(6)↑	—	(1)			

T.P cost = $2\times20 + 3\times30 + 5\times10 + 9\times10 + 6\times30$

$= 450$

∴ T.P cost = $4\times10 + 2\times10 + 3\times30 + 5\times10 + 4\times10 + 6\times30$

$=$

Unbalanced T.P

A T.P. is said to be unbalanced, when sum of demand is not equal to supply. In this situation we have to make it balance by adding a dummy row or column as per the requirement, with proper supply and demand.

The entries of dummy row or dummy column are 0.

	D_1	D_2	D_3	Supply
O_1	2	3	4	70
O_2	5	6	7	50
demand	0	0	0	10
	20	30	80	

sum of supply = 120
" of demand = 130

	D_1	D_2	D_3	
O_1	2	3	4	70
O_2	5	6	7	50
O_3	0	0	0	10
	20	30	80	

→

	D_1	D_2	D_3	
O_1	2	1	0	30
O_2	3	2	0	40
O_3	4	3	0	70
	50	60	80	

	D_1	D_2	D_3
O_1	2	2	3
O_2	4	1	2
O_3	1	3	1

Amit
Amit

Initial Basic Feasible Solution

The Sol^n of a ~~trans~~ TP using NWCR, LCM or VAM is knows as IBFS.

optimal Sol^n

Optimal sol of a TP is is the best Sol^n

Step-1 Optimal Sol^n is solved by MODI method (Modified Distributed Method)

Step-2 First find out the Sol^n of the T.P using anyone of the IBFS method.

Step-3 check whether the Sol^n is degenerate or non degenerate.

(i) A IBFS is said to be degenerate or degeneracy if the no. of allocation is less than $m+n-1$)

where m is no. of row

and n is no. of column.

(ii) An IBFS is said to be non-degenerate if it is equal to $m+n-1$)

Step-4 The MODI method is applicable when a TP is non-degenerate.

	D_1	D_2	D_3	D_4	Supply
O_1	21	16	25	13 (11)	11
O_2	17 (6)	18	14 (3)	23 (4)	13
O_3	32	17 (10)	18 (9)	41	19
Dem	6	10	12	15	

$\Delta ij = c_{ij} - (u_i + v_j)$

Allocated cell

$c_{14} = u_1 + v_4 = 13$

$c_{21} = u_2 + v_1 = 17$

$c_{23} = u_2 + v_3 = 14$

$c_{24} = u_2 + v_4 = 23$

$c_{32} = u_3 + v_2 = 17$

$c_{33} = u_3 + v_3 = 18$

Non allocated cell

c_{11} $\Delta_{11} = c_{11} - (u_1 + v_1)$
$= 21 - (-10 + 17) = 14$

c_{12} $\Delta_{12} = c_{12} - (u_1 + v_2)$ > 0
$= 16 - (-10 + 13) = 13$

c_{13} $\Delta_{13} = c_{13} - (u_1 + v_3)$ > 0
$= 25 - (-10 + 14) = 21$ > 0

c_{22} $\Delta_{22} = 18 - (0 + 13) = 5$

c_{31} $\Delta_{31} = 32 - (4 + 17) = 11$ > 0

c_{34} $\Delta_{34} = 41 - (4 + 23)$
$= 14 > 0$

Define a eq^n $c_{ij} = u_i + v_j$, where u_i represent row
v_j " column.

Let $u_2 = 0$, so $v_4 = 13$, $v_1 = 17$, $v_3 = 14$, $v_4 = 23$, $v_2 = 17$, $v_3 = 18$

$u_1 = -10$, $u_2 = 0$, $u_3 = 4$

$v_1 = 17$, $v_2 = 13$, $v_3 = 14$, $v_4 = 23$

To solve u_i & v_j of the allocated cell take anyone of the variable as 0, where no. of allocation is more.

when all Δij are +ve over Sol^n is optimal

T.P cost = $11 \times 13 + 17 \times 6 + 14 \times 3 + 23 \times 4 + 17 \times 10 + 18 \times 9$

Solve the T.P using MODI method.

8.

	D_1	D_2	D_3	D_4	Supply
O_1	2	2	2	1	3
O_2	10	8	5	4	7
O_3	7	6	6	8	5
De	4	3	4	4	

→

	D_1	D_2	D_3	D_4	Supply	P_1	P_2	P_3
O_1	2 [3]	2	2	1	3	(1)	–	–
O_2	10	8	5 [3]	4 [4]	7	(1)	(1)	(3) ←
O_3	7 [1]	6 [3]	6 [1]	8	5	(1)	(1)	(1)
Demand	4	3	4	4				

P_1 (5)↑ (4) (4) (3)

P_2 (3) (2) (1) (4)↑

P_3 (3) (2) (1) –

The T.P. Cost = $2\times3 + 5\times3 + 4\times4 + 7\times1 + 6\times3 + 6\times1$

= 68

Here $m = 3$ and $n = 4$, so $m+n-1 = 6$

and also no. of occupied cell is 6

Thus this sol^n is non-degenerate.

As the 3rd contain more no. of occupied cell, thus let $U_3 = 0$

Allocated cell

$c_{11} = U_1 + V_1 = 2$, $U_1 = -5$

$c_{23} = U_2 + V_3 = 5$, $U_2 = -1$

$c_{24} = U_2 + V_4 = 4$, $V_4 = 5$

$c_{31} = U_3 + V_1 = 7$, $V_1 = 7$

$c_{32} = U_3 + V_2 = 6$, $V_2 = 6$

$c_{33} = U_3 + V_3 = 6$, $V_3 = 6$

Non Allocated cell

~~$c_{21} = \Delta_{21} = c_{21} - (U_2+V_1) = 2-(-1+7) =$~~

$c_{12} = \Delta_{12} = c_{12} - (U_1+V_2) = 2-(-5+6) = 1 > 0$

$c_{13} = \Delta_{13} = c_{13} - (U_1+V_3) = 2-(-5+6) = 1 > 0$

$c_{14} = \Delta_{14} = c_{14} - (U_1+V_4) = 1-(-5+5) = 1 > 0$

$c_{21} = \Delta_{21} = c_{21} - (U_2+V_1) = 10-(-1+7) = 4 > 0$

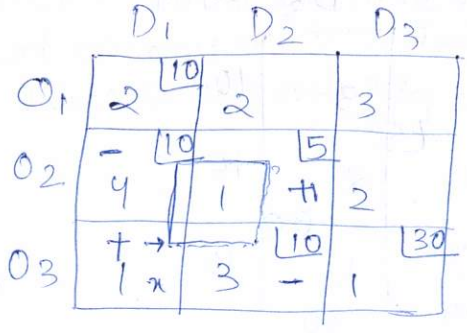
$c_{22} = \Delta_{22} = c_{22} - (U_2+V_2) = 8-(-1+6) = 3 > 0$

$c_{34} = \Delta_{34} = c_{34} - (U_3+V_4) = 8-(0+5) = 3 > 0$

∴ Since all $\Delta_{ij} > 0$, the ~~solution is optimum~~ T.P. has been reached the optimum solution is $x_{11}=3$, $x_{23}=3$, $x_{24}=4$, $x_{31}=1$, $x_{32}=3$, $x_{33}=1$ and the T.P. cost = 68 respectively

Q.

	D_1	D_2	D_3	Supply
O_1	2 (10)	2	3	~~10~~ 0
O_2	4 (10)	1 (5)	2	~~15~~ 5 0
O_3	1	3 (10)	1 (30)	40
Demand	~~20~~ ~~10~~ 0	~~15~~ 10	30	



T.P cost = 20 + 40 + 5 + 30 + 30

= 125

~~Allocated cell~~ This soln is non-degenerency

As u_2 & v_3 contains maximum occupied cell, we choose u_2 and let $u_2 = 0$

Allocated cell

$c_{11} = u_1 + v_1 = 2, u_1 = -2$

$c_{21} = u_2 + v_1 = 4, v_1 = 4$

$c_{22} = u_2 + v_2 = 1, v_2 = 1$

$c_{32} = u_3 + v_2 = 3, u_3 = 2$

$c_{33} = u_3 + v_3 = 1, v_3 = -1$

Non Allocated cell

$c_{12}, A_{12} = c_{12} - (u_1 + v_2) = 2 - (-2 + 1) = 3 > 0$

$c_{13}, A_{13} = c_{13} - (u_1 + v_3) = 3 - (-2 - 1) = 6 > 0$

$c_{23}, A_{23} = c_{23} - (u_2 + v_3) = 2 - (0 + -1) = 3 > 0$

$c_{31}, A_{31} = c_{31} - (u_3 + v_1) = 1 - (2 + 4) = -5 < 0$

As we know that when $A_{ij} \geq 0$, our soln is optimal, but in this case $A_{31} < 0$, so the soln is not optimal.

Loop

Find out the highest (-ve) from Aij and start a loop, the loop moves horizontally and vertically where the corner of the loop must be an allocated cell and again it will comeback to its initial cell for when it has started

Assign (+ve) and (-ve) ~~sign~~ sign in the corner of the loop, start with +ve sign

Define a parameter θ = ~~max (negative~~ min{Allocated -ve cell} =

θ = min{10, 10} = 10

Add θ to the +ve cell and subtract it from the -ve cell.

	D_1	D_2	D_3	Supply
O_1	[10] 2	2	3	10
O_2	4	[15] 1	2	15
O_3	[10] 1	3	[30] 1	10
Demand	20	15	30	

Now the no. of occupied cell (4) is not equal to m+n-1.

So this T.P is degeneracy.

Now to make this T.P non-degeneracy, we have to make a cell occupied, So for that we choose an empty cell i.e. C(1,2) which is the smallest among the unoccupied cell and also the cell is in closed path.

And we allocate the small +ve value ε to the cell.

	D_1	D_2	D_3
O_1	[10] 2	[ε] 2	3
O_2	4	[15] 1	2
O_3	[10] 1	3	[30] 1

Now the T.P. is non-degeneracy and to find the optimal sol^n for this T.P, we use UV method.

As 1st row contains maximum no. of occupied cell, so we choose $U_1 = 0$

Allocated cell

$C_{11} = U_1 + V_1 = 2$, $V_1 = 2$

$C_{12} = U_1 + V_2 = 2$, $V_2 = 2$

$C_{22} = U_2 + V_2 = 1$, $U_2 = -1$

$C_{31} = U_3 + V_1 = 1$, $U_3 = -1$

$C_{33} = U_3 + V_3 = 1$, $V_3 = 2$

Non Allocated cell (Net evalution)

$\Delta_{13} = C_{13} - (U_1 + V_3) = 3 - (0+2) = 1 > 0$

$\Delta_{21} = C_{21} - (U_2 + V_1) = 4 - (-1+2) = 3 > 0$

$\Delta_{23} = C_{23} - (U_2 + V_3) = 2 - (-1+2) = 1 > 0$

$\Delta_{32} = C_{32} - (U_3 + V_2) = 3 - (-1+2) = 2 > 0$

$\therefore$ Since all $\Delta_{ij} > 0$, the T.P has been reached the optimal solution.

$x_{11} = 10$, $x_{12} = \varepsilon$, $x_{22} = 15$, $x_{31} = 10$, $x_{33} = 30$

and the T.P. Cost $= 2\times10 + 2\times\varepsilon + 1\times15 + 1\times10 + 1\times30$

$= 75 + 2\varepsilon$

$= 75$ ($\because \varepsilon$ is very small +ve no.)

when there is degeneracy occur in a T.P, to convert into non-degenerate we have to assign a very small value ε to the smallest non-allocated cell present in the matrix and that cell must be in independent position means from that particular cell there is no possibility of forming a loop.

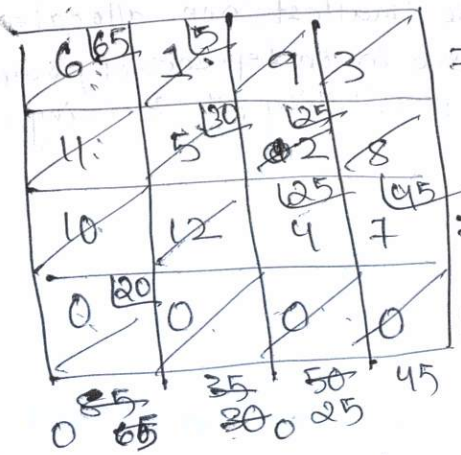
Q.

	D_1	D_2	D_3	D_4	Supply
O_1	6	1	9	3	70
O_2	11	5	2	8	55
O_3	10	12	4	7	70
Demand	85	35	50	45	195 / 215

$O_1 \rightarrow O_2$

	D_1	D_2	D_3	D_4		P_1	P_2	P_3
O_1	6 (65)	1 (5)	9	3	~~70~~ 5	(2)	(2)	(2)
O_2	11	5	2	8	55	(3)	(3)	(3)
O_3	10	12	4	7	70	(3)	(3)	(3)
O_4	0 (20)	0	0	0	~~20~~ 0	(0)	—	—
	~~85~~ 65	35	50	45				

P_1	(4) ↑ (4)	(2)	(4)	
P_2	(4) ↑ (4)	(2)	(4)	
P_3	—	(4) ↑ (2)	(4)	



	P_1	P_2	P_3	P_4	P_5	
70	(2)	(2)	(2)	—	—	0
55	(3)	(3)	(3)	(3)	(6)	250
70	(3)	(3)	(3)	(3)	(3)	
20	(0)	—	—	—	—	0

P_1 (6) (1) (4) (3)

P_2 (4) (4) (2) (4)

P_3 — (4) (2) (4)

P_4 — (7) (2) (1)

P_5 — — (2) (1)

T.P cost $= 6\times65 + 1\times5 + 5\times30 + 2\times25 + 0 + 4\times25 \times 7\times45$

$= 1010$

This T.P is non-degeneracy.

Now let $U_1 = 0$

Allocated cell

$C_{11} = (U_1+V_1) = 6$, $V_1 = 6$

$C_{12} = (U_1+V_2) = 1$, $V_2 = 1$

$C_{22} = (U_2+V_2) = 5$, $U_2 = 4$

$C_{23} = (U_2+V_3) = 2$, $V_3 = -2$

$C_{33} = (U_3+V_3) = 4$, $U_3 = 6$

$C_{34} = (U_3+V_4) = 7$, $V_4 = 1$

$C_{41} = (U_4+V_1) = 0$, $U_4 = -6$

Non Allocated cell (Net ev

$\Delta_{13} = C_{13} - (U_1+V_3) = 9+2 = 11 > 0$

$\Delta_{14} = C_{14} - (U_1+V_4) = 3-(0+1) = 2 > 0$

$\Delta_{21} = C_{21} - (U_2+V_1) = 11-(4+6) = 1 > 0$

$\Delta_{24} = C_{24} - (U_2+V_4) = 8-(4+1) = 3 > 0$

$\Delta_{31} = C_{31} - (U_3+V_1) = 10-(6+6) = -2 < 0$

$\Delta_{32} = C_{32} - (U_3+V_2) = 12-(6+1) = 5 > 0$

$\Delta_{42} = C_{42} - (U_4+V_2) = 0-(-6+1) = 5 > 0$

$\Delta_{43} = C_{43} - (U_4+V_3) = 0-(-6-2) = 8 > 0$

$\Delta_{44} = C_{44} - (U_4+V_4) = 0-(-6+1) = 5 > 0$

65	6	1	9	3
30	11	5	2	8
25	10	12	4	7
45	0	0	0	0
20				

	6	1	9	3
	11	5	2	8
	10	12	4	7
	0	0	0	0

$\min(65, 30, 25) = 25$

40	6	1	9	3
30	11	5	2	8
25	10	12	4	7
45	0	0	0	0
20				

Let $v_1 = 0$

T.P is non-degeneracy

$c_{11} = (v_1 + v_1) = 6$, $v_1 = 6$

$c_{12} = (v_1 + v_2) = 1$, $v_2 = 1$

$c_{22} = (v_2 + v_2) = 5$, $v_2 = 4$

$c_{23} = (v_2 + v_3) = 2$, $v_3 = -2$

$c_{31} = (v_3 + v_1) = 10$, $v_3 = 4$

$c_{34} = (v_3 + v_4) = 7$, $v_4 = 3$

$c_{41} = (v_4 + v_1) = 0$, $v_4 = -6$

Non Allocated (net evalution)

$A_{13} = c_{13} - (v_1 + v_3) = 9 - (0 - 2) = 11 > 0$

$A_{14} = c_{14} - (v_1 + v_4) = 3 - (0 + 3) = 0$

$A_{21} = c_{21} - (v_2 + v_1) = 11 - (4 + 6) = 1 > 0$

$A_{24} = c_{24} - (v_2 + v_4) = 8 - (4 + 3) = 1 > 0$

$A_{32} = c_{32} - (v_3 + v_2) = 12 - (4 + 1) = 7 > 0$

$A_{33} = c_{33} - (v_3 + v_3) = 4 - (4 - 2) = 2 > 0$

$A_{42} = c_{42} - (v_4 + v_2) = 0 - (-6 + 1) = 5 > 0$

$A_{43} = c_{43} - (v_4 + v_3) = 0 - (-6 - 2) = 8 > 0$

$A_{44} = c_{44} - (v_4 + v_4) = 0 - (-6 + 3) = 3 > 0$

Since all $A_{ij} > 0$, the T.P has reached its optimal solution.

and the T.P cost $= 6 \times 40 + 1 \times 30 + 5 \times 5 + 2 \times 50 + 10 \times 25 + 7 \times 45 + 0$

$= 240 + 30 + 25 + 100 + 250 + 315$

$= 960$

Q.

	D_1	D_2	D_3	D_4	Supply
O_1	3	1	7	4	250
O_2	2	6	5	9	350
O_3	8	3	3	2	400
Demand	200	300	350	150	

As this T.P is a balanced, so ~~use~~ we use VAM method direct.

	D_1	D_2	D_3	D_4		P_1	P_2	P_3	P_4
O_1	3	1 (250)	7	4	~~250~~ 0	(2)	(3) ←	—	—
O_2	2 (200)	6	5 (150)	9	~~350~~ 150	(3) ←	(1)	(1)	(1)
O_3	8	3 (150)	3 (200)	2 (150)	~~400~~ ~~250~~ 200	(1)	(1)	(1)	(0)
	~~200~~ 0	~~300~~ 150 0	350	150 0					

P_1	(1)	(2)	(2)	(2)
P_2	—	(2)	(2)	(2)
P_3	—	(3)	(2)	(7) ↑
P_4	—	(3) ↑	(2)	—

The T.P. Cost = $1\times250 + 2\times200 + 5\times150 + 3\times150 + 3\times200 + 2\times150$

= 2450

Now using MODI method to find out optimal solution, (This sol? is non-degeneracy)

Here, 3rd contains maximum no. of allocation, so we choose $U_3 = 0$ to find out values of ~~U_i and~~ U_i, V_j and A_{ij}

Allocated cell

$C_{12} = U_1 + V_2 = 1$, $U_1 = -2$

$C_{21} = U_2 + V_1 = 2$, $V_2 = 0$

$C_{23} = U_2 + V_3 = 5$, $U_2 = 2$

$C_{32} = U_3 + V_2 = 3$, $V_2 = 3$

$C_{33} = U_3 + V_3 = 3$, $V_3 = 3$

$C_{34} = U_3 + V_4 = 2$, $V_4 = 2$

Non Allocated cell (Net evalution)

$A_{11} = C_{11} - (U_1 + V_1) = 3 - (-2 + 0) = 5 > 0$

$A_{13} = C_{13} - (U_1 + V_3) = 7 - (-2 + 3) = 6 > 0$

$A_{14} = C_{14} - (U_1 + V_4) = 4 - (-2 + 2) = 4 > 0$

$A_{22} = C_{22} - (U_2 + V_2) = 6 - (2 + 3) = 1 > 0$

$A_{24} = C_{24} - (U_2 + V_4) = 9 - (2 + 2) = 5 > 0$

$A_{31} = C_{31} - (U_3 + V_1) = 8 - (0 + 0) = 8 > 0$

Since $A_{ij} > 0$, So the T.P. has been reached the optimal solution and

$x_{12} = 1$, $x_{21} = 2$, $x_{23} = 5$, $x_{32} = 3$, $x_{33} = 3$, $x_{34} = 2$

and the T.P cost = $1\times250 + 2\times200 + 5\times150 + 3\times150 + 3\times200 + 2\times150$

= 2450

Q.2

	I	II	III	IV	V	Supply
A	8	6 (15)	6 (85)	9	7	~~100~~ 15
B	7	8 (20)	9	6 (105)	10	~~125~~ ~~20~~ 0
C	5 (60)	6 (45)	8	5	6 (70)	~~175~~ ~~115~~ 45
	~~60~~ 0	~~80~~ 60 0	~~85~~ 0	~~105~~ 0	~~70~~ 0	

	P_1	P_2	P_3	P_4	P_5
	(2)	(1)	(1)	(1)	(1)
	(1)	(2) ←	(1)	(2) ←	-
	(1)	(1)	(2)	(0)	(0)

P_1	(2) ↑	(2)	(2)	(1)	(1)
P_2	-	(2)	(2)	(1)	(1)
P_3	-	(2)	(2) ↑	-	(1)
P_4	-	(2)	-	-	(1)
P_5	-	(0)	-	-	(1) ↑

	I	II	III	IV	V	
A	8	6 (15)	6 (85)	9	7	100
B	7	8 (20)	9	6 (105)	10	125
C	5 (60)	6 (45)	8	5	6 (70)	175
	60	80	85	105	70	

→ The T.P. cost = $6\times15 + 6\times85 + 8\times20 + 5\times60 + 6\times45 + 6\times105 + 6\times70$

= 2380

Here the no. of occupied cell = $m+n-1 = 7$, so this solution is non-degenerecy. Now to find optimal sol^n for this T.P, we use MODI method ~~and~~.

Let $U_3 = 0$, because 3rd row contains maximum no. of occupied cell.

Allocated cell

$c_{12} = U_1 + V_2 = 6, U_1 = 0$

$c_{13} = U_1 + V_3 = 6, V_3 = 6$

$c_{22} = U_2 + V_2 = 8, U_2 = 2$

$c_{24} = U_2 + V_4 = 6, V_4 = 4$

$c_{31} = U_3 + V_1 = 5, V_1 = 5$

$c_{32} = U_3 + V_2 = 6, V_2 = 6$

$c_{35} = U_3 + V_5 = 6, V_5 = 6$

Non Allocated cell (Net evalution)

$\Delta_{11} = c_{11} - (U_1 + V_1) = 8 - (0+5) = 3 > 0$

$\Delta_{14} = c_{14} - (U_1 + V_4) = 9 - (0+4) = 5 > 0$

$\Delta_{15} = c_{15} - (U_1 + V_5) = 7 - (0+6) = 1 > 0$

$\Delta_{21} = c_{21} - (U_2 + V_1) = 7 - (2+5) = 0$

$\Delta_{23} = c_{23} - (U_2 + V_3) = 9 - (2+6) = 1 > 0$

$\Delta_{25} = c_{25} - (U_2 + V_5) = 10 - (2+6) = 2 > 0$

$\Delta_{33} = c_{33} - (U_3 + V_3) = 8 - (0+6) = 2 > 0$

$\Delta_{34} = c_{34} - (U_3 + V_4) = 5 - (0+4) = 1 > 0$

Since all $\Delta_{ij} \geq 0$, the T.P. has been reached the optimal solution and

$x_{12} = 6, x_{13} = 6, x_{22} = 8, x_{24} = 6, x_{31} = 5, x_{32} = 6, x_{35} = 6$

and the T.P. cost = $6\times15 + 6\times85 + 8\times20 + 5\times60 + 6\times45 + 6\times105 + 6\times70$

= 2380

Q → New T.P after adding variable cost to the Transportation cost, ~~is~~ given below.

	I	II	III	IV	V	
A	22	20	20	23	21	100
B	23	24	25	22	26	125
C	20	21	23	20	21	175
	60	80	85	105	70	

As this T.P is balanced, so we use MODI method directly,

	I	II	III	IV	V	supply	P_1	P_2	P_3	P_4
A	22	20	85 20	23	15 21	~~100~~ 15	(0)	(1)	(1)	(1)
B	23	80 24	25	22	45 26	~~125~~ 45	(1)	(1)	(2)	(2) ←
C	60 20	21	23	105 20	10 21	~~175~~ ~~115~~ 10	(0)	(0)	(1)	(0)
Demand	~~60~~ 0	~~80~~ 0	~~85~~ 0	~~105~~ 0	70					

	I	II	III	IV	V
P_1	(2)	(1)	(3) ↑	(2)	(0)
P_2	(2) ↑	(1)	–	(2)	(0)
P_3	–	(1)	–	(2) ↑	(0)
P_4	–	(1)	–	–	(0)

The T.P. Cost according to this route = $20\times85 + 21\times15 + 24\times80 + 26\times45 + 20\times60 + 20\times105 + 21\times10$

$= 8615$

Also this ~~P.P.~~ solution is non-degenerecy (No. of allocated cell = 7 = m+n-1)

Now to find optimal solution, we use MODI method / UV method.

And the 3rd row contains maximum no. of allocated cell, so we take $U_3 = 0$.

Allocated cell

$C_{13} = U_1 + V_3 = 20$, $V_3 = 20$

$C_{15} = U_1 + V_5 = 21$, $V_1 = 0$

$C_{22} = U_2 + V_2 = 24$, $V_2 = 19$

$C_{25} = U_2 + V_5 = 26$, $U_2 = 5$

$C_{31} = U_3 + V_1 = 20$, $V_1 = 20$

$C_{34} = U_3 + V_4 = 20$, $V_4 = 20$

$C_{35} = U_3 + V_5 = 21$, $V_5 = 21$

Non Allocated cell (Net evalution)

$\Delta_{11} = C_{11} - (U_1 + V_1) = 22 - (0+20) = 2 > 0$

$\Delta_{12} = C_{12} - (U_1 + V_2) = 20 - (0+19) = 1 > 0$

$\Delta_{14} = C_{14} - (U_1 + V_4) = 23 - (0+20) = 3 > 0$

$\Delta_{21} = C_{21} - (U_2 + V_1) = 23 - (5+20) = -2 < 0$

$\Delta_{23} = C_{23} - (U_2 + V_3) = 25 - (5+20) = 0$

$\Delta_{24} = C_{24} - (U_2 + V_4) = 22 - (5+20) = -3 < 0$

$\Delta_{32} = C_{32} - (U_3 + V_2) = 21 - (0+19) = 2 > 0$

Hence Δ_{21} and Δ_{24} are less than zero, so the sol^n not reached optimal.

Hence we need to ~~create~~ start a loop from C_{24} cell as its value is highest -ve.

	I	II	III	IV	V
A	22	20	20 [85]	23	21 [15]
B	23	24 [80]	25	22 [+ve]	26 [45] [-ve]
C	20 [60]	21	23	20 [105] [-ve]	21 [10] [+ve]

⇒

	I	II	III	IV	V
A	22	20	20 [85]	23	21 [15]
B	23	24 [80]	25	22 [45]	26
C	20 [60]	21	23	20 [60]	21 [55]

θ = min {Allocated -ve cell}
= min {105, 45} = 45

And the new T.P is non-degeneracy as No. of occupied cell = 7 = (m+n-1).

Again to find optimal soln for the new. TP we use UV method.
and $v_3 = 0$ (3rd row contains max. allocated cell)

Allocated cell

$c_{13} = u_1 + v_3 = 20, v_3 = 20$

$c_{15} = u_1 + v_5 = 21, u_1 = 0$

$c_{22} = u_2 + v_2 = 24, v_2 = 22$

$c_{24} = u_2 + v_4 = 22, u_2 = 2$

$c_{31} = u_3 + v_1 = 20, v_1 = 20$

$c_{34} = u_3 + v_4 = 20, v_4 = 20$

$c_{35} = u_3 + v_5 = 21, v_5 = 21$

Non Allocated cell (Net evalution)

$\Delta_{11} = c_{11} - (u_1 + v_1) = 22 - (0+20) = 2 > 0$

$\Delta_{12} = c_{12} - (u_1 + v_2) = 20 - (0+22) = -2 < 0$

$\Delta_{14} = c_{14} - (u_1 + v_4) = 23 - (0+20) = 3 > 0$

$\Delta_{21} = c_{21} - (u_2 + v_1) = 23 - (2+20) = 1 > 0$

$\Delta_{23} = c_{23} - (u_2 + v_3) = 25 - (2+20) = 3 > 0$

$\Delta_{25} = c_{25} - (u_2 + v_5) = 26 - (2+21) = 3 > 0$

$\Delta_{32} = c_{32} - (u_3 + v_2) = 21 - (0+22) = -1 < 0$

$\Delta_{33} = c_{33} - (u_3 + v_3) = 23 - (0+20) = 3 > 0$

Since Δ_{12} and Δ_{32} are negative, the soln not reached optimal.
We need to start loop from cell (1,2) as it is the highest -ve value.

	I	II	III	IV	V
A	22	20 [85] (+)	20	23	21 [15] (-)
B	23	24 [80] (-)	25	22 [45] (+)	26
C	20 [60]	21	23	20 [60] (-)	21 [55] (+)

⇒

	I	II	III	IV	V
A	22	20 [15]	20 [100]	23	21
B	23	24 [65]	25	22 [60]	26
C	20 [60]	21	23	20 [45]	21 [70]

$\theta = \{80, 60, 15\} = 15$

Allocated cell

$c_{12} = u_1 + v_2 = 20$, $u_1 = -2$

$c_{13} = u_1 + v_3 = 20$, $v_3 = 22$

$c_{22} = u_2 + v_2 = 24$, $v_2 = 22$

$c_{24} = u_2 + v_4 = 22$, $u_2 = 2$

$c_{31} = u_3 + v_1 = 20$, $v_1 = 20$

$c_{34} = u_3 + v_4 = 20$, $v_4 = 21$

$c_{35} = u_3 + v_5 = 21$, $v_5 = 21$

22	20	20	23	21
23	24	25	22	26
20	21	23	20	21

Non Allocated cell (Net evalution)

$\Delta_{11} = c_{11} - (u_1 + v_1) = 22 - (-2 + 20) = 4 > 0$

$\Delta_{14} = c_{14} - (u_1 + v_4) = 23 - (-2 + 20) = 5 > 0$

$\Delta_{15} = c_{15} - (u_1 + v_5) = 21 - (-2 + 21) = 2 > 0$

$\Delta_{21} = c_{21} - (u_2 + v_1) = 23 - (2 + 20) = 1 > 0$

$\Delta_{23} = c_{23} - (u_2 + v_3) = 25 - (2 + 22) = 1 > 0$

$\Delta_{25} = c_{25} - (u_2 + v_5) = 26 - (2 + 21) = 3 > 0$

$\Delta_{32} = c_{32} - (u_3 + v_2) = 21 - (0 + 22) = -1 < 0$

$\Delta_{33} = c_{33} - (u_3 + v_3) = 23 - (0 + 22) = 1 > 0$

Q

	D_1	D_2	D_3	D_4	D_5
O_1					
O_2					
O_3					

Maximization

Generally T.P is a minimization problem. If the given problem is in maximization form of a T.P then first convert the max. to minimization by subtracting all the element of the T.P from the highest element of T.P table, then follow as usual method to find the sol;

Q. Find the T.P cost where the matrix is a maximizat

	D_1	D_2	D_3	D_4	supply
O_1	2	2	2	4	1000
O_2	3	5	3	2	700 →
O_3	4	3	2	1	900
Dem	900	800	500	400	

P_1	D_1	D_2	D_3	D_4	P_1	P_2	P_3
O_1	3	3 100	3 500	1 400	~~1000~~ 600 ←	(2)	(0)	(0)
O_2	2	0 700	2	3	~~700~~ 0	(2)	(2)	–
O_3	1 900	2	3	4	~~900~~ 0	(1)	(1)	(1)
	~~900~~ 0	~~800~~ 100	500	~~400~~ 0				
P_1	(1)	(2)	(1)	(2)				
P_2	(1)	(2) ↑	(1)	–				
P_3	(2) ↑	(1)	(0)	–				

T.P Cost = $3\times100 + 3\times500 + 1\times400 + 0\times700 + 1\times900$

$= 300 + 1500 + 400 + 900$

$= 3100$

Q.

2 (100)	2	2 (500)	4 (400)
3	5 (700)	3	2
4 (900)	3	2	1

T.P cost = $2\times100 + 2\times500 + 4\times400 + 5\times700 + 4\times900$

= 9900

Q. Solve the

	D_1	D_2	D_3	D_4	Suppl
O_1	15	1	42	33	23
O_2	80	42	26	81	44 →
O_3	90	40	66	60	33
	23	31	16	30	

	(23)			23	P1 (9) P2 (18) P3 (18) P4 (32)
75	89	48	57		
(6)	(8)		(30)	44	(1) (1) (1) (39)
10	48	64	9	38 8	
(17)		(16)			
0	50	24	30	33 17 0	(24) (30) ←
23 0	31	16 0	38		

P1	(10)	(2)	(24)↑	(21)
P2	(10)	(2)	–	(21)
P3	(65)↑	(41)	–	(48)
P4	–	(41)	–	(48)↑

	(23)			23
15	1	42	33	
(6)	(8)		(30)	44 = 5855
80	42	26	81	
(17)		(16)		33
90	40	66	60	

Q.

	A	B	C	D	
1	40	25	22	33	100
2	44	35	30	30	30 →
3	38	38	28	30	70
	40	20	60	30	

	A	B	C	D	E	
1	40	25	22	33	0	100
2	44	35	30	30	0	30
3	38	38	28	30	0	70
	40	20	60	30	50	

	A	B	C	D	E	P1
			20	30	50	
1	4	19	22	11	44	~~100~~ 70
	30					
2	0	9	14	14	44	~~30~~ 0 ←
	10	20	40			
3	6	6	16	14	44	~~70~~ 50 40 0
	40	20	60	30	50	
P1	~~(4)~~	~~(3)~~	~~(2)~~	~~(3)~~	~~(0)~~	
	~~40~~ 0	~~10~~ 0	~~20~~ 0	~~60~~ 20	~~30~~ 0	50

	P1	P2	P3	P4	P5
	(7)	(7)	(7)	(11) ←	(22)
	(9)	–	–	–	–
	(0)	(0)	(8) ←	(2)	(28) ←

P1 (4) (3) (2) (3) (0)

P2 (2) (13) (6) (3) (0)

P3 (2) – (6) (3) (0)

P4 – – (6) (3) (0)

P5 – – (6) – (0)

→

		20	30	50
40	25	22	33	0
30				
44	35	30	30	0
10	20	40		
38	38	28	30	0

= 22×20 + 33×30 + 0×50 + 44×30 + 38×10 + 38×20 + 28×40

= 440 + 990 + 0 + 1320 + 380 + 760 + ~~920~~ 1120

= 5010

4	19	22 (20)	11 (30)	44 (50)
0 (30)	9	14	14	44
6 (10)	6 (20)	16 (40)	14	44
40	20	60	30	50

This is a non-degeneracy T.P.
as $m+n-1 = 7$ = occupied cell

Let $U_1 = 0$

11 lakh
470,000
12.6 lakh
7.54.000

Allocated cell

$C_{13} = U_1 + V_3 = 22, V_3 = 22$

$C_{14} = U_1 + V_4 = 11, V_4 = 11$

$C_{15} = U_1 + V_5 = 44, V_5 = 44$

$C_{21} = U_2 + V_1 = 0, U_2 = -12$

$C_{31} = U_3 + V_1 = 6, V_1 = 12$

$C_{32} = U_3 + V_2 = 6, V_2 = 12$

$C_{33} = U_3 + V_3 = 16, U_3 = -6$

Non Allocated cell (Net evalution)

$\Delta_{11} = C_{11} - (U_1 + V_1) = 4 - (0+12) = -8 < 0$

$\Delta_{12} = C_{12} - (U_1 + V_2) = 19 - (0+12) = 7 > 0$

~~$\Delta_{32} = C_{32} - (U_3 + V_2) = 9 - (-6+12) = 3 > 0$~~

~~$\Delta_{33} = C_{33} - (U_3 + V_3) = 14 - (-6+22) = -2 < 0$~~

~~$\Delta_{34} = C_{34} - (U_3 + V_4) = 14 - (-6+11) = 9 > 0$~~

~~$\Delta_{35} = C_{35} - (U_3 + V_5) = 44 - (-6+44) = 6 > 0$~~

$\Delta_{22} = C_{22} - (U_2 + V_2) = 9 - (-12+12) = 9 > 0$

$\Delta_{23} = C_{23} - (U_2 + V_3) = 14 - (-12+22) = 4 > 0$

$\Delta_{24} = C_{24} - (U_2 + V_4) = 14 - (-12+11) = 13 > 0$

$\Delta_{25} = C_{25} - (U_2 + V_5) = 44 - (-12+44) = 12 > 0$

$\Delta_{34} = C_{34} - (U_3 + V_4) = 14 - (-6+11) = 9 > 0$

$\Delta_{35} = C_{35} - (U_3 + V_5) = 44 - (-6+44) = 6 > 0$

As $\Delta_{11} < 0$ so the T.P has not reached optimal solution.

Hence we need to start loop from C_{11} cell

+ →		(-) 22 (20)	11 (30)	44 (50)
4 ↓	19			
30		↑		
0 ↓	9	14	14	44
10	←	(+)		
6 (-)	6 →	16 (20)	14 (40)	44

→

10		10	30	50
4	19	22	11	44
30				
0	9	14	14	44
	20	50		
6	6	16	14	44

$(U_1 = 0)$

20			30	50
4	19	22	11	44
10				
0	9	14	14	44
30	ε	60		
6	6	16	14	44

$(U_1 = 0)$

Here $\theta = \min\{\cancel{30, 20, 20}\ 10, 20\} = \cancel{20}\ 10$

Allocated

$C_{11} = U_1 + V_1 = 4, V_1 = 4$

$C_{14} = U_1 + V_4 = 11, V_4 = 11$

$C_{15} = U_1 + V_5 = 44, V_5 = 44$

$C_{21} = U_2 + V_1 = 0, U_2 = -4$

~~$C_{31} = U_3 + V_1 = 6, U_3 = 2$~~

$C_{32} = U_3 + V_2 = 6, V_2 = \cancel{12}\ 12$

$C_{33} = U_3 + V_3 = 16, V_3 = \cancel{-6}\ -6$

$C_{13} = U_1 + V_3 = 22, V_3 = 22$

Non Allocated

~~$\Delta_{12} = C_{12} - (U_1 + V_2) = 19 - (0+4) = 15 > 0$~~

~~$\Delta_{13} = C_{13} - (U_1 + V_3) = 22 - (0+14) = 8 > 0$~~

~~$\Delta_{21} = C_{21} - (U_2 + V_2) = 9 - (-4+4) = 9 > 0$~~

$\Delta_{22} = C_{22} - (U_2 + V_3) = 14 - (-4+14) = 4 > 0$

$\Delta_{24} = C_{24} - (U_2 + V_4) = 14 - (-4+11) = 7 > 0$

$\Delta_{25} = C_{25} - (U_2 + V_5) = 44 - (-4+44) = 4 > 0$

$\Delta_{34} = C_{34} - (U_3 + V_4) = 14 - (2+11) = 1 > 0$

$\Delta_{35} = C_{35} - (U_3 + V_5) = 44 - (-2+44) = 2 > 0$

$19 - (0+12) = 7 > 0$

$\Delta_{22} = 9 - (-4+12) = 9 - 8 = 1 > 0$

$\Delta_{23} = 14 - (-4+22) = -4 < 0$

$\Delta_{24} = 14 - (-4+11) = 7 > 0$

$\Delta_{25} = 44 - (-4+44) = 4 > 0$

$\Delta_{31} = 6 - (-6+4) = 8 > 0$

$\Delta_{34} = 14 - (-6+11) = 9 > 0$

$\Delta_{35} = 44 - (-6+44) = 6 > 0$

20			30	50
40	25	22	33	0
10				
44	35	30	30	0
30	ε	60		
38	38	28	30	0

T.P. cost = $40\times20 + 33\times30 + 0\times50 + 44\times10 + 38\times30 + 38\times\varepsilon + 28\times60$

$= 800 + 990 + 0 + 440 + 1140 + 38\varepsilon + 1680$

$= 5050 + 38\varepsilon$

$= 5050$

Q.

40	25	22	33	100
44	35	30	30	30 →
38	38	28	30	70
40	20	60	30	

4	19	22	11	100
0	9	14	14	30 →
6	6	16	14	70
40	20	60	30	

4	19	22	11	0	
0	9	14	14	0	
6	6	16	14	0	
40	20	60	30	50	

Ans)

10		60	30	
4	19	22	11	0
30				
0	9	14	14	0
	20			50
6	6	16	14	0
~~40~~ 10	~~20~~ 0	60	30	~~50~~ 0

	P_1	P_2	P_3
100	(4)	(7)	(7)
~~30~~ 0	(0)	(9) ←	
~~70~~ 20 ①	(6) ←	(0)	(0)

P_1	(4)	(3)	(2)	(3)	(0)
P_2	(4)	(3)	(2)	(3)	(-)
P_3	(2)	(13)↑	(6)	(3)	(-)

10		60	30	
4	19	22	11	0
30				
0	9	14	14	0
	20			50
6	6	16	14	0

~~The T.P cost~~

Allocated cell

~~$c_{11} = u_1 + v_1 = 4, v_1 = 4$~~

~~$c_{13} = u_1 + v_3 = 22, v_3 = 22$~~

~~$c_{14} = u_1 + v_4 = 11, v_4 = 11$~~

~~$c_{21} = u_2 + v_1 = 0, u_2 = -4$~~

~~$c_{31} = u_3 + v_1 = 6, u_3 = -4$~~

~~$c_{32} = u_3 + v_2 = 6, v_2 = 10$~~

~~$c_{35} = u_3 + v_5 = 0, v_5 = 4$~~

10		60	30	
4	19	22	11	0
30				
0	9	14	14	0
E	20			50
6	6	16	14	0

($u_1 = 0$)

Allocated cell

$c_{11} = u_1 + v_1 = 4, v_1 = 4$

$c_{13} = u_1 + v_3 = 22, v_3 = 22$

$c_{14} = u_1 + v_4 = 11, v_4 = 11$

$c_{21} = u_2 + v_1 = 0, u_2 = -4$

$c_{31} = u_3 + v_1 = 6, u_3 = 2$

$c_{32} = u_3 + v_2 = 6, v_2 = 4$

$c_{35} = u_3 + v_5 = 0, v_5 = -2$

Non Allocated Net evolution

$\Delta_{12} = c_{12} - (u_1 + v_2) = 19 - (0+4) = 15 > 0$

$\Delta_{15} = c_{15} - (u_1 + v_5) = 0 - (0-2) = 2 > 0$

$\Delta_{22} = c_{22} - (u_2 + v_2) = 9 - (-4+4) = 9 > 0$

$\Delta_{23} = c_{23} - (u_2 + v_3) = 14 - (-4+22) = -4 < 0$

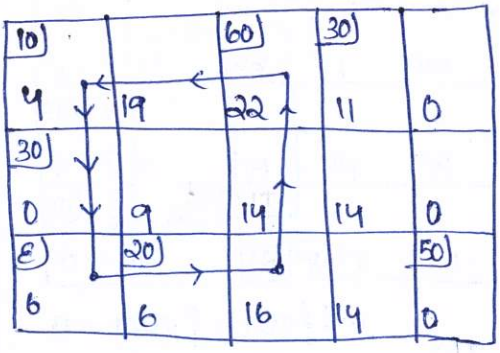
$\Delta_{24} = c_{24} - (u_2 + v_4) = 14 - (-4+11) = 7 > 0$

$\Delta_{25} = c_{25} - (u_2 + v_5) = 0 - (-4-2) = 6 > 0$

$\Delta_{33} = c_{33} - (u_3 + v_3) = 16 - (2+22) = -8 < 0$ ✓

$\Delta_{34} = c_{34} - (u_3 + v_4) = 14 - (2+11) = 1 > 0$

$\Delta_{35} = c_{35} - (u_3 + v_5) = 0 - (2-2) = 0$



		[20]	[30]	[50]	100
4	19	22	11	44	
[30]					30
0	9	14	14	44	
[10]	[20]	[40]			70
6	6	16	14	44	
40	20	60	30	50	

This is a non-degeneracy T.P as $m+n-1 = 7$ = occupied cell

Let $U_1 = 0$

Allocated cell ($U_1 = 0$)

$C_{13} = U_1 + V_3 = 22, V_3 = 22$

$C_{14} = U_1 + V_4 = 11, V_4 = 11$

$C_{15} = U_1 + V_5 = 44, V_5 = 44$

$C_{21} = U_2 + V_1 = 0, U_2 = -12$

$C_{31} = U_3 + V_1 = 6, V_1 = 12$

$C_{32} = U_3 + V_2 = 6, V_2 = 12$

$C_{33} = U_3 + V_3 = 16, U_3 = -6$

Non Allocated cell

$\Delta_{11} = C_{11} - (U_1 + V_1) = 4 - (0+12) = -8 < 0$

$\Delta_{12} = 19 - (0+12) = 7 > 0$

$\Delta_{22} = 9 - (-12+2) = 9 > 0$

$\Delta_{23} = 14 - (-12+22) = 4 > 0$

$\Delta_{24} = 14 - (-12+11) = 15 > 0$

$\Delta_{25} = 44 - (-12+44) = 12 > 0$

$\Delta_{34} = 14 - (-6+11) = 9 > 0$

$\Delta_{35} = 44 - (-6+44) = 6 > 0$

As $\Delta_{11} < 0$, so the T.P has not reached optimal, hence we need to start a loop from C_{11}

(+)		(-) [20]	[30]	[50]
4	19	22	11	44
[30]				
0	9	14	14	44
[10]	[20]	(+)		
6 (-)	6	16	[40]	14	44

$\theta = \min\{10, 20\} = 10$

→

[10]		[10]	[30]	[50]
4	19	22	11	44
[30]				
0	9	14	14	44
	[20]	[50]		
6	6	16	14	44

This New T.P is Non-degeneracy

Allocated cell ($U_1 = 0$)

$C_{11} = U_1 + V_1 = 4, V_1 = 4$

$C_{13} = U_1 + V_3 = 22, V_3 = 22$

$C_{14} = U_1 + V_4 = 11, V_4 = 11$

$C_{15} = U_1 + V_5 = 44, V_5 = 44$

$C_{21} = U_2 + V_1 = 0, U_2 = -4$

$C_{32} = U_3 + V_2 = 6, V_2 = 12$

$C_{33} = U_3 + V_3 = 16, U_3 = -6$

Non Allocated

$\Delta_{12} = C_{12} - (U_1 + V_2) = 19 - (0+12) = 7 > 0$

$\Delta_{22} = 9 - (-4+12) = 1 > 0$

$\Delta_{23} = 14 - (-4+22) = -4 < 0$

$\Delta_{24} = 14 - (-4+11) = 7 > 0$

$\Delta_{25} = 44 - (-4+44) = 4 > 0$

$\Delta_{31} = 6 - (-6+4) = 8 > 0$

$\Delta_{34} = 14 - (-6+11) = 9 > 0$

$\Delta_{35} = 44 - (-6+44) = 6 > 0$

As $\Delta_{23} < 0$ so the T.P has not reached optimal and we have to start a loop from C_{23}

(+) 10		(-) 10	30	50
4	19	22	11	44
30				
0 (-)	9	14 (+)	14	44
	20	50		
6	6	16	14	44

$\theta = \min\{30, 10\} = 10$

→

20			30	50
4	19	22	11	44
20		10		
0	9	14	14	44
	20	50		
6	6	6	14	44

The New T.P is Non-degenerecy,

Allocated cell ($U_1 = 0$)

$c_{11} = U_1 + V_1 = 4, V_1 = 4$

$c_{14} = U_1 + V_4 = 11, V_4 = 11$

$c_{15} = U_1 + V_5 = 44, V_5 = 44$

$c_{21} = U_2 + V_1 = 0, U_2 = -4$

$c_{23} = U_2 + V_3 = 14, V_3 = 18$

$c_{32} = U_3 + V_2 = 6, V_2 = 8$

$c_{33} = U_3 + V_3 = 16, U_3 = -2$

Non Allocated cell (Net evalution)

$\Delta_{12} = c_{12} - (U_1 + V_2) = 19 - (0+8) = 11 > 0$

$\Delta_{13} = 22 - (0+18) = 4 > 0$

$\Delta_{22} = 9 - (-4+8) = 5 > 0$

$\Delta_{24} = 14 - (-4+11) = 7 > 0$

$\Delta_{25} = 44 - (-4+44) = 4 > 0$

$\Delta_{31} = 6 - (-2+4) = 4 > 0$

$\Delta_{34} = 14 - (-2+11) = 5 > 0$

$\Delta_{35} = 44 - (-2+44) = 2 > 0$

Since all $\Delta ij \geq 0$, the ~~solution~~ T.P has reached to optimal solution and the Profit cost = $40\times20 + 33\times30 + 0\times50 + 44\times20 + 30\times10 + 38\times20 + 28\times50$

$= 5130$

The Assignment Problem is a minimization problem, where we have to assign 'n' job to 'n' person in such a way that the total time will be minimum or total cost will be minimum.

To solve an Assignment Method, we have to follow Hungarian Method

Step-1 Row Reduction

Step-2 Column Reduction

Step-3 Assignment

Row Reduction

Find out the smallest element in each row and subtract it from every element of that row.

Column Reduction

After row reduction, find out the smallest element in each column and subtract from every element of that column.

Assignment Process

(Row Assignment) → Find out single zero in a row make an rectangle there and cross all other zeros of that corresponding column, and continue this process from first row to last row.

(Column Assignment) – After row assignment, find out a single zero in a column, make a rectangle there and cross all other zeros of that corresponding row and continue this process from first column to last column

Then if the no. of assignment are equal to the order of the matrix, then our soln is optimal

N.B: Check the no. of assignment must be 1 in each row and each column, otherwise you are wrong.

Q.

	J_1	J_2	J_3	J_4	J_5
P_1	10	3	3	2	8
P_2	9	7	8	2	7
P_3	7	5	6	2	4
P_4	3	5	8	2	4
P_5	9	10	9	6	10

row reduction →

	8	1	1	0	6
	7	5	6	0	5
	5	3	4	0	2
	1	3	6	0	2
	3	4	3	0	4

Column reduction

7	[0]	✗	✗	4	
6	4	5	[0]	3	
4	2	3	✗	[0]	
[0]	2	5	✗	✗	
2	1	2	✗	2	

↓

9	[0]	✗	2	6	
6	2	3	[0]	3	
4	✗	1	✗	[0]	
[0]	✗	3	✗	✗	
2	1	[0]	✗	2	

→

	Assi]	Cost
P_1	J_2	3
P_2	J_4	2
P_3	J_5	4
P_4	J_1	3
P_5	J_3	9
		21

In this problem, after assignment process is over the no. of assignments are 4 which is not equal to the order of matrix, so over sol^n is not optimal

Step-4 If the sol^n is not optimal, draw minimum no. of lines (horizontal & vertical) to cover all the zeros of the matrix Then find out the smallest uncovered element by the lines subtract it from uncovered ; and add it to the intersection elements of the line, Then we have a new Table and start the as usual assignment process to find the sol^n.

P_1
P_2
P_3
P_4
P_5

Here the no. of assignments are 5, and we have assigned

To solve an assignment problem, whether it is balanced or unbalanced. An assignment problem is said to be balanced if no. of rows or no. of column are equal. If we have an unbalance assignment problem then make it balance by adding a dummy row or dummy column as per the requirement, where the entries of dummy rows or column are Zero

Row Reduction

Column Reduction

$$\begin{array}{c} \\ M_1 \\ M_2 \\ M_3 \\ M_4 \end{array}\begin{array}{c} \begin{array}{cccc} J_1 & J_2 & J_3 & J_4 \end{array} \\ \begin{bmatrix} 5 & 8 & 4 & 10 \\ 7 & 5 & 7 & 4 \\ 11 & 9 & 10 & 8 \\ 6 & 6 & 7 & 3 \end{bmatrix} \end{array} \rightarrow \begin{bmatrix} 1 & 7 & 0 & 6 \\ 3 & 1 & 3 & 0 \\ 3 & 1 & 2 & 0 \\ 3 & 3 & 4 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 0 & 6 & 0 & 6 \\ 2 & 0 & 3 & 0 \\ 2 & 0 & 2 & 0 \\ 2 & 2 & 4 & 0 \end{bmatrix}$$

Assignment Process

Drawing lines

$$\rightarrow \begin{bmatrix} [0] & 6 & \cancel{0} & 6 \\ 2 & [0] & 3 & \cancel{0} \\ 2 & \cancel{0} & 2 & \cancel{0} \\ 2 & 2 & 4 & [0] \end{bmatrix} \quad \begin{bmatrix} \cancel{[0]} & \cancel{6} & \cancel{0} & \cancel{6} \\ 2 & [0] & 3 & \cancel{0} \\ 2 & \cancel{0} & 2 & \cancel{0} \\ 2 & 2 & 4 & [0] \end{bmatrix} \rightarrow \begin{bmatrix} 0 & 8 & 0 & 8 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 2 & 2 & 0 \end{bmatrix}$$

Row Reduction

Column Reduction

	M_1	M_2	M_3	M_4
J_1	5	7	11	6
J_2	8	5	9	6
J_3	4	7	10	7
J_4	10	4	8	3

$\Rightarrow$

0	2	6	1
3	0	4	1
0	3	6	3
7	1	5	0

$\rightarrow$

0	2	2	1
3	0	0	1
0	3	2	3
7	1	1	0

Assignment Process

[0]	2	2	1
3	[0]	✗	1
✗	3	2	3
7	1	1	[0]

(Not optimal)

(Drawing lines)

[0]	2	2	1
~~3~~	~~[0]~~	~~✗~~	~~1~~
✗	3	2	3
~~7~~	~~1~~	~~1~~	~~[0]~~

(New Table)

0	1	1	0
4	0	0	1
0	2	1	2
8	1	1	0

Row Reduction of New Table

0	1	1	0
4	0	0	1
0	2	1	2
8	1	1	0

Column Reduction

0	1	1	0
4	0	0	1
0	2	1	2
8	1	1	0

Assignment Process

✗	1	1	✗
~~4~~	~~[0]~~	~~✗~~	~~1~~
[0]	2	1	2
8	1	1	[0]

0	1	1	0
5	0	0	2
0	1	0	2
8	0	0	0

Q.

row reduction → column reduction → Assignment Process

$$\begin{bmatrix} 5 & 7 & 11 & 6 \\ 8 & 5 & 9 & 6 \\ 4 & 7 & 10 & 7 \\ 10 & 4 & 8 & 3 \end{bmatrix} \rightarrow \begin{bmatrix} 0 & 2 & 6 & 1 \\ 3 & 0 & 4 & 1 \\ 0 & 3 & 6 & 3 \\ 7 & 1 & 5 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 0 & 2 & 2 & 1 \\ 3 & 0 & 0 & 1 \\ 0 & 3 & 2 & 3 \\ 7 & 1 & 1 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} [0] & 2 & 2 & 1 \\ 3 & [0] & \cancel{0} & 1 \\ \cancel{0} & 3 & 2 & 3 \\ 7 & 1 & 1 & [0] \end{bmatrix}$$

Drawing lines → New Table → Assignment Process → Drawing lines

$$\begin{bmatrix} [0] & 2 & 2 & \cancel{1} \\ \cancel{3} & [0] & \cancel{0} & \cancel{1} \\ \cancel{0} & 3 & 2 & 3 \\ \cancel{7} & 1 & 1 & [0] \end{bmatrix} \rightarrow \begin{bmatrix} 0 & 1 & 1 & 1 \\ 4 & 0 & 0 & 2 \\ 0 & 2 & 1 & 3 \\ 7 & 0 & 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} [0] & 1 & 1 & 1 \\ 4 & [0] & \cancel{0} & 2 \\ \cancel{0} & 2 & 1 & 3 \\ 7 & \cancel{0} & \cancel{0} & [0] \end{bmatrix} \rightarrow \begin{bmatrix} [0] & 1 & 1 & 1 \\ \cancel{4} & [0] & \cancel{0} & \cancel{2} \\ \cancel{0} & 2 & 1 & 3 \\ \cancel{7} & \cancel{0} & \cancel{0} & [0] \end{bmatrix}$$

$$\begin{bmatrix} [0] & \cancel{0} & \cancel{0} & \cancel{0} \\ 5 & [0] & \cancel{0} & 2 \\ \cancel{0} & 1 & [0] & 2 \\ 8 & \cancel{0} & \cancel{0} & [0] \end{bmatrix} \rightarrow \begin{matrix} J_1 \rightarrow M_1 \\ J_2 \rightarrow M_2 \\ J_3 \rightarrow M_3 \\ J_4 \rightarrow M_4 \end{matrix}$$

and Cost = 5 + 5 + 10 + 3
= 23

When there is no single zero in row or column in a assignment table, make an assignment arbitrarily in such a way that there must be single assignment in each row and each column.

Maximization case in Assignment Problem

Like T.P., assignment problem is also a minimization problem, but sometime the given matrix will be a profit matrix, in that case we have to convert the maximization matrix to minimization matrix by subtracting all the elements of that matrix from the largest element of matrix, and then follow the hungarian procedure to find the solution, after getting the optimal assignment choose exact value of the initial matrix to get the solution.

	D_1	D_2	D_3	D_4	D_5
S_1	32	38	40	28	40
S_2	40	24	28	21	36
S_3	41	27	33	30	37
S_4	28	38	41	36	36
S_5	29	33	40	35	39

→ Minimization

$$\begin{bmatrix} 9 & 3 & 1 & 13 & 1 \\ 1 & 17 & 13 & 20 & 5 \\ 0 & 14 & 8 & 11 & 4 \\ 13 & 3 & 0 & 5 & 5 \\ 12 & 8 & 1 & 6 & 2 \end{bmatrix}$$

→ row reduction

$$\begin{bmatrix} 8 & 2 & 0 & 12 & 0 \\ 0 & 16 & 12 & 19 & 4 \\ 0 & 14 & 8 & 11 & 4 \\ 10 & 3 & 0 & 5 & 5 \\ 11 & 7 & 0 & 5 & 1 \end{bmatrix}$$

→ Column reduction

$$\begin{bmatrix} 8 & 0 & 0 & 7 & 0 \\ 0 & 14 & 12 & 14 & 4 \\ 0 & 12 & 8 & 6 & 4 \\ 13 & 1 & 0 & 0 & 5 \\ 11 & 5 & 0 & 0 & 1 \end{bmatrix}$$

→ Assignment Process

$$\begin{bmatrix} 8 & [0] & 7 & 7 \\ [0] & 14 & 12 & 14 & 4 \\ 12 & 8 & 6 & 4 \\ 13 & 1 & [0] & 5 \\ 11 & 5 & [0] & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & [0] & 1 & 8 \\ [0] & 13 & 12 & 14 & 3 \\ 11 & 8 & 6 & 3 \\ 13 & [0] & 4 \\ 11 & 4 & [0] \end{bmatrix}$$

$$\begin{bmatrix} 12 & [0] & 1 & 8 \\ 10 & 9 & 11 & [0] \\ [0] & 8 & 5 & 3 \\ 16 & [0] & 4 \\ 14 & 4 & [0] \end{bmatrix} \rightarrow$$

$S_1 \rightarrow D_2 \rightarrow 38 + 36 + 41 + 36 + 40$

$S_2 \rightarrow D_5 \quad = 191$

$S_3 \rightarrow D_1$

$S_4 \rightarrow D_4$

$S_5 - D_3$

Q: Sec-A (Central Tendency)

Q1 Find the sum of 'n' natural no.
Q2 Find the S.D of 'n' natural no. } (2 marks)
Q3 Correct mean

Sec-B Q1 missing frequency (mean, median, mode)
Q2 combined mean

S.Q check the consistency bet' two series,
Diff' between absolute measure & relative measure
Rel' bet' MD, SD, QD and range
Combined S.D
Correct S.D
Range of correlation of coefficient Types (Population correlation coefficient (π ± PE)

L.Q. Table
calculation of Q_1 and Q_3 of continuous series
L.Q from correlation

Let x_1 and x_2 ...

$\frac{n(n+1)}{2}$

Assignment

Q.1

	A	B	C	D
W	41	72	39	52
X	22	29	49	65
Y	27	39	60	51
Z	45	50	48	52

Row Reduction

2	33	0	13
0	7	27	43
0	12	33	24
0	5	3	7

Column Reduction

2	28	0	6
0	2	27	56
0	7	33	17
0	0	3	0

Assignment Process

~~2~~	~~28~~	[0]	6
[0]	2	27	56
~~0~~	7	33	17
~~0~~	[0]	~~3~~	~~0~~

New Table

4	28	[0]	6
~~0~~	[0]	25	54
[0]	5	31	15
2	~~0~~	3	[0]

Cost = 39 + 29 + 27 + 52
= 147

Q.1

	A	B	C	D
W	~~41~~	~~72~~	39	52
X	22	29	49	65
Y	27	39	60	51
Z	45	50	48	52

~~31~~	~~0~~	~~33~~	~~20~~
~~50~~	~~43~~	~~23~~	7
~~45~~	~~33~~	~~12~~	~~21~~
~~27~~	~~22~~	~~24~~	~~20~~

~~31~~	~~0~~	~~33~~	~~20~~
~~43~~	~~36~~	~~16~~	~~0~~
~~33~~	~~24~~	~~0~~	~~9~~
~~7~~	~~2~~	~~4~~	~~0~~

Q.2

1000	1200	400	7000
600	500	300	800
200	300	400	500
600	700	300	1000

Row Reduction

600	800	0	300
300	200	0	500
0	100	200	300
300	400	0	700

Column Reduction

600	700	0	0
300	100	0	200
0	0	200	0
300	300	0	400

Assignment Process

600	700	~~0~~	[0]
300	100	[0]	200
[0]	~~0~~	~~200~~	~~0~~
300	300	~~0~~	400

500	600	~~0~~	[0]
200	[0]	~~0~~	200
[0]	~~0~~	300	100
200	200	[0]	400

Cost = 700 + 500 + 200 + 300
= 1700

Q3

$$\begin{bmatrix} 7 & 9 & 3 & 3 & 2 \\ 6 & 1 & 6 & 6 & 5 \\ 1 & 5 & 2 & 2 & 4 \\ 6 & 6 & 9 & 4 & 2 \end{bmatrix} \rightarrow \begin{bmatrix} 5 & 7 & 1 & 1 & 0 \\ 5 & 0 & 5 & 5 & 4 \\ 0 & 4 & 6 & 7 & 4 \\ 4 & 4 & 7 & 2 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 5 & 7 & [0] & \cancel{4} & \cancel{4} \\ 5 & [0] & 4 & 4 & 4 \\ [0] & 1 & 5 & \cancel{6} & 4 \\ 4 & 4 & 6 & \cancel{1} & [0] \end{bmatrix}$$

Q3

$$\begin{bmatrix} 7 & 9 & 3 & 3 & 2 \\ 6 & 1 & 6 & 6 & 5 \\ 1 & 5 & 2 & 2 & 4 \\ 6 & 6 & 9 & 4 & 2 \end{bmatrix} \rightarrow \begin{bmatrix} 5 & 7 & 1 & 1 & 0 \\ 5 & 0 & 5 & 5 & 4 \\ 0 & 4 & 1 & 1 & 3 \\ 4 & 4 & 7 & 2 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 5 & 7 & [0] & \cancel{3} & \cancel{3} \\ 5 & [0] & 4 & 4 & 4 \\ [0] & 4 & \cancel{1} & [0] & 3 \\ 4 & 4 & 6 & 1 & [0] \end{bmatrix}$$

Cost = 3 + 1 + 1 +

Q3

$$\begin{bmatrix} 7 & 9 & 3 & 3 & 2 \\ 6 & 1 & 6 & 6 & 5 \\ 3 & 4 & 9 & 10 & 7 \\ 1 & 5 & 2 & 2 & 4 \\ 6 & 6 & 9 & 4 & 2 \end{bmatrix} \rightarrow \begin{bmatrix} 5 & 7 & 1 & 1 & 0 \\ 5 & 0 & 5 & 5 & 4 \\ 0 & 1 & 6 & 7 & 4 \\ 0 & 4 & 1 & 1 & 3 \\ 4 & 4 & 7 & 2 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 5 & 7 & [0] & \cancel{3} & \cancel{3} \\ 5 & [0] & 4 & 4 & 4 \\ [0] & 1 & 5 & 6 & 4 \\ \cancel{3} & 4 & \cancel{1} & [0] & 3 \\ 4 & 4 & 6 & 1 & [0] \end{bmatrix}$$

Cost = 3 + 1 + 3 + 2 + 2
= 11

Q4

$$\begin{bmatrix} 48 & 48 & 50 & 44 \\ 56 & 60 & 60 & 68 \\ 96 & 94 & 90 & 85 \\ 42 & 44 & 54 & 46 \end{bmatrix} \rightarrow \begin{bmatrix} 4 & 4 & 6 & 0 \\ 0 & 4 & 4 & 12 \\ 11 & 9 & 5 & 0 \\ 0 & 2 & 12 & 4 \end{bmatrix} \rightarrow \begin{bmatrix} 4 & 2 & 2 & [0] \\ \cancel{2} & 2 & [0] & 12 \\ 11 & 7 & 1 & \cancel{0} \\ \cancel{0} & [0] & 8 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 1 & 1 & [0] \\ [0] & 2 & \cancel{0} & 3 \\ 10 & 6 & [0] & \cancel{0} \\ \cancel{0} & [0] & 8 & 5 \end{bmatrix}$$

Cost = 44 + 56 + 90 + 44
= 234

Regression Analysis

In statistical study, regression analysis is a set of statistical process for estimating the relationships between a dependent variable and one or more independent variables.

This technique was developed by the British Biometrician Sir Francis Galton in 1877. He studied the average relationship between the two variables graphically, and named the line describing the relationship, the "Line of Regression"

Definition

Taro Yamane - "One of the most frequently used techniques in economics and business research to find a relation between two or more variables that are related is known as Regression Analysis."

Characteristics

- It consists of mathematical terms that are used to measure the average relationship between two or more closely related variables.
- It is used for estimating the unknown values of some dependent variable with reference to the known values of its related independent variables.
- It provides a mechanism for prediction or forecast of the values of one variables in terms of the other variables.
- It consists of two lines of equations.
  i.e. (i) Eqⁿ of X on Y
  (ii) Eqⁿ of Y on X

Regression Lines

(1) Y on X :

Here, x = given , y = to be determined
x = independent y = dependent

The eqⁿ of regression line Y on X is

$$\boxed{(y-\bar{y}) = b_{yx}(x-\bar{x})}$$

where, b_{yx} = regression coefficient Y on X

$$\Rightarrow b_{yx} = r \cdot \frac{\sigma_y}{\sigma_x}$$

$\bar{y}$ = mean of series Y
$\bar{x}$ = mean of series X
σ_y = S.D. of series Y
σ_x = S.D. of series X

(2) X on Y :

Here, y = given , x = to be determined
y = independent x = dependent

The eq^n of regression line X on Y is

$$(X - \bar{X}) = b_{xy}(Y - \bar{Y})$$

where, b_{xy} = regression coefficient X on Y

$$\Rightarrow b_{xy} = r \cdot \frac{\sigma_x}{\sigma_y}$$

Note :- Multiplying the regression coefficient of line Y on X and the regression coefficient of line X on Y, we get -

$$b_{yx} \cdot b_{xy} = r \cdot \frac{\sigma_y}{\sigma_x} \times r \cdot \frac{\sigma_x}{\sigma_y}$$

$$= r^2$$

$$\Rightarrow r^2 = b_{yx} \cdot b_{xy}$$

$$\Rightarrow r = \pm\sqrt{b_{yx} \cdot b_{xy}}$$

- The sign of both b_{yx} and b_{xy} should be either positive or negative.
- r^2 is positive or negative that depends upon the sign of regression coefficients.

Direct Method :-

$$b_{yx} = \frac{n\Sigma xy - \Sigma x \cdot \Sigma y}{n\Sigma x^2 - (\Sigma x)^2}, \quad b_{xy} = \frac{n\Sigma xy - \Sigma x \cdot \Sigma y}{n\Sigma y^2 - (\Sigma y)^2}$$

then find the regression line.

Shortcut Method

This method is useful for obtaining the equations of the lines of regression if the mean values $\bar{x}$ and $\bar{y}$ come out to be in fractions or if the values of x and y are large.

$$b_{yx} = \frac{cov(x,y)}{\sigma_x^2}$$

$\because b_{yx} = r \cdot \frac{\sigma_x}{\sigma_y}$

$$\Rightarrow b_{yx} = \frac{cov(x,y)}{\sigma_x \cdot \sigma_y} \cdot \frac{\sigma_y}{\sigma_x}$$

$$\Rightarrow b_{yx} = \frac{cov(x,y)}{\sigma_x^2}$$

$$\Rightarrow b_{yx} = \frac{\frac{\Sigma (X-\bar{X})(Y-\bar{Y})}{N}}{\frac{\Sigma (X-\bar{X})^2}{N}}$$

$$\Rightarrow \boxed{b_{yx} = \frac{\Sigma (X-\bar{X})(Y-\bar{Y})}{\Sigma (X-\bar{X})^2}}$$

We can also write it as,

$$\Rightarrow \boxed{b_{yx} = \frac{\Sigma dx \cdot dy}{\Sigma dx^2}}$$

Similarly, $b_{xy} = \frac{cov(x,y)}{\sigma_y^2}$

$$\Rightarrow b_{xy} = \frac{\frac{\Sigma (x-\bar{x})(y-\bar{y})}{N}}{\frac{\Sigma (y-\bar{y})^2}{N}}$$

$$\Rightarrow \boxed{b_{xy} = \frac{\Sigma (x-\bar{x})(y-\bar{y})}{\Sigma (y-\bar{y})^2}}$$

we can also convert it as

$$\boxed{b_{xy} = \frac{\Sigma dx \cdot dy}{\Sigma dy^2}}$$

$\because b_{xy} = r \cdot \frac{\sigma_x}{\sigma_y}$

$$\Rightarrow b_{xy} = \frac{cov(x,y)}{\sigma_x \cdot \sigma_y} \times \frac{\sigma_x}{\sigma_y}$$

$$\Rightarrow b_{xy} = \frac{cov(x,y)}{\sigma_y^2}$$

Q. Given the following values of X and Y

X: 3 5 6 8 9 11

Y: 2 3 4 6 5 8

Find the eq^n of regression line (i) Y on X (ii) X on Y by direct method.

X	Y	X^2	Y^2	XY
3	2	9	4	6
5	3	25	9	15
6	4	36	16	24
8	6	64	36	48
9	5	81	25	45
11	8	121	64	88
42	28	336	154	226

Here $\bar{X} = \frac{42}{6} = 7$

$\bar{Y} = \frac{28}{6} = 4.66$

According to Direct method,

$$b_{yx} = \frac{n\Sigma XY - \Sigma X \cdot \Sigma Y}{n\Sigma X^2 - (\Sigma X)^2}$$

$$= \frac{(6 \times 226) - (42 \times 28)}{(6 \times 336) - (42)^2} = \frac{1356 - 1176}{2016 - 1764}$$

$$= \frac{180}{252} = 0.71$$

And $b_{xy} = \dfrac{n\Sigma xy - \Sigma x \cdot \Sigma y}{n\Sigma y^2 - (\Sigma y)^2}$

$= \dfrac{(6 \times 226) - (42 \times 28)}{(6 \times 154) - (28)^2}$

$= \dfrac{180}{940 - 784}$

$= \dfrac{180}{156} = 1.15$

Now, Eq^n of regression line y on x is

$(y - \bar{y}) = b_{yx}(x - \bar{x})$

$\Rightarrow (y - 4.66) = 0.71 (x - 7)$

$\Rightarrow y - 4.66 = 0.71x - 4.97$

$\Rightarrow 0.71x - y = 0.31$

$\Rightarrow 0.71x - y - 0.31 = 0$

And Eq^n of regression line x on y is

$(x - \bar{x}) = b_{xy}(y - \bar{y})$

$\Rightarrow (x - 7) = 1.15 (y - 4.66)$

$\Rightarrow x - 7 = 1.15y - 5.35$

$\Rightarrow x - 1.15y = 1.65$

$\Rightarrow x - 1.15y - 1.65 = 0$

Q: Find the regression equation of y on x and x on y. where x and y are the marces obtained by 10 students as given below. (By shortcut method)

→

x	y	dx	dy	dx^2	dy^2	dxdy
20	20	-23	-26.5	529	702.25	609.5
45	60	2	13.5	4	182.25	27
65	55	22	8.5	484	72.25	187
40	45	-3	-1.5	9	2.25	4.5
55	75	12	28.5	144	812.25	342
35	35	-8	-11.5	64	132.25	92
15	25	-28	-21.5	784	462.25	602
80	90	37	43.5	7369	1892.25	1609.5
25	10	-18	-36.5	324	1332.25	657
50	50	7	3.5	49	12.25	24.5
430	465			3760	5528.5	4155

$\bar{x} = \dfrac{430}{10} = 43$

$\bar{y} = \dfrac{465}{10} = 46.5$

According to shortcut method,

$$b_{yx} = \frac{\Sigma dx.dy}{\Sigma dx^2} = \frac{4155}{3760} = 1.10$$

and $$b_{xy} = \frac{\Sigma dx.dy}{\Sigma dy^2} = \frac{4155}{5528.5} = 0.75$$

Now, Eq^n of regression line y on x is

$$(y-\bar{y}) = b_{yx}(x-\bar{x})$$

$$\Rightarrow (y-46.5) = 1.10(x-43)$$

$$\Rightarrow y - 46.5 = 1.10x - 47.3$$

$$\Rightarrow 1.1x - y = -0.8$$

$$\Rightarrow 1.1x - y + 0.8 = 0$$

Eq^n of regression line x on y is

$$(x-\bar{x}) = b_{xy}(y-\bar{y})$$

$$\Rightarrow (x-43) = 0.75(y-46.5)$$

$$\Rightarrow x - 43 = 0.75y - 34.87$$

$$\Rightarrow x - 0.75y = 8.13$$

$$\Rightarrow x - 0.75y - 8.13 = 0$$

Q. Out of the following two lines of regression, which one is the regression line of x only and which one is the regression line of y on x ?

$4x - 5y + 30 = 0$ and $20x - 9y - 107 = 0$.

→ Let us consider that,

$$4x - 5y + 30 = 0 \quad \text{——(1)}$$

$$20x - 9y - 107 = 0 \quad \text{——(2)}$$

Assume that,

Eq^n(1) is the regression line of x on y

Eq^n(2) is the regression line of y on x.

Then according to the form of the regression line of x on y, eq^n(1) becomes-

$$4x = 5y - 30$$

$$\Rightarrow x = \frac{5}{4}y - \frac{15}{2} \quad \text{——(3)}$$

Similarly, according to the form of the regression line of y on x, eq^n(2) becomes-

$$9y = 20x - 107$$

$$\Rightarrow y = \frac{20}{9}x - \frac{107}{9} \quad \text{——(4)}$$

From eq^n(3), $b_{xy} = \frac{5}{4}$

From eq^n(4), $b_{yx} = \frac{20}{9}$

Now, we know that,

$$r^2 = b_{xy} \cdot b_{yx}$$

Putting the values in above, we get -

$$r^2 = \frac{5}{4} \times \frac{20}{9} = \frac{25}{9}$$

$$\Rightarrow r^2 = 2.77$$

$$\Rightarrow r = 1.66$$

Here $r^2 > 1$, so that $r \notin [-1, 1]$

But, from Correlation Analysis, we know that,

'r' always lies between $[-1, 1]$ $\left[\because -1 \leq r \leq r\right]$

Hence, it is clear that,

Our assumption is wrong

So again we have to rewrite the given equations by assuming as below:

Eq^n(1) is the regression line of y on x

Eq^n(2) " " " " of x on y

Then, according to the forms of the regression line of y on x & x on y, we can write,

eq^n(1) becomes, $5y = 4x + 30$

$$\Rightarrow y = \frac{4}{5}x + 6 \quad \text{——— (5)}$$

eq^n(2), becomes, $20x = 9y + 107$

$$\Rightarrow x = \frac{9}{20}y + \frac{107}{20} \quad \text{——— (6)}$$

Now, From eq^n(5), $b_{yx} = \frac{4}{5}$

From eq^n(6), $b_{xy} = \frac{9}{20}$

we have, $r^2 = b_{yx} \cdot b_{xy}$

$$\Rightarrow r^2 = \frac{4}{5} \times \frac{9}{20} = \frac{9}{25}$$

$\Rightarrow r^2 = 0.36$

$\Rightarrow r = \pm 0.6$

Here $r^2 < 1$ so that, $r \in [-1, 1]$

whether, r is positive or negative that depends upon the sign of regression coefficients.

Here we have to take the value of r in positive

[$\because$ b_{xy} & b_{yx} both are positive]

So, our second assumption is right.

i.e $4x - 5y + 30 = 0$ is the regression line of y on x and $20x - 9y - 107 = 0$ is the regression line of x on y.

Q. What is Game Theory.

→ Game is defined as the activity between two or more person which involves with a certain set of rules. At the end of the game each person recieves some benefits or sufferes loss. The set of rules and procedures defines the game. A competitive situation is called game. The term game represents a conflict between two or more parties.

Game Theory

Module-II — Linear Programming Problem — (Quantitative Technique)

Linear Programming Problem is an optimization problem, where there is an objective function and subject to constraints and non-negative restriction.

There are 2 parts in LPP —

(I) Formulation of LPP

(II) Solution of LPP → (i) Graphical solution
(ii) Simplex Method

* Formulation of LPP

A manufacturing unit for a production company produce 2 types of products M_1 & M_2. Product M_1 requires, 4 hours of grinding and 2 hours of polishing, where product M_2 requires 2 hours of grinding and 5 hours of polishing. Then the company has 2 grinders and 3 polishers, each grinders works 40 hrs a week and each polishers works 60 hrs a week. After production the product M_1 is Rs. 3 profit and M_2 is Rs. 4 profit per unit. Formulate a mathematical modle to maximize the profit.

→ Step-1 Define the decision variables

Step-2 Construct the objective function

Step-3 Formulate the subject to constraint

Step-4 Declare the non-negative constraint-

→ (Step-1) Let X_1 and X_2 are two decision variables.

X_1 stands for product M_1

X_2 " " " M_2

Let X_1 & X_2 units of products produced in that company for M_1 and M_2

(Step-2) Since profit on 1 unit for product M_1 is 3 rupees

So total product profit on X_1 is $3X_1$

Similarly total product M_2 profit is $4X_2$

The objective function is here $Z = 3X_1 + 4X_2$

$$\text{Max } Z = 3X_1 + 4X_2$$

Step-3

	x_1	x_2	
Grinderer	4	2	$\leq$ 80 hrs
Polisherr	2	5	$\leq$ 180 hrs

Step-4

$x_1 \geq 0,\ x_2 \geq 0$

→ Max $z = 3x_1 + 4x_2$

$4x_1 + 2x_2 \leq 80$

$2x_1 + 5x_2 \leq 180$

$x_1, x_2 \geq 0$

Graphical Solution

If the LPP having two variables only, then it can be solve by graphical method, if the LPP having more than 2 variables or 2 variables, then it can be solve by simplex method.

There 4 types of Graphical solution

(1) Unique solution
(2) No solution
(3) Infinetly many solution
(4) Unbounded solution

→ Step-1 Express all the inequality constraints into equation

$4x_1 + 2x_2 = 80$

x_1	0	20
x_2	40	0

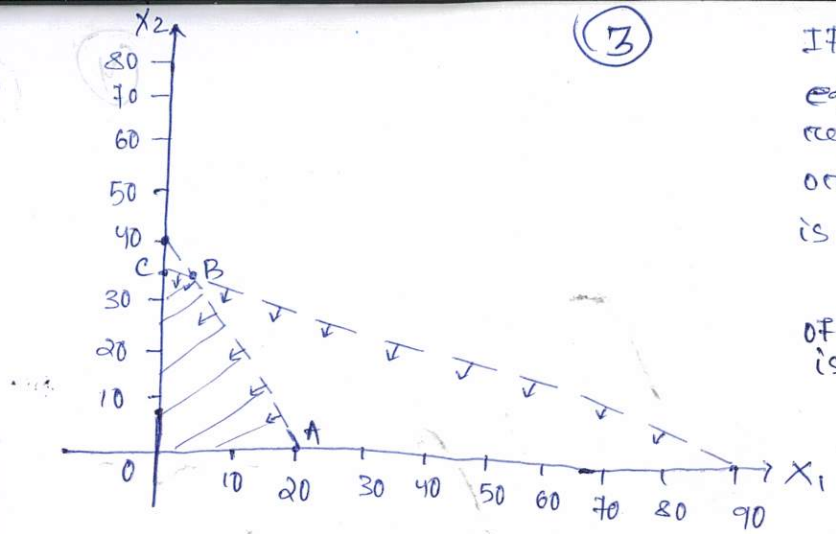
$(x_1, x_2) = (0, 40), (20, 0)$

$2x_1 + 5x_2 = 180$

x_1	0	90
x_2	36	0

$(x_1, x_2) = (0, 36)$ & $(90, 0)$

The solution of LPP in graphical technique will be found at the corner points of the feasible region.



If the constraint having less than equal to sign, then consider the region below the line towards the origin. Here the common region is OABC

The common region, which satisfy of the constraints of the given LPP is known common feasible region.

Points	Max $Z = 3X_1 + 4X_2$
O (0,0)	$Z = 0$
A (20,0)	$Z = 60$
B (2.5, 35)	$Z = 7.5 + 140 = 147.5$
C (0, 36)	$Z = 144$

Z is maximum at point B(2.5, 35)
So the optimal solution is $X_1 = 2.5$, $X_2 = 35$
Max $Z = 147.5$

$4X_1 + 2X_2 = 80$ — (1)
$2X_1 + 5X_2 = 180$ — (2)

multiplying (2) on eq^n (2) & subtract it with eq^n (1)

$4X_1 + 2X_2 = 80$
$4X_1 + 10X_2 = 360$
$-8X_2 = -280$
$X_2 = 35$

Putting X_2 on eq^n (1), we get -

$4X_1 + 2 \times 35 = 80$
$\Rightarrow 4X_1 = 80 - 70 = 10$
$\Rightarrow X_1 = \frac{10}{4} = 2.5$
$(X_1, X_2) = (2.5, 35)$

Q. Min $Z = 20X_1 + 10X_2$

$X_1 + 2X_2 \leq 40$
$3X_1 + X_2 \geq 30$
$4X_1 + 3X_2 \geq 60$
$X_1, X_2 \geq 0$

$X_1 + 2X_2 = 40$

X_1	0	40
X_2	20	0

(X_1, X_2) (0,20), (40,0)

$3X_1 + X_2 = 30$

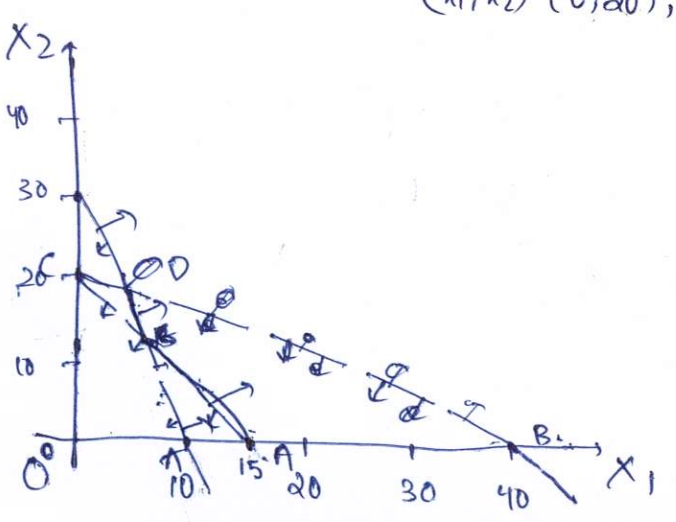
X_1	0	10
X_2	30	0

(0,30) (10,0)

$4X_1 + 3X_2 = 60$

X_1	0	15
X_2	20	0

(0,20), (15,0)



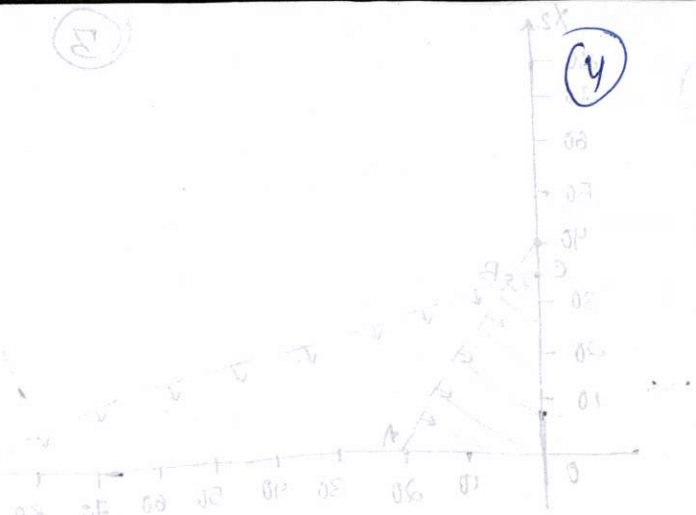
Points	Max Z
O (0,0)	0
A (10,0)	200
B (6,6)	180
C (0,20)	200

$3X_1 + 6X_2 = 120$
$3X_1 + X_2 = 30$
$5X_2 = 90$
$X_2 = 18$
$X_1 =$

$3X_1 + 6X_2 = 90$
$4X_1 + 3X_2 = 60$
$5X_1 = 30$
$X_1 = 6$
$24 + 3X_2 = 60$
$\Rightarrow 3X_2 = 36$
$\Rightarrow X_2 = 12$

(4)

Points	Max Z
A(15,0)	300
B(40,0)	800
C(4,18)	260
D(6,12)	240



Max Z = 240

$x_1 = 6, x_2 = 12$

Assignments

Q.1 Max $Z = X_1 - 3X_2$

$X_1 + X_2 \leq 300$

$X_1 - 2X_2 \leq 200$

$2X_1 + X_2 \leq 100$

$X_2 \leq 200$

$X_1, X_2 \geqslant 0$

→ $X_1 + X_2 = 300$

X_1	0	300
X_2	300	0

(0,300) & (300,0)

$X_1 - 2X_2 = 200$

X_1	0	200
X_2	−100	0

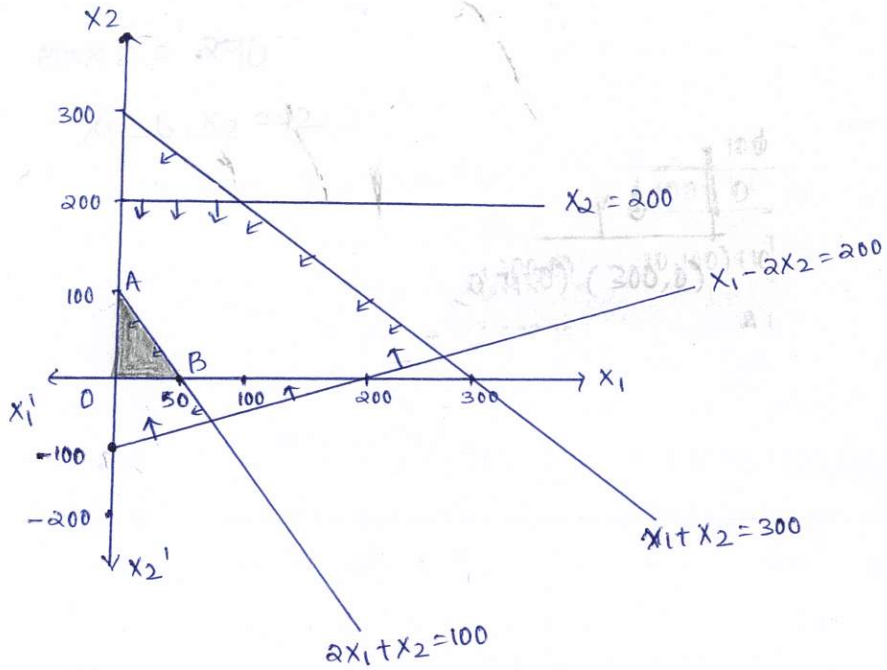
(0,−100) & (200,0)

$2X_1 + X_2 = 100$

X_1	0	50
X_2	100	0

(0,100) & (50,0)

and $X_2 = 200$ is ~~the~~ a line itself.



Here OAB is the common feasible region.

Points	Max $Z = X_1 - 3X_2$
O (0,0)	$Z = 0$
A (0,100)	$Z = 0 - 3\times100 = -300$
B (50,0)	$Z = 50 - 3\times0 = 50$ (Maximum)

Z is maximum at point B(50,0)

optimal solution is $X_1 = 50, X_2 = 0$

Max $Z = 50$

Q.2

$\text{Max } Z = 5x + 8y$

$3x + 2y \leq 36$

$x + 2y \leq 20$

$3x + 4y \leq 42$

$x, y \geq 0$

$3x+2y=36$

x	0	12
y	18	0

(0,18) & (12,0)

$x+2y=20$

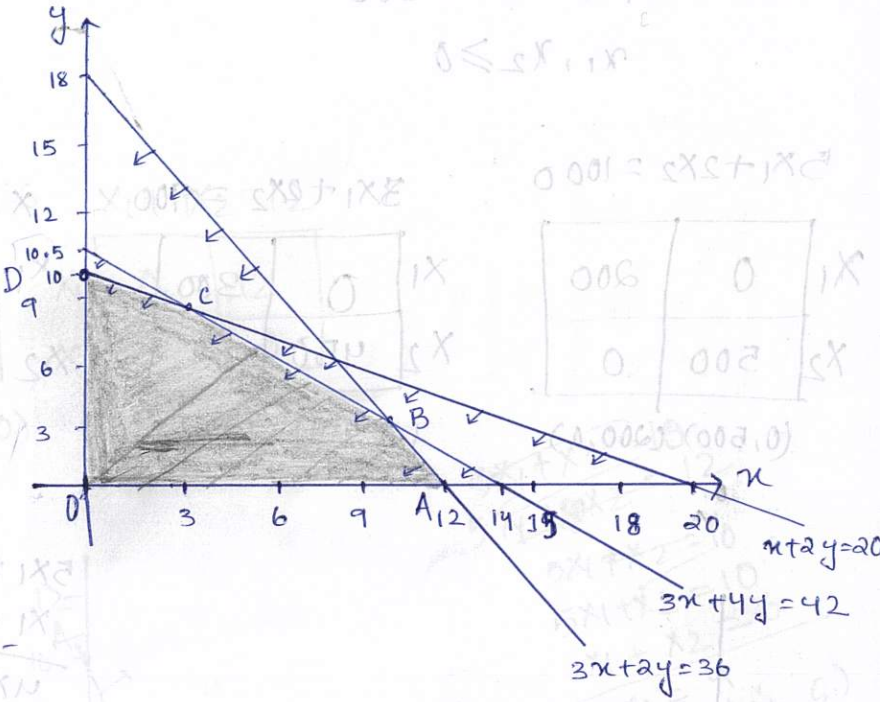
x	0	20
y	10	0

(0,10) & (20,0)

$3x+4y=12$

x	0	14
y	10.5	0

(0,10.5) & (14,0)



- The common region is OABCD

Now point B is intersection point of $(3x+2y=36)$ & $(3x+4y=42)$

Thus substracting both the eq^n, we get -

$$3x + 2y = 36$$
$$\underline{3x + 4y = 42}$$
$$-2y = -6$$
$$y = 3$$

Putting 'y' value on eq^n (1), we get -

$3x + 2 \times 3 = 36$

$\Rightarrow 3x = 30$

$\Rightarrow x = 10$

$\therefore B(10, 3)$

Similarly 'C' is intersection point of $(x+2y=20)$ and $(3x+4y=42)$

By solving them, we get

$3 \times$ eq^n (1) $\rightarrow$ $3x + 6y = 60$

$$\underline{3x + 4y = 42}$$
$$2y = 18$$
$$y = 9$$

Putting 'y' on eq^n (2), we get -

$3x + 36 = 42$

$\Rightarrow 3x = 6$

$\Rightarrow x = 2$

$\therefore C(2, 9)$

Points	Max $Z = 5x + 8y$
O (0,0)	$Z = 0$
A (12,0)	$Z = 60$
B (10,3)	$Z = 50 + 24 = 74$
C (2,9)	$Z = 10 + 72 = 82$ (Maximum).
D (0,10)	$Z = 80$

Z is maximum at point C(2,9),

optimal solution is $x = 2$, $y = 9$ and Max $Z = 82$

& the LPP has a unique solution.

Q. Max $z = 100x_1 + 40x_2$

s.t $5x_1 + 2x_2 \leq 1000$

$3x_1 + 2x_2 \leq 900$

$x_1 + 2x_2 \leq 500$

$x_1, x_2 \geq 0$

$5x_1 + 2x_2 = 1000$

x_1	0	200
x_2	500	0

(0,500) (200,0)

$3x_1 + 2x_2 = 900$

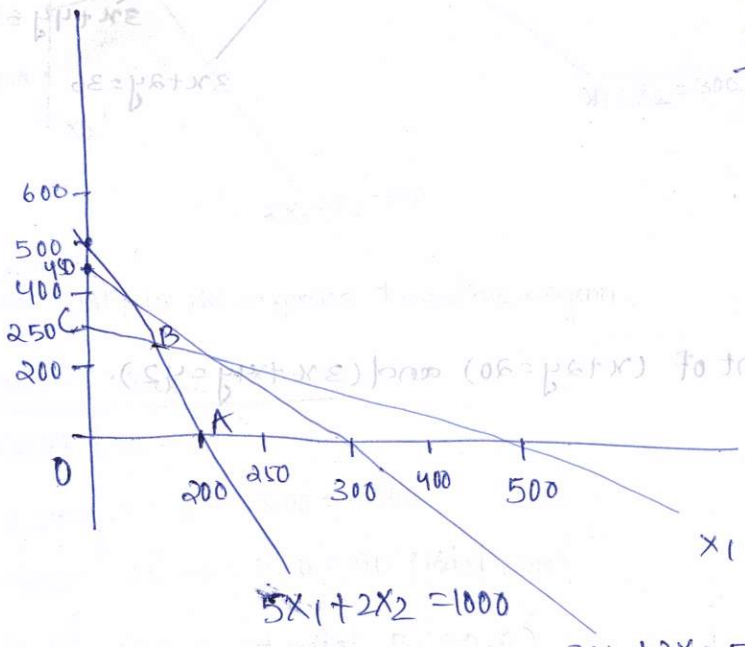
x_1	0	300
x_2	450	0

(0,450) (300,0)

$x_1 + 2x_2 = 500$

x_1	0	500
x_2	250	0

(0,250) (500,0)



$5x_1 + 2x_2 = 1000$

$x_1 + 2x_2 = 500$

$4x_1 = 500$

$x_1 = \frac{500}{4} = 125$

$125 + 2x_2 = 500$

$2x_2 = 375$

$x_2 = 187.5$

point	Max $z = 100x_1 + 40x_2$
0 (0,0)	$z = 0$
A (200,0)	$z = 20{,}000$
B (125, 187.5)	$z = 100 \times 125 + 40 \times 187.5 = 12500 + 7500 = 20{,}000$
C (0,250)	$z = 18{,}000$

This problem has infinetly many solution.

(8)

Min $Z = 3X_1 + 2X_2$

S.t $5X_1 + X_2 \geq 10$

$X_1 + X_2 \geq 6$

$X_1 + 4X_2 \geq 12$

$X_1, X_2 \geq 0$

~~$5X_1 + X_2 = 10$~~

X_1	1	0.2
X_2	5	1

1, -5

$X_1 + X_2 = 6$

X_1	0	6
X_2	6	0

(0,6) (6,0)

$X_1 + 4X_2 = 12$

X_1	0	12
X_2	3	0

(0,3) (12,0)

X_1	0	2
X_2	10	0

(0,10), (2,0)

$5X_1 + X_2 = 10$

$5X_1 + 4X_2 = 12$

$5X_1 + X_2 = 10$

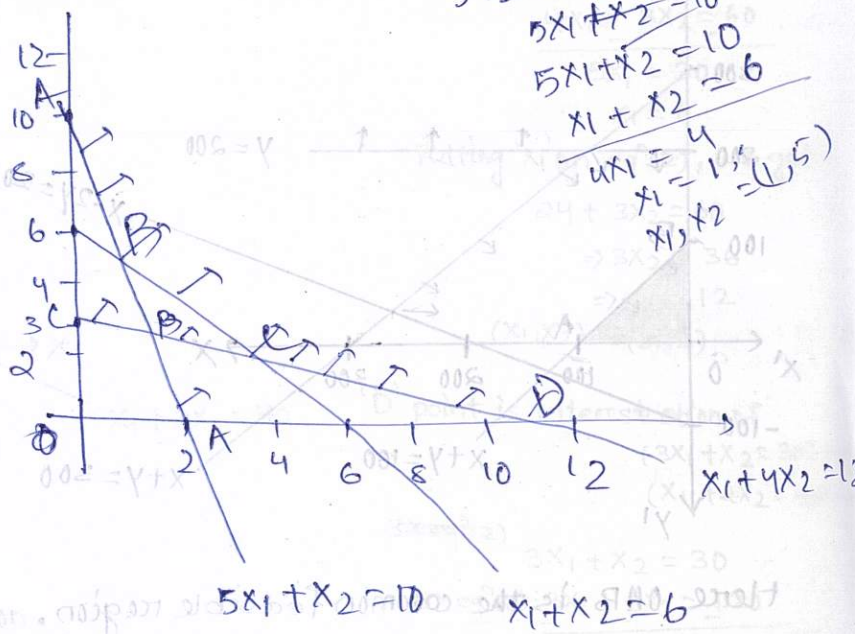
$5X_1 + X_2 = 10$

$X_1 + X_2 = 6$

$4X_1 = 4$

$X_1 = 1$; (1,5)

X_1, X_2



~~ETP~~	Max $Z = 3X_1 + 2X_2$
A(10,0)	$Z = 30$
B(1,5)	$Z = 3 + 10 = 13$ (Min)
C(4,2)	$Z = 16$
D(12,0)	$Z = 36$

In this case if the objective function is a maximization one, then the soln is unbounded because the feasible region has no boundary.

Q. Max $Z = x - 3y$

s.t $x + y \leq 300$

$x - 2y \leq 200$

$x + y \leq 100$

$y \geqslant 200$

$x, y \geqslant 0$

→ $x + y = 300$

x	0	300
y	300	0

(0,300) (300,0)

$x - 2y = 200$

x	0	200
y	-100	0

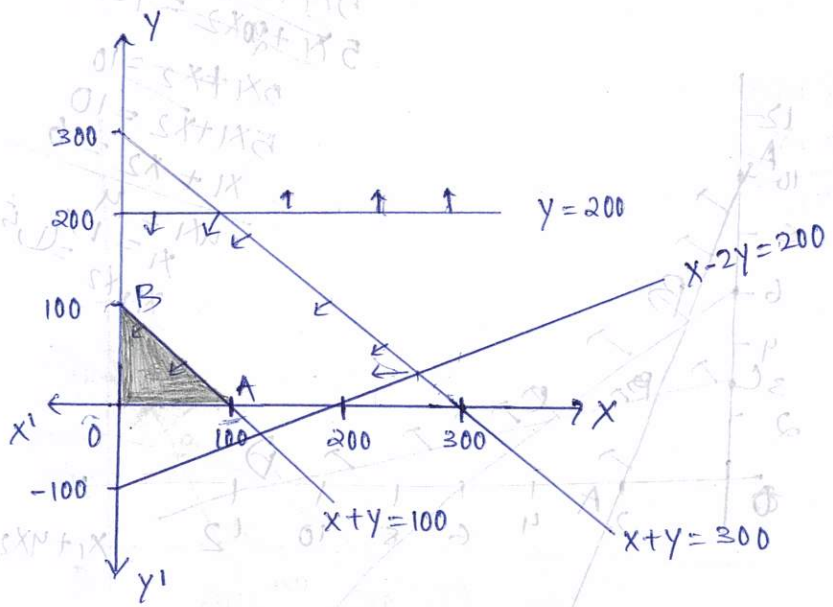
(0,-100) (200,0)

$x + y = 100$

x	0	100
y	100	0

(0,100) (100,0)

$y = 200$ line



Here OAB is the common feasible region.

Points	Max $Z = x - 3y$
O(0,0)	$Z = 0$
A(100,0)	$Z = 100$ (Maximum)
B(0,100)	$Z = -300$

∴ Z is maximum at point A(100,0). Optimal solution is $x = 100$, $y = 0$ & Max $Z = 100$.

Q.2

(10)

Max $Z = 20X_1 + 10X_2$

s.t. $X_1 + 2X_2 \leq 40$

$3X_1 + X_2 \geq 30$

$4X_1 + 3X_2 \geq 60$

$X_1, X_2 \geq 0$

→

$X_1 + 2X_2 = 40$

X_1	0	40
X_2	20	0

(0,20) (40,0)

$3X_1 + X_2 = 30$

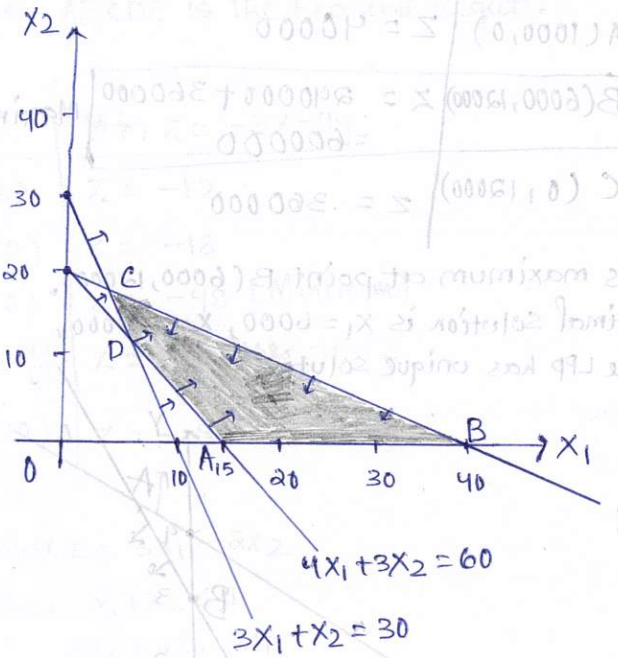
X_1	0	10
X_2	30	0

(0,30) (10,0)

$4X_1 + 3X_2 = 60$

X_1	0	15
X_2	20	0

(0,20) (15,0)



Here ABCD is the feasible region.

Points	Max $Z = 20X_1 + 10X_2$
A(15,0)	Z = 300
B(40,0)	Z = 800 (Maximum)
C(6,12)	Z = 120 + 120 = 240
D(4,18)	Z = 80 + 180 = 260

'C' point is intersection of $(3X_1 + X_2 = 30) - (1)$

$(4X_1 + 3X_2 = 60) - (2)$

3 × eq'(1) $9X_1 + 3X_2 = 90$

$4X_1 + 3X_2 = 60$

$5X_1 = 30$

$X_1 = 6$

Putting X_1 on eq'(2), we get -

$24 + 3X_2 = 60$

$\Rightarrow 3X_2 = 36$

$\Rightarrow X_2 = 12$

$(X_1, X_2) = (6, 12)$

'D' point is intersection of

$(3X_1 + X_2 = 30) - (1)$

$(X_1 + 2X_2 = 40) - (2)$

~~3 × eq'(2)~~ $3X_1 + X_2 = 30$

3 × eq'(2) $3X_1 + 6X_2 = 120$

$-5X_2 = -90$

$X_2 = 18$

~~$(X_1 + X_2) =$~~

Putting X_2 on eq'(1), we get -

$3X_1 + 18 = 30$

$\Rightarrow 3X_1 = 12$

$\Rightarrow X_1 = 4$

$(X_1, X_2) = (4, 18)$

∴ Z is maximum at point B(40,0). The LPP has a unique solution, whence optimal solution is $X_1 = 40$, $X_2 = 0$ & Max $Z = 800$.

Q.6

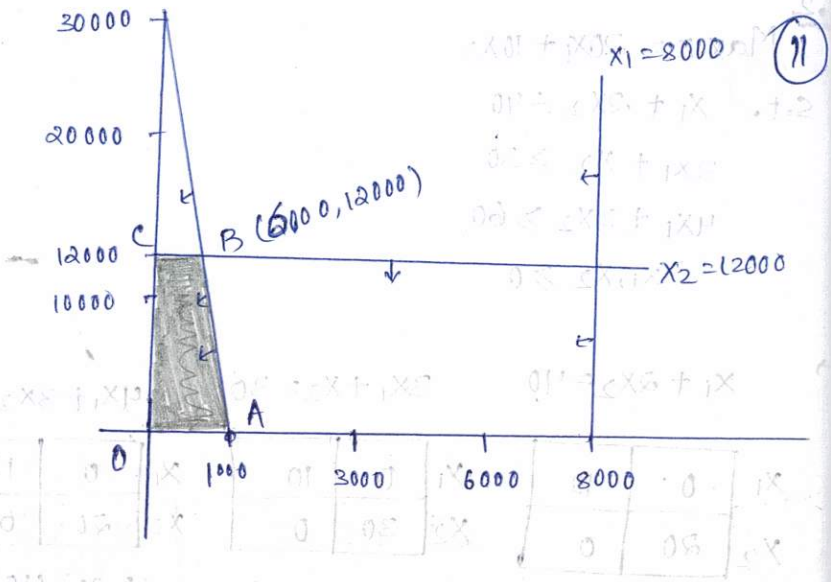
→ Max $Z = 40x_1 + 30x_2$

s.t. $3x_1 + x_2 \leq 30000$

$x_1 \leq 8000$

$x_2 \leq 12000$

$x_1, x_2 \geq 0$



$3x_1 + x_2 = 30000$

x_1	0	10000
x_2	30000	0

$x_1 = 8000$ and $x_2 = 12000$

Point 'B' is the intersection of $3x_1 + x_2 = 30000$ and $x_2 = 12000$

Thus $3x_1 + 12000 = 30000$

$\Rightarrow 3x_1 = 18000$

$\Rightarrow x_1 = 6000$

$(x_1, x_2) = (6000, 12000)$

Points	Max $Z = 40x_1 + 30x_2$	
A(1000, 0)	$Z = 40000$	
B(6000, 12000)	$Z = 240000 + 360000 = 600000$	Maximum
C(0, 12000)	$Z = 360000$	

∴ Z is maximum at point B(6000, 12000).
optimal solution is $x_1 = 6000$, $x_2 = 12000$.
The LPP has unique solution.

Q.8 Min $Z = 3x - 2y$

s.t. $-2x + 3y \leq 9$

$x - 5y \geq -20$

$x, y \geq 0$

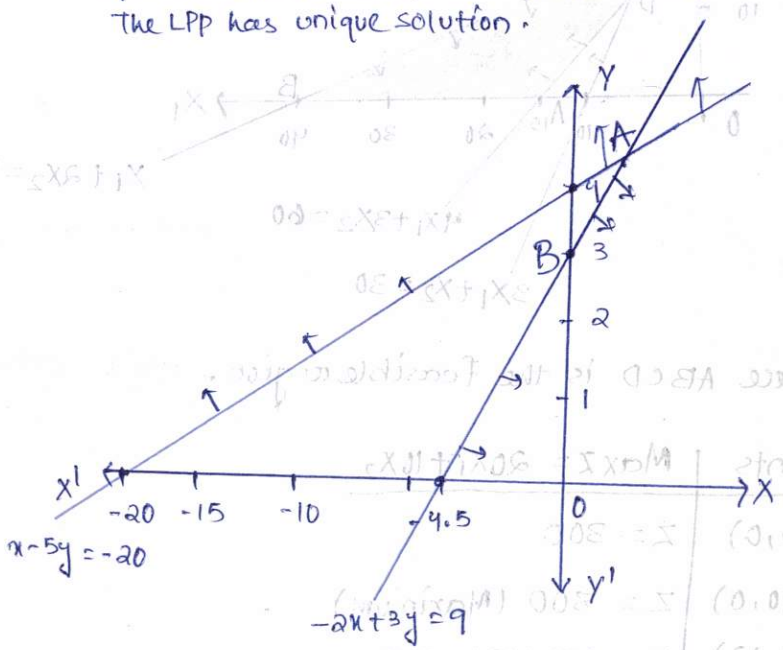
→ $-2x + 3y = 9$ $\quad x - 5y = -20$

x	0	-4.5
y	3	0

(0, 3) (-4.5, 0)

x	0	-20
y	4	0

(0, 4) (-20, 0)



A is the intersection of $x - 5y = -20$ and $-2x + 3y = 9$

2 × eq^n(1) $\quad 2x - 10y = -40$

$-2x + 3y = 9$

$-7y = -31$

$y = 4.42$

$x - 22.14 = -20$

$\Rightarrow x = 2.14$

$(x, y) = (2.14, 4.42)$

Points	Max $Z = 3x - 2y$
A(2.14, 4.42)	$Z = 6.42 - 8.84 = -2.42$
B(0, 3)	$Z = -6$

Q.8

Min $Z = -6x_1 - 4y$

s.t. $2x + 3y \geq 30$

$3x + 2y \leq 24$

$x + y \geq 3$

$x, y \geq 0$

→ $2x + 3y = 30$, $3x + 2y = 24$, $x + y = 3$

x	0	15
y	10	0

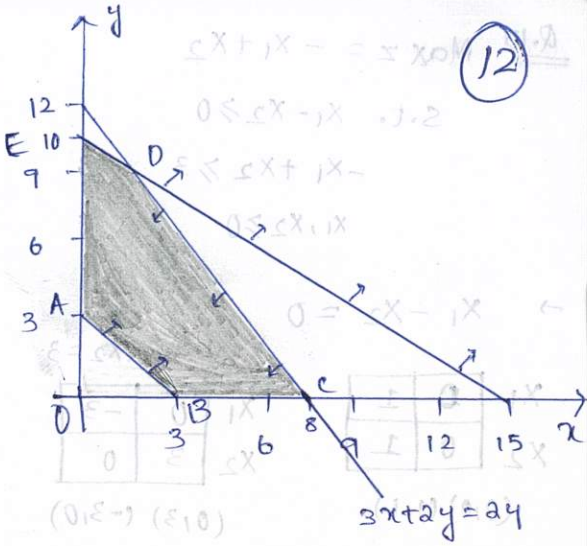
(0,10) (15,0)

x	0	8
y	12	0

(0,12) (8,0)

x	0	3
y	3	0

(0,3) (3,0)



Hence ABCDE is the feasible region.

Points	Min $Z = -6x - 4y$
A(0,3)	$Z = -12$
B(3,0)	$Z = -18$
C(8,0)	$Z = -48$ (Minimum)
D($\frac{12}{5}$, $\frac{42}{5}$)	$Z = \frac{-48}{5} - \frac{168}{5} = -43.2$
E(0,10)	$Z = -40$

Point D is the intersection point of $2x + 3y = 30$ and $3x + 2y = 24$,

3 × eq'n(1) = $6x + 9y = 90$

2 × eq'n(2) = $6x + 4y = 48$

$5y = 42$

$y = \frac{42}{5}$,

Putting 'y' value on eq'n(1), we get-

$2x + \frac{126}{5} = 30$, $\Rightarrow 2x = \frac{150 - 126}{5}$

$\Rightarrow 2x = \frac{24}{5} \Rightarrow x = \frac{12}{5}$

$\therefore (x, y) = (\frac{12}{5}, \frac{42}{5})$

Q.9 Max $Z = 3x_1 - 2x_2$

s.t. $x_1 + x_2 \leq 1$

$2x_1 + 2x_2 \geq 4$

$x_1, x_2 \geq 0$

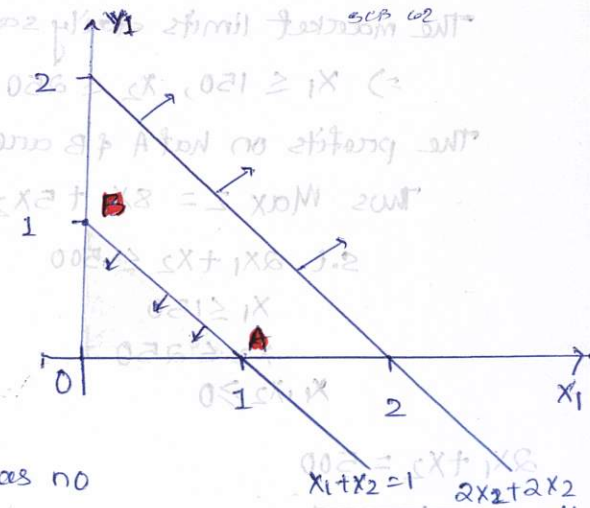
→ $x_1 + x_2 = 1$, $2x_1 + 2x_2 = 4$

x_1	0	1
x_2	1	0

(0,1) (1,0)

x_1	0	2
x_2	2	0

(0,2) (2,0)



~~Hence DAB is the feasible region.~~

This LPP has no feasible region.

Hence this an infeasible solution.

~~Points~~	~~Max $Z = 3x_1 - 2x_2$~~
~~O(0,0)~~	~~$Z = 0$~~
~~A(1,0)~~	~~$Z = 3$ (Maximum)~~
~~B(0,1)~~	~~$Z = -2$~~

~~The LPP has unique solution, Z is maximum at point A(1,0).~~

~~Optimal solution $x_1 = 1$, $x_2 = 0$.~~

Q.10 Max $z = -x_1 + x_2$

s.t. $x_1 - x_2 \geq 0$

$-x_1 + x_2 \geq 3$

$x_1, x_2 \geq 0$

→ $x_1 - x_2 = 0$

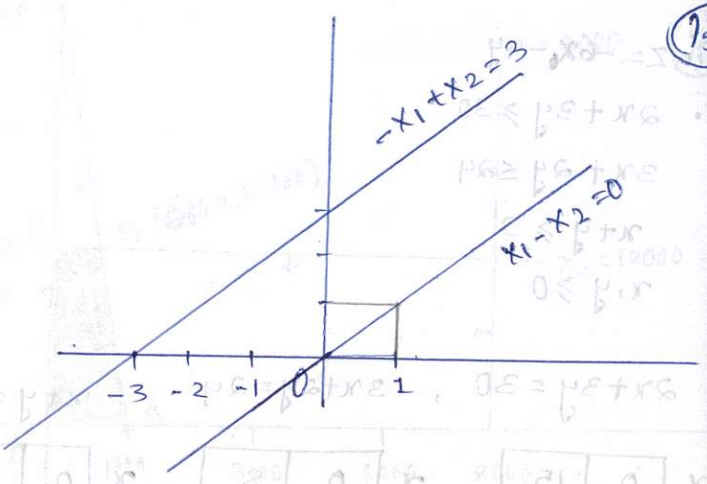
x_1	0	1
x_2	0	1

(0,0) (1,1)

$-x_1 + x_2 = 3$

x_1	0	-3
x_2	3	0

(0,3) (-3,0)



This LPP has no feasible region, hence no solution.

Q. → A company produces 2 types of hats A and B.

Let x_1 for total production for hat A

& x_2 " " " " " B.

Every hat A requires twice as much labour time as the second hat B.

Thus $2x_1 + x_2$.

If the company only hat B, then it can produce a total of 500 hats per day. that means in one day, company can also produce twice of hat A compared to hat B, and the total hat is equal to 500.

That implies $2x_1 + x_2 \leq 500$.

The market limits daily sales of hat A & B to 150 & 250 respectively.

$\Rightarrow x_1 \leq 150, \; x_2 \leq 250$

The profits on hat A & B are Rs.8 & Rs.5 respectively.

Thus Max $Z = 8x_1 + 5x_2$

s.t $2x_1 + x_2 \leq 500$

$x_1 \leq 150$

$x_2 \leq 250$

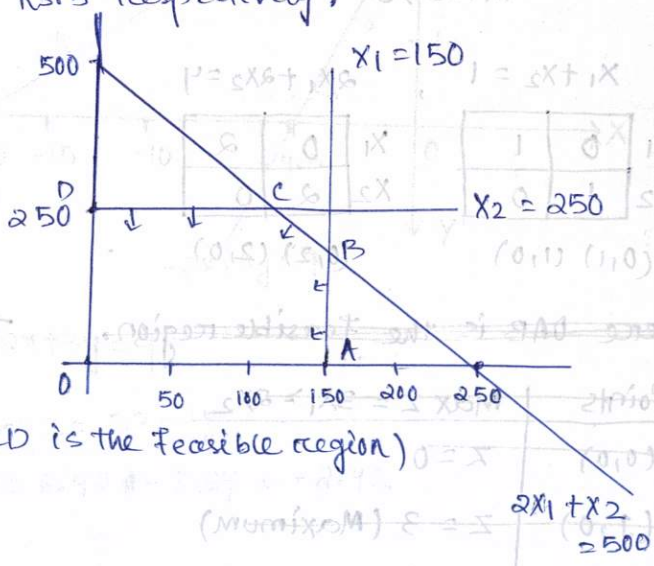
$x_1, x_2 \geq 0$

$2x_1 + x_2 = 500$

x_1	0	250
x_2	500	0

$x_1 = 150$

$x_2 = 250$



(OABCD is the feasible region)

Points	Max $Z = 8x_1 + 5x_2$
O (0,0)	$Z = 0$
A (150,0)	$Z = 1200$
B (150,200)	$Z = 2200$
C (125, 250)	$Z = 2250$ (Maximum)
D (0,250)	$Z = 1250$

Assignment

d.

Given AM of 5 observations, $\bar{x} = 6$

and $\sigma_x^2 = 8$

Also three observations are 2, 6 and 10

let others two are x_1 and x_2

Thus $\frac{2+6+10+x_1+x_2}{5} = 6$ (Given)

$\Rightarrow x_1 + x_2 + 18 = 30$

$\Rightarrow x_1 + x_2 = 12$ ——(1)

Now $\sigma_x^2 = 8$

$\Rightarrow \frac{\Sigma x^2}{n} - (\bar{x})^2 = 8$

$\Rightarrow \frac{x_1^2 + x_2^2 + 2^2 + 6^2 + 10^2}{5} - 6^2 = 8$

$\Rightarrow \frac{x_1^2 + x_2^2 + 140}{5} = 8 + 36$

$\Rightarrow x_1^2 + x_2^2 + 140 = 220$

$\Rightarrow x_1^2 + x_2^2 = 80$ ——(11)

Again from eq^n(1), we have $x_2 = (12 - x_1)$

Now putting x_2 value on eq^n(2), we get -

$x_1^2 + (12 - x_1)^2 = 80$

$\Rightarrow x_1^2 + 144 + x_1^2 - 24x_1 - 80 = 0$

$\Rightarrow 2x_1^2 - 24x_1 + 64 = 0$

$\Rightarrow x_1^2 - 12x_1 + 32 = 0$

$x_1 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{12 \pm \sqrt{144 - 128}}{2}$

$= \frac{12 \pm \sqrt{16}}{2}$

$= \frac{12 \pm 4}{2}$

$= \frac{12+4}{2}$ or $\frac{12-4}{2}$

$= 8$ or 4

Now putting x_1 value on eq^n(1), we get -

If $x = 8$, then eq^n(1) will be -

$8 + x_2 = 12$

$\Rightarrow x_2 = 4$

If $x = 4$, then eq^n(1) will be -

$4 + x_2 = 12$

$\Rightarrow x_2 = 8$

$\therefore (x_1, x_2) = (8, 4)$ or $(4, 8)$

(e)

→ Given $f(x,y) = 0.5$

$n = 10$

As $f(x,y) = 0.5$

$$\Rightarrow 1 - \frac{6\Sigma d^2}{n(n^2-1)} = 0.5$$

$$\Rightarrow -\frac{6\Sigma d^2}{10(10^2-1)} = 0.5 - 1$$

$$\Rightarrow \frac{6\Sigma d^2}{990} = 0.5$$

$$\Rightarrow 6\Sigma d^2 = 495$$

$$\Rightarrow \Sigma d^2 = \frac{495}{6} = 82.5$$

Later on it was found that the difference between the ranks (d) in the two subject was wrongly taken as 3 in place of 7.

Hence correct Σd^2 = Wrong Σd^2 - (wrongly taken observation)^2 + (right observation)^2

$$\text{correct } \Sigma d^2 = 82.5 - 3^2 + 7^2$$

$$\text{correct } \Sigma d^2 = 82.5 - 9 + 49 = 122.5$$

So correct $f(x,y) = 1 - \frac{6\Sigma d^2}{n(n^2-1)} = 1 - \frac{6 \times 122.5}{990}$

$$= 1 - \frac{735}{990}$$

$$= 1 - 0.74$$

$$= 0.25$$

(9)

	A	B	C	D	E
1	7	9	13	8	9
2	10	7	7	8	7
3	8	9	12	9	5
4	12	6	10	4	6
5	0	0	0	0	0

$\rightarrow$

0	2	6	1	2
3	0	0	1	0
3	4	7	4	0
8	2	6	0	2
0	0	0	0	0

(Row reduction)

$\rightarrow$

0	2	6	1	2
3	0	0	1	0
3	4	7	4	0
8	2	6	0	2
0	0	0	0	0

(Column reduction)

[0]	2	6	1	2
3	[0]	~~0~~	1	~~0~~
3	4	7	4	[0]
8	2	6	[0]	2
~~0~~	~~0~~	[0]	~~0~~	~~0~~

Assignment Process

$\rightarrow$

Machine		Job	cost
1	$\rightarrow$	A	7
2	$\rightarrow$	B	7
3	$\rightarrow$	E	5
4	$\rightarrow$	D	4
			23

SIMPLEX METHOD

A LPP having two variable or more than two variable can be solved by simplex method.

Step-1 Put the LPP into standard form

1. The objective function must be in maximize form If it is minimize, then convert to maximize by multiplying (-) ve sign to both the sign.
2. Convert all the inequality constraints into eq^n by adding slack variable and subtract surplus variables.
3. The right hand side constants of the constraints must be positive, if not then multiply a (-ve) sign to both the side of constraints, ~~then~~.

Slack Variable

When the inequality constraints having ($\leq$) then add a slack variable to make it equation and the coefficient of slack variable to the objective function is zero.

$C_B \rightarrow$ Coefficient of B.V in objective function
$Br \rightarrow$ Basic Variables
$X_B \rightarrow$ Right hand side of the constraint

Simplex Table

			C_j 3	2	0	0	
C_B	Br	X_B	x_1	x_2	s_1	s_2	leaving min+ve ($\frac{X_B}{Ec}$)
0	s_1	4	1	1	1	0	4/1 = 4
0	s_2	2	1	-1	0	1	2/1 = 2
			-3	-2	0	0	

The most (-ve) net evalution value corresponding to the variable is known as entering variable and min +ve ratio of $\frac{X_B}{EC}$ corresponding to the variable is known as leaving variable.

Max $Z = 3x_1 + 2x_2 + 0s_1 + 0s_2$

s.t. $x_1 + x_2 \leq 4$
$x_1 - x_2 \leq 2$
$x_1, x_2 \geq 0$

$\rightarrow$ $x_1 + x_2 + s_1 = 4$
$x_1 - x_2 + s_2 = 2$

$$\begin{matrix} x_1 & x_2 & s_1 & s_2 \end{matrix}$$
$$\begin{pmatrix} 1 & 1 & 1 & 0 \\ 1 & -1 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ s_1 \\ s_2 \end{pmatrix}$$

Pivot element is the intersection element of entering column and leaving row. Pivot element must be one (1), if not then make it one by dividing the same amount throughout the ~~row~~ pivot row.

		Cj	3	2	0	0	XB/EC
CB	BV	XB	x_1	x_2	s_1	s_2	
0	s_1	1	0	2	1	-1	1 ←
3	x_1	2	1	-1	0	1	-2
		Zj	3	-3	0	3	
		Zj-Cj	0	-5 ↑	0	3	

$2 + 2P_2$
$2 +$

		Cj	3	2	0	0
CB	BV	XB	x_1	x_2	s_1	s_2
2	x_2	1	0	1	$\frac{1}{2}$	$\frac{-1}{2}$
3	x_1	3	1	0	$\frac{1}{2}$	$\frac{1}{2}$
		Zj	3	2	$\frac{5}{2}$	$\frac{1}{2}$
		Zj-Cj	0	0	$\frac{5}{2}$	$\frac{1}{2}$

$1 + \frac{3}{2}$

$-1 + \frac{3}{2}$

$-2 + \frac{3}{2}$

the solution is optimal and $x_1 = 3$, $x_2 = 1$ and max $Z = 11$

Assignments

Q. Max $Z = X_1 + X_2 + 3X_3$

s.t. $3X_1 + 2X_2 + X_3 \leq 2$

$2X_1 + X_2 + 2X_3 \leq 2$

$X_1, X_2, X_3 \geq 0$

→ Max $Z = X_1 + X_2 + 3X_3 + 0S_1 + 0S_2$

s.t. $3X_1 + 2X_2 + X_3 + S_1 + 0S_2 = 2$

$2X_1 + X_2 + 2X_3 + 0S_1 + S_2 = 2$

Simplex Table

CB	BV	Cj / XB	1 / X_1	1 / X_2	3 / X_3	0 / S_1	0 / S_2	XB / EC
0	S_1	2	3	2	1	1	0	2
0	S_2	2	2	1	2	0	1	1 ←
	Z_j	0	0	0	0	0	0	
	~~$Z_j - C_j$~~ $Z_j - C_j$		-1	-1	-3 ↑	0	0	

Here there are three $Z_j - C_j$ are (-)ve and the most (-)ve is "-3".

Therefore X_3 is the entering variable.

and S_2 is the leaving variable.

Now we will convert the key element as unity and all other entries in that column to zero.

Now the revised simplex table as follows.

$R_2 \rightarrow \frac{R_2}{\text{Key element}}$

$R_1 \rightarrow R_1 - \frac{R_2}{2}$

CB	BV	Cj / XB	1 / X_1	1 / X_2	3 / X_3	0 / S_1	0 / S_2
0	S_1	1	2	$\frac{1}{2}$	0	1	$-\frac{1}{2}$
3	X_3	1	1	$\frac{1}{2}$	1	0	$\frac{1}{2}$
	Z_j	3	3	$\frac{3}{2}$	3	0	$\frac{3}{2}$
	$Z_j - C_j$		2	$\frac{1}{2}$	0	0	$\frac{3}{2}$

As all $Z_j - C_j \geq 0$, so the optimum solution to the LPP has been reached.

The solution is, $X_1 = 0$, $X_2 = 0$, $X_3 = 1$ and Max $Z = 3$

Q. Max $Z = 10x_1 + 6x_2$

s.t. $x_1 + x_2 \leq 2$

$2x_1 + x_2 \leq 4$

$3x_1 + 8x_2 \leq 12$

$x_1, x_2 \geq 0$

→ Max $Z = 10x_1 + 6x_2 + 0s_1 + 0s_2 + 0s_3$

$x_1 + x_2 + s_1 + 0s_2 + 0s_3 = 2$

$2x_1 + x_2 + 0s_1 + s_2 + 0s_3 = 4$

$3x_1 + 8x_2 + 0s_1 + 0s_2 + 8s_3 = 12$

		Cj	10	6	0	0	0	XB/EC
CB	BV	XB	x_1	x_2	s_1	s_2	s_3	
0	s_1	2	1	1	1	0	0	2 ←
0	s_2	4	2	1	0	1	0	2
0	s_3	12	3	8	0	0	1	4
	Z_j	0	0	0	0	0	0	
	$Z_j - C_j$		↑ −10	−6	0	0	0	

Here, there are two $Z_j - C_j$ are (-ve) and the most (-) ve is "-10".

Therefore x_1 is the entering variable

and the min +ve ratio of $\frac{XB}{EC} = \min\{2, 2, 4\} = 2$

So we choose s_1 as the leaving variable.

Now as the key element is equals to 1, we only have to convert all other entries in that column to zero.

$R_2 \rightarrow$	$R_1 - \frac{R_2}{2}$
$R_3 \rightarrow$	$R_1 - \frac{R_3}{3}$

Hence Revised simplex table as follows.

		Cj	10	6	0	0	0
CB	BV	XB	x_1	x_2	s_1	s_2	s_3
10	x_1	2	1	1	1	0	0
0	s_2	0	0	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	0
0	s_3	−2	0	$-\frac{5}{3}$	1	0	$-\frac{1}{3}$
	Z_j	20	10	10	10	0	0
	$Z_j - C_j$		0	4	10	0	0

Since all $Z_j - C_j \geq 0$, our optimal solution to the LPP has been reached.

The solution is $x_1 = 2, x_2 = 0$ and Max $Z = 20$

Q. Max $Z = 30x_1 + 23x_2 + 29x_3$

s.t. $6x_1 + 5x_2 + 3x_3 \leq 52$

$6x_1 + 2x_2 + 5x_3 \leq 14$

$x_1, x_2, x_3 \geq 0$

→ Max $Z = 30x_1 + 23x_2 + 29x_3 + 0s_1 + 0s_2$

$6x_1 + 5x_2 + 3x_3 + s_1 + 0s_2 \leq 52$

$6x_1 + 2x_2 + 5x_3 + 0s_1 + s_2 = 14$

Simplex Table

CB	BV	Cj XB	30 x_1	23 x_2	29 x_3	0 s_1	0 s_2	XB / EC
0	s_1	52	6	5	3	1	0	$\frac{52}{6} = 8.66$
0	s_2	14	6	2	5	0	1	$\frac{14}{6} = 2.33$ ←
	Z_j	0	0	0	0	0	0	
	$Z_j - C_j$		-30↑	-23	-29	0	0	

Here, there are three $Z_j - C_j$ are (−)ve and the most (−)ve is "−30".

Therefore x_1 is the entering variables.

and the min = +ve $\{8.66, 2.33\} = 2.33$

So s_2 is the leaving variable.

Now we will convert the key element as unity and all other entries in that column to zero.

Now the revised simplex table as follows.

CB	BV	Cj XB	30 x_1	23 x_2	29 x_3	0 s_1	0 s_2	XB / EC
0	s_1	38	0	3	-2	1	-1	$38/3 = 12.66$
30	x_1	$\frac{14}{6}$	1	$\frac{1}{3}$	$\frac{5}{6}$	0	$\frac{1}{6}$	7 ←
	Z_j	70	30	10	25	0	5	
	$Z_j - C_j$		0	-13↑	-4	0	5	

$R_2 \rightarrow \frac{R_2}{\text{Key element}}$

$R_1 \rightarrow R_1 - R_2$

$R_2 \rightarrow 3R_2$

$R_1 \rightarrow R_1 - 9R_2$

CB	BV	Cj XB	30 x_1	23 x_2	29 x_3	0 s_1	0 s_2
0	s_1	17	-9	0	$\frac{-19}{2}$	1	$\frac{-5}{2}$
23	x_2	7	3	1	$\frac{5}{2}$	0	$\frac{1}{2}$
	Z_j	161	69	23	57.5	0	11.5
	$Z_j - C_j$		39	0	28.5	0	11.5

Since all $Z_j - C_j \geq 0$, the optimal solution to the LPP has been reached.

The solution is $x_1 = 0$, $x_2 = 7$, $x_3 = 0$ and Max $Z = 161$.

Q. Max $Z = x_1 + 2x_2 + x_3$

s.t. $2x_1 + x_2 - x_3 \geq -2$

$-2x_1 + x_2 - 5x_3 \leq 6$

$4x_1 + x_2 + x_3 \leq 6$

$x_1, x_2, x_3 \geq 0$

→ Max $Z = x_1 + 2x_2 + x_3 + 0s_1 + 0s_2 + 0s_3$

Multiplying (-) into first constraints, which is $-2x_1 - x_2 + x_3 \leq 2$

~~$-2x_1 - x_2 + x_3 \leq 2$~~

$-2x_1 - x_2 + x_3 + s_1 + 0s_2 + 0s_3 = 2$

~~$4x_1 + x_2 - 5x_3 + 0s_1 + s_2 + 0s_3$~~

$-2x_1 + x_2 - 5x_3 + 0s_1 + s_2 + 0s_3 = 6$

$4x_1 + x_2 + x_3 + 0s_1 + 0s_2 + s_3 = 6$

CB	BV	c_j / XB	1 / x_1	2 / x_2	1 / x_3	0 / s_1	0 / s_2	0 / s_3	XB / EC
0	s_1	2	-2	-1	1	1	0	0	-2
0	s_2	6	-2	1	-5	0	1	0	6 ←
0	s_3	6	4	1	1	0	0	1	6
	Z_j	0	0	0	0	0	0	0	
	$z_j - c_j$	-1	-2↑	-1	0	0	0		

$R_1 \rightarrow R_2 + R_1$

$R_3 \rightarrow R_2 - R_3$

$R_1 \rightarrow R_1 + R_2$

Here, there are three $z_j - c_j$ are (-)ve and the most (-)ve is "-2".

Therefore x_2 is the entering variable.

also as $\min\{\frac{XB}{EC}\} = 6$, we choose s_3 as leaving variable.

Now as the key element is equal to 1, we only have to convert all other entries in that column to zero. Hence revised simplex table as follows.

CB	BV	c_j / XB	1 / x_1	2 / x_2	1 / x_3	0 / s_1	0 / s_2	0 / s_3	XB / EC
0	s_1	8	-4	0	-4	1	1	0	
2	x_2	6	-2	1	-5	0	1	0	
0	s_3	0	-6	0	-6	0	1	-1	
	Z_j	12	-4	2	-10	0	2	0	
	$z_j - c_j$		-5	0	-11↑	0	2	0	

unbounde

		Cj	1	2	1	0	0	0
CB	BV	XB	x_1	x_2	x_3	s_1	s_2	s_3
0	s_1	8	2	0	2	1	0	1
0	s_2	0	6	0	6	0	-1	1
2	x_2	6	4	1	1	0	0	1
	Zj	12	8	2	2	0	0	2
	Zj-Cj		7	0	1	0	0	2

$R_1 \rightarrow R_1 + R_3$

$R_2 \rightarrow$

Since all $Z_j - C_j \geq 0$, the optimum solution to this LPP has been reached.

The solution is $x_1 = 0$, $x_2 = 6$, $x_3 = 0$ and Max $Z = 12$.

		Cj	1	2	1	0	0	0	
CB	BV	XB	x_1	x_2	x_3	s_1	s_2	s_3	$\frac{XB}{EC}$
0	s_1	8	-4	0	-4	1	1	0	-2
2	x_2	6	-2	1	-5	0	1	0	$-\frac{6}{5}$
0	s_3	0	6	0	6	0	-1	1	
	Zj	12	-4	2	-10	0	2	0	
	Zj-Cj		-5	0	-11 ↑	0	2	0	

Q. Max $Z = x_1 + 2x_2 + x_3$

s.t. $2x_1 + x_2 - x_3 \geq -2$

$-2x_1 + x_2 - 5x_3 \leq 6$

$4x_1 + x_2 + x_3 \leq 6$

$x_1, x_2, x_3 \geq 6$

→ Multiplying (-) into first constraints, which will be $-2x_1 - x_2 + x_3 \leq 2$

Max $Z = x_1 + 2x_2 + x_3 + 0s_1 + 0s_2 + 0s_3$

$-2x_1 - x_2 + x_3 + s_1 + 0s_2 + 0s_3 = 2$

$-2x_1 + x_2 - 5x_3 + 0s_1 + s_2 + 0s_3 = 6$

$4x_1 + x_2 + x_3 + 0s_1 + 0s_2 + s_3 = 6$

		Cj	1	2	1	0	0	0	XB/EC
CB	BV	XB	x_1	x_2	x_3	s_1	s_2	s_3	
0	s_1	2	-2	-1	1	1	0	0	-2
0	s_2	6	-2	1	-5	0	1	0	6
0	s_3	6	4	1	1	0	0	1	6 ←
		Zj	0	0	0	0	0	0	
		Zj-Cj	-1	-2↑	-1	0	0	0	

Here, there are three Zj-Cj are (-)ve and the most (-)ve is "-2". Therefore x_2 is the entering variable.

also as min +ve $\left\{\frac{XB}{EC}\right\} = 6$, we choose s_3 as leaving variable.

Now as the key element is equal to 1, we only have to convert all other entries in that column to zero. Hence revised simplex table as follows.

		Cj	1	2	1	0	0	0
CB	BV	XB	x_1	x_2	x_3	s_1	s_2	s_3
0	s_1	8	2	0	2	1	0	1
0	s_2	0	-6	0	-6	0	1	-1
2	x_2	6	4	1	1	0	0	1
	Zj	12	8	2	2	0	0	2
	Zj-Cj		7	0	1	0	0	2

CB.	BV	c_j / XB	1 / x_1	2 / x_2	1 / x_3	0 / s_1	0 / s_2	0 / s_3	XB/EC
0	s_1	8	-4	0	-4	1	1	0	-2
2	x_2	6	-2	1	-5	0	1	0	$-\frac{6}{5}$
0	s_3	0	6	0	6	0	-1	1	0 ←
	z_j	12	-4	2	-10	0	2	0	
	$z_j - c_j$		-5	0	-11↑	0	2	0	

CB	BV	c_j / XB	1 / x_1	2 / x_2	1 / x_3	0 / s_1	0 / s_2	0 / s_3	XB/EC
0	s_1	8	0	0	0	1	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{8}{3}$
2	x_2	6	3	1	0	0	$\frac{1}{6}$	$\frac{5}{6}$	1
1	x_3	0	1	0	1	0	-1	0	0 ←
	z_j	12	7	2	1	0	$-\frac{2}{3}$	$\frac{5}{3}$	
	$z_j - c_j$		6	0	0	0	$-\frac{2}{3}$↑	$\frac{5}{3}$	

		c_j / XB	1 / x_1	2 / x_2	1 / x_3	0 / s_1	0 / s_2	0 / s_3
CB	BV							
0	s_1							
2	x_2							

* When all the elements of entering are (-)ve, the sol^n of LPP possess unbounded solution.
* when there is tie occures in min +ve ratio column, this is known as degeneracy.
* Alternative optimal sol^n
When the net evaluation of a non basic variable is zero in that case alternative optimal solution arises.

Q. Max $Z = 3x_1 + 5x_2$

s.t. $x_1 \le 4$

$2x_2 \le 12$

$3x_1 + 2x_2 \le 18$

$x_1, x_2 \ge 0$

→ Max $Z = 3x_1 + 5x_2 + 0s_1 + 0s_2 + 0s_3$

$x_1 + 0x_2 + s_1 + 0s_2 + 0s_3 = 4$

$0x_1 + 2x_2 + 0s_1 + s_2 + 0s_3 = 12$

$3x_1 + 2x_2 + 0s_1 + 0s_2 + s_3 = 18$

		Cj	3	5	0	0	0	XB/EC
CB	BV	XB	x_1	x_2	s_1	s_2	s_3	
0	s_1	4	1	0	1	0	0	4/0 ∴
0	s_2	12	0	2	0	1	0	6 ←
0	s_3	18	3	2	0	0	1	9
	Zj	0	0	0	0	0	0	
	Zj-Cj		-3	-5↑	0	0	0	

		Cj	3	5	0	0	0	XB/EC
CB	BV	XB	x_1	x_2	s_1	s_2	s_3	
0	s_1	4	1	0	1	0	0	4
5	x_2	6	0	1	0	1/2	0	6/0
0	s_3	6	3	0	0	-1	1	2 ←
	Zj	30	0	5	0	5/2	0	
	Zj-Cj		-3↑	0	0	5/2	0	

		Cj	3	5	0	0	0
CB	BV	XB	x_1	x_2	s_1	s_2	s_3
0	s_1	2	0	0	1	1/3	-1/3
5	x_2	6	0	1	0	1/2	0
3	x_1	2	1	0	0	-1/3	1/3
	Zj	36	3	5	0	3/2	1
	Zj-Cj		0	0	0	0	0

→ $x_1 = 2$, $x_2 = 6$, Max $Z = 36$

Q. Max $Z = 12x_1 + 16x_2$

s.t. $10x_1 + 20x_2 \leq 120$

$8x_1 + 8x_2 \leq 80$

x_1 and $x_2 \geq 0$

→ Max $Z = 12x_1 + 16x_2 + 0S_1 + 0S_2$

$10x_1 + 20x_2 + S_1 + 0S_2 = 120$

$8x_1 + 8x_2 + 0S_1 + S_2 = 80$

→

		Cj	12	16	0	0	
CB	BV	XB	x_1	x_2	S_1	S_2	$\frac{XB}{EC}$
0	S_1	120	10	20	1	0	6
0	S_2	80	8	8	0	1	10
		Zj-Cj	-12	-16↑	0	0	

		Cj	12	16	0	0	
CB	BV	XB	x_1	x_2	S_1	S_2	$\frac{XB}{EC}$
16	x_2	6	1/2	1	1/20	0	3 ←
0	S_2	32	4	0	$-\frac{8}{20}$	1	8
		Zj	96	8	16	$\frac{16}{20}$	0
		Zj-Cj	-4↑	0	$\frac{16}{20}$	0	

		Cj	12	16	0	0
CB	BV	XB	x_1	x_2	S_1	S_2
12	x_1	12	1	2	$\frac{1}{10}$	0
0	S_2	-4	0	-2	$-\frac{2}{5}$	1
	Zj	144	12	24	0	0
	Zj-Cj	0	8	0	0	

The LPP has been reached optimum solution,

solution is $x_1 = 12$, $x_2 = 0$ and Max $Z = 144$

Q. Max $Z = 3x_1 + 2x_2 + 5x_3$

s.t. $x_1 + 2x_2 + x_3 \le 430$

$3x_1 + 2x_3 \le 460$

$x_1 + 4x_2 \le 420$

$x_1, x_2, x_3 \ge 0$

→ Max $Z = 3x_1 + 2x_2 + 5x_3 + 0S_1 + 0S_2 + 0S_3$

$x_1 + 2x_2 + x_3 + S_1 + 0S_2 + 0S_3 = 430$

$3x_1 + 2x_2 + 2x_3 + 0S_1 + S_2 + 0S_3 = 460$

$x_1 + 4x_2 + 0x_3 + 0S_1 + 0S_2 + S_3 = 420$

		Cj	3	2	5	0	0	0	
CB	BV	XB	x_1	x_2	x_3	S_1	S_2	S_3	XB/EC
0	S_1	430	1	2	1	1	0	0	430
0	S_2	460	3	0	2	0	1	0	230 ←
0	S_3	420	1	4	0	0	0	1	(420/0)
		Zj	0	0	0	0	0	0	
		Zj-Cj	-3	-2	-5↑	0	0	0	

		Cj	3	2	5	0	0	0	
CB	BV	XB	x_1	x_2	x_3	S_1	S_2	S_3	XB/EC
0	S_1	200	-1/2	1	0	1	-1/2	0	200 ←
5	x_3	230	3/2	0	1	0	1/2	0	(230/0)
0	S_3	1	4	0	0	0	0	1	(1/0)
		Zj	1150	15/2	0	5	0	5/2	0
		Zj-Cj	9/2	-2↑	0	0	5/2	0	

		Cj	3	2	5	0	0	0	
CB	BV	XB	x_1	x_2	x_3	S_1	S_2	S_3	
2	x_2	200	-1/2	1	0	1	-1/2	0	
5	x_3	230	3/2	0	1	0	1/2	0	
0	S_3	1	4	0	0	0	0	1	
		Zj	1550	13/2	2	5	2	3/2	0
		Zj-Cj	7/2	0	0	2	3/2	0	

→ $x_1 = 0$

$x_2 = 200$

$x_3 = 230$

Max $Z = 1550$

Big-M Method

When a LPP having "$\geq$" inequality constraints and equal to "$=$" constraints, in that case to get intial basis matrix, we have to use artificial variable to get the basis matrix.

So when an artificial variable is used in a LPP then that LPP can be solved by two methods,

(1) Big-M Method

(2) Two Phase Method

Q. Max $Z = x_1 + 5x_2$

s.t. $4x_1 + 4x_2 \leq 6$

$x_1 + 3x_2 \geq 2$

$x_1, x_2 \geq 0$

$x_1 + 5x_2 + 0S_1 + 0S_2 - MA_1$

$4x_1 + 4x_2 + S_1 + 0S_2 + 0A_1 = 6$

$x_1 + 3x_2 + 0S_1 - S_2 + A_1 = 2$

x_1	x_2	S_1	S_2	A_1
4	4	1	0	0
1	3	0	-1	1

		c_j	1	5	0	0	-M	
CB	BV	XB	x_1	x_2	S_1	S_2	A_1	$\frac{XB}{EC}$
0	S_1	6	4	4	1	0	0	3/2 ←
-M	A_1	2	1	3	0	-1	1	2/3 ←
		Z_j	-2M	-M -3M	0	M	-M	
		$Z_j - c_j$	-M-1	-3M-5+0	M	0		

When an artificial variable left the basis, removed that artificial variable column from next table onwards.

		c_j	1	5	0	0	-M	
CB	BV	XB	x_1	x_2	s_1	s_2	A_1	$\frac{XB}{EC}$
0	s_1	10/3	8/3	0	1	4/3	-4/3	$\frac{10}{4}$ ←
5	x_2	2/3	1/3	1	0	-1/3	1/3	-2
	z_j	10/3	5/3	5	0	-5/3	5/3	
	$z_j - c_j$		2/3	0	0	-5/3 ↑	5/3+M	

		c_j	1	5	0	0	-M
CB	BV	XB	x_1	x_2	s_1	s_2	~~A~~
0	s_2	$\frac{10}{4}$	2	0	$\frac{3}{4}$	1	
5	x_2	$\frac{3}{2}$	1	1	1/4	0	
	z_j	$\frac{15}{2}$	5	5	5/4	0	
	$z_j - c_j$		4	0	5/4	0	

~~A~~ → $x_1 = 0$, $x_2 = \frac{3}{2}$ Max $z = \frac{15}{2}$

Assignment

Q. what do you mean by artificial variable?
Q. what is unbounded sol^n in simplex method?
Q. what is degeneracy in simplex method?
Solve the following LPP using Big M method

Max $z = 3x_1 + 2x_2 + 6x_3$
s.t. $4x_1 + x_2 + x_3 \leq 100$
$x_1 + x_2 \geq 40$
$x_2 + x_3 \leq 30$
$x_1, x_2, x_3 \geq 0$

Max $z = 3x_1 + 2x_2 + 6x_3 + 0s_1 + 0s_2 + 0s_3 - MA_1$
$4x_1 + x_2 + x_3 + s_1 + 0s_2 + 0s_3 + 0A_1$
$0x_1 + x_2 + 0x_3 + 0s_1 - s_2 + 0s_3 + A_1$
$0x_1 + x_2 + x_3 + 0s_1 + 0s_2 + 0s_3 + 0A_1$

		Cj	3	2	6	0	0	0	-M	
CB	BV	XB	x_1	x_2	x_3	S_1	S_2	S_3	A_1	XB/EC
0	S_1	100	4	1	1	1	0	0	0	100
0	S_3	40	1	1	0	0	0	0	1	40/0
-M	A_1	30	0	1	1	0	0	1	0	30 ←
		Zj	0	-M	-M	0	0	-M	0	
		Zj-Cj	-3	-M-2	-M-6	0	0	-M	M	
					↑					

		Cj	3	2	6	0	0	0	
CB	BV	XB	x_1	x_2	x_3	S_1	S_2	S_3	XB/EC
0	S_1	70	4	0	1	0	-1	0	70/4 ←
0	S_3	40	1	1	0	0	0	0	40
6	x_3	30	0	1	1	0	0	1	30/0
	Zj	180	0	6	6	0	0	6	
	Zj-Cj	-3 ↑	4	0	0	0	0	6	

		Cj	3	2	6	0	0	0	
CB	BV	XB	x_1	x_2	x_3	S_1	S_2	S_3	XB/EC
3	x_1	70/4	1	0	1/4	0	-1/4	0	(←)
0	S_3	90/4	0	1	-1/4	0	1/4	0	90
6	x_3	30	0	1	1	0	0	1	30 ←
	Zj	930/4	3	6	27/4	0	-3/4	6	
	zj-cj	0	4	3/4	0	-3/4 ↑	6		

CB	BV	Cj / XB	3 x_1	2 x_2	6 x_3	0 s_1	0 s_2	0 s_3
3	x_1						0	
0	s_3						0	
0	s_2	31	1	2	2	0	1	2

$4x_1 + x_2 + x_3 + s_1 = 100$
$x_1 + x_2 - s_2 + A_1 = 40$
$x_2 + x_3 + s_3 = 30$

	x_1	x_2	x_3	s_1	s_2	s_3	A_1
s_1	4	1	1	1	0	0	0
A_1	1	1	0	0	−1	0	1
s_3	0	1	1	0	0	1	0

Q. $3x_1 + 2x_2 + 6x_3 + 0s_1 + 0s_2 + 0s_3 - MA_1$

s.t. $4x_1 + x_2 + x_3 + 0s_1 + 0s_2 + 0s_3 + 0A_1 = 100$
$x_1 + x_2 + 0x_3 + 0s_1 - s_2 + 0s_3 + A_1 = 40$
$0x_1 + x_2 + x_3 + 0s_1 + 0s_2 + 0s_3 + 0A_1 = 30$

CB	BV	Cj / XB	3 x_1	2 x_2	6 x_3	0 s_1	0 s_2	0 s_3	−M A_1	XB/EC
0	s_1	100	4	1	1	1	0	0	0	25 ←
−M	A_1	40	1	1	0	0	−1	0	1	40
0	s_3	30	0	1	1	0	0	1	0	(−)
		Z_j	−M	−M	0	0	M	0	−M	
		$Z_j - C_j$	−M−3	−M−2	−6	0	M	0	0	

↑

CB	BV	Cj / XB	3 x_1	2 x_2	6 x_3	0 s_1	0 s_2	0 s_3	−M A_1	XB/EC
3	x_1	25	1	1/4	1/4	1/4	0	0	0	100
−M	A_1	15	0	3/4	−1/4	−1/4	−1	0	1	20
0	s_3	30	0	1	1	0	0	1	0	30
		Z_j	3	$\frac{3}{4}-\frac{3M}{4}$	$\frac{3}{4}+\frac{M}{4}$	$\frac{3}{4}+\frac{3M}{4}$	M	0	−M	
		$Z_j - C_j$	0	$-\frac{3M}{4}-\frac{5}{2}$						

↑

CB	BV	XB	3 x_1	2 x_2	6 x_3	0 s_1	0 s_2	0 s_3	XB/EC
3	x_1	20	1	0	1/3	1/3	−1/3	0	60
2	x_2	20	0	1	−1/3	−1/3	4/3	0	−60
0	s_3	10	0	0	4/3	1/3	4/3	1	30/4 ←
		Z_j	100	3	2	7/3	7/3	−5/3	0
		$Z_j - C_j$	0	0	−11/3 ↑	7/3	−5/3	0	

CB	BV	C_j XB	3 x_1	2 x_2	6 x_3	0 s_1	0 s_2	0 s_3
3	x_1	70/4	1	0	0	1/4	0	-1/4
2	x_2	90/4	0	1	0	-3/4	-1	1/4
6	x_3	30/4	0	0	1	1/4	1	3/4
	Z_j	570/4	3	2	6	7/4	4	17/4
	$Z_j - C_j$		0	0	0	7/4	4	17/4

$$\text{Max } Z = \frac{570}{4}$$

$$\left[x_1 = \frac{70}{4},\ x_2 = \frac{90}{4},\ x_3 = \frac{30}{4}\right]$$

① $4x_1 + x_2 + x_3 \le 100$

$$4 \times \frac{70}{4} + \frac{90}{4} + \frac{30}{4}$$

$$= 70 + \frac{90+30}{4}$$

$$= \frac{280 + 90 + 30}{4}$$

$$= \frac{400}{4}$$

$$= 100$$

② $x_1 + x_2 \ge 40$

$$\Rightarrow \frac{70}{4} + \frac{90}{4}$$

$$= \frac{70+90}{4} = \frac{160}{4} = 40$$

③ $x_2 + x_3 \le 30$

$$= \frac{90}{4} + \frac{30}{4}$$

$$= \frac{90+30}{4} = \frac{120}{4} = 30$$

maximize

$Z = X_1 + 5X_2$

Subject to

$4X_1 + 4X_2 \leq 6$

$X_1 + 3X_2 \geq 2$

$X_1, X_2 \geq 0$

$4X_1 + 4X_2 + S_1 = 6$

$X_1 + 3X_2 - S_2 + A_1 = 2$

$Z = X_1 + 5X_2 + 0S_1 - 0S_2 - MA_1$

	Cj		1	5	0	0	−M	
Cont^n	variable	const	X_1	X_2	S_1	S_2	A_1	
0	S_1	6	4	4	1	0	0	6/4 = 1.5
−M	A_1	2	1	3	0	−1	1	2/3 = 0.6 ← KR
	Zj	−2M	−M	−3M	0	M	−M	
	Cj − Zj		1+M	5+3M	0	−M	0	

↓ KC

value for main row :−

$\frac{2}{3}, \frac{1}{3}, \frac{3}{3}, \frac{0}{3}, -\frac{1}{3}, \frac{1}{3}$

Q.4.

$$\text{Max } z = 4x_1 + x_2 + 3x_3 - x_4$$

s.t. $x_1 + 2x_2 + 3x_3 = 15$

$2x_1 + x_2 + 5x_3 = 20$

$x_1 + 2x_2 + x_3 + x_4 = 10$

$x_1, x_2, x_3, x_4 \geq 0$

→

$$\text{Max } z = 4x_1 + x_2 + 3x_3 - x_4 - MA_1 - MA_2 - MA_3$$

s.t. $x_1 + 2x_2 + 3x_3 + A_1 = 15$

$2x_1 + x_2 + 5x_3 + A_2 = 20$

$x_1 + 2x_2 + x_3 + x_4 + A_3 = 10$

	x_1	x_2	x_3	x_4	A_1	A_2	A_3	
	1	2	3	0	1	0	0	15
	2	1	5	0	0	1	0	20
	1	2	1	1	0	0	1	10

	cj	4	1	3	-1	-M	-M	-M	XB/EC
CB	BV	XB	x_1	x_2	x_3	x_4	A_1	A_2	A_3	
-M	A_1	15	1	2	3	0	1	0	0	5
-M	A_2	20	2	1	5	0	0	1	0	4 ←
-M	A_3	10	1	2	1	1	0	0	1	10
	z_j		-4M	-5M	-9M	-M	-M	-M	-M	
	$z_j - c_j$		(-4M-4)	(-5M-1)	(-9M-3) ↑	(-M+1)	0	0	0	

	cj	4	1	3	-1	-M	-M	-M	XB/EC
CB	BV	XB	x_1	x_2	x_3	x_4	A_1	A_2	A_3	
-M	A_1	3	-1/5	7/5	0	0	1	-3/5	0	15/7 ←
3	x_3	4	2/5	1/5	1	0	0	1/5	0	20
-M	A_3	6	3/5	9/5	0	1	0	-1/5	1	30/9
	z_j		$\frac{-2M+6}{5}$	$\frac{-16M+3}{5}$	0	-M	-M	$\frac{4M+3}{5}$	-M	
	$z_j - c_j$		$\frac{-2M-14}{5}$	$\frac{-16M-2}{5}$ ↑	-3	-M+1	0		0	

CB	BV	Cj / XB	4 / x_1	1 / x_2	3 / x_3	-1 / x_4	-M / A_3	XB/EC
1	x_2	15/7	-1/7	1	0	0	0	(-)
3	x_3	25/7	3/7	0	1	0	0	25/3
-M	A_3	15/7	6/7	0	0	1	1	15/6 ←
		z_j	$\frac{8-6M}{7}$	1	3	-M	-M	
		$z_j - c_j$	$\frac{-6M-20}{7}$ ↑	0	0	-M+1	0	

CB	BV	Cj / XB	4 / x_1	1 / x_2	3 / x_3	-1 / x_4
1	x_2	95/6	0	1	0	-1/6
3	x_3	105/6	0	0	1	-1/2
4	x_1	15/6	1	0	0	7/6
		z_j	4	1	3	$\frac{20}{6}$
		$z_j - c_j$	0	0	0	$\frac{14}{6}$

All $z_j - c_j \geq 0$, the sol^n is optimal,

and sol^n is $x_1 = \frac{15}{6}$, $x_2 = \frac{95}{6}$, $x_3 = \frac{105}{6}$

and Max $z = \frac{95}{6} + 3 \times \frac{105}{6} + 4 \times \frac{15}{6}$

$= \frac{470}{6}$

Q. $4x_1 + x_2 + 3x_3 - x_4 = \text{Max } Z$

s.t. $x_1 + 2x_2 + 3x_3 = 15$

$2x_1 + x_2 + 5x_3 = 20$

$x_1 + 2x_2 + x_3 + x_4 = 10$

$x_1, x_2, x_3 \geq 0$

$\rightarrow$

$\text{Max } Z = 4x_1 + x_2 + 3x_3 - x_4 - MA_1 - MA_2$

$x_1 + 2x_2 + 3x_3 + A_1 = 15$

$2x_1 + x_2 + 5x_3 + A_2 = 20$

$x_1 + 2x_2 + x_3 + x_4 = 10$

		Cj	4	1	3	-1	-M	-M	XB/EC
CB	BV	XB	x_1	x_2	x_3	x_4	A_1	A_2	
-M	A_1	15	1	2	3	0	1	0	$\frac{15}{3}=5$
-M	A_2	20	2	1	5	0	0	1	$\frac{20}{5}=4$ ←
-1	x_4	10	1	2	1	1	0	0	10
		Zj	-3M-1	-3M-2	-8M-1	-1	-M	-M	
		Zj-Cj	-3M-5	-3M-3	-8M-4 ↑	0	0	0	

		Cj	4	1	3	-1	-M	XB/EC
CB	BV	XB	x_1	x_2	x_3	x_4	A_1	
-M	A_1	3	-1/5	7/5	0	0	1	$\frac{15}{7}$ ←
3	x_3	4	2/5	1/5	1	0	0	20
-1	x_4	6	3/5	9/5	0	1	0	$\frac{30}{9}$
		Zj	$\frac{M+3}{5}$	$\frac{-7M-6}{5}$	3	-1	-M	
		Zj-Cj	$\frac{M-17}{5}$	$\frac{-7M-11}{5}$ ↑	0	0	0	

		Cj	4	1	3	-1	XB/EC
CB	BV	XB	x_1	x_2	x_3	x_4	
1	x_2	$\frac{15}{7}$	$\frac{-1}{7}$	1	0	0	(-)
3	x_3	$\frac{25}{7}$	$\frac{15}{7}$	0	1	0	5/3
-1	x_4	$\frac{15}{7}$	$\frac{30}{7}$	0	0	1	1/2 ←
		Zj	2	-1	3	-1	
		Zj-Cj	-2 ↑	-2	0	0	

C_j			4	1	3	-1
C_B	BV	X_B	x_1	x_2	x_3	x_4
1	x_2	95/6	0	1	0	1/30
3	x_3	35/6	0	0	2	-1
4	x_1	1/2	1	0	0	7/30
	Z_j		4	1	6	

$\text{Max } Z = 4x_1 + x_2 + 3x_3 - x_4$

s.t. $x_1 + 2x_2 + 3x_3 = 15$ →

$2x_1 + x_2 + 5x_3 = 20$

$x_1 + 2x_2 + x_3 + x_4 = 10$

$x_1, x_2, x_3, x_4 \geq 0$

→ $\text{Max } Z = 4x_1 + x_2 + 3x_3 - x_4 - MA_1 - MA_2 - MA_3$

s.t. $x_1 + 2x_2 + 3x_3 + A_1 = 15$

$2x_1 + x_2 + 5x_3 + A_2 = 20$

$x_1 + 2x_2 + x_3 + x_4 + A_3 = 10$

	cj	4	1	3	-1	-M	-M	-M	
CB	BV	XB	x_1	x_2	x_3	x_4	A_1	A_2	A_3	XB/EC
-M	A_1	15	1	2	3	0	1	0	0	5
-M	A_2	20	2	1	5	0	0	1	0	4 ←
-M	A_3	10	1	2	1	1	0	0	1	10
	zj	-4M	-5M	-9M	-M	-M	-M	-M		
	zj-cj	(-4M-4)	(-5M-1)	(-9M-3) ↑	(-M+1)	0	0	0		

$R_1 \to R_1 - \frac{3}{5} R_2$

$R_3 \to R_3 - \frac{1}{5} R_2$

	cj	4	1	3	-1	-M	-M	
CB	BV	XB	x_1	x_2	x_3	x_4	A_1	A_3	XB/EC
-M	A_1	3	-1/5	7/5	0	0	1	0	15/7 ←
3	x_3	4	2/5	1/5	1	0	0	0	20
-M	A_3	6	3/5	9/5	0	1	0	1	10/3
	zj	$\frac{-2M+6}{5}$	$\frac{-16M+3}{5}$	3	-M	-M	-M		
	zj-cj	$\frac{-2M+4}{5}$	$\frac{-16M-2}{5}$ ↑	0	-M+1	0	0		

$R_2 \to R_2 - \frac{1}{7} R_1$

$R_3 \to R_3 - \frac{9}{7} R_1$

	cj	4	1	3	-1	-M	
CB	BV	XB	x_1	x_2	x_3	x_4	A_3	XB/EC
1	x_2	15/7	-1/7	1	0	0	0	(-)
3	x_3	25/7	3/7	0	1	0	0	25/3
-M	A_3	3/5	6/7	0	0	1	1	7/10 ←
	zj	$\frac{-6M+8}{7}$	1	3	-M	-M		
	zj-cj	$\frac{-6M-20}{7}$ ↑	0	0	-M+1	0		

CB	BV	c_j / x_B	4 / x_1	1 / x_2	3 / x_3	-1 / x_4
2	x_2	$\frac{157}{70}$	0	1	0	1/6
3	x_3	$\frac{229}{70}$	0	0	1	-1/2
4	x_1	$\frac{7}{10}$	1	0	0	7/6
		Z_j	4	2	3	$\frac{105}{18}$
		$Z_j - c_j$	0	1	0	$\frac{87}{18}$

$R_1 \to R_1 + \frac{1}{6} R_3$

$R_2 \to R_2 - \frac{1}{2} R_3$

Q. What do you mean by Game?

→ Game is defined ~~by~~ as an activity between two or more persons according to a set of rules at the end of which each person receives some benefit or suffers loss.

Q.2 What do you mean by pure strategy & Mixed?

→ A force of action taken by a player is called strategy.

Pure strategy :- If a player select the same strategy each time or if one knows in advance of the play that is certain to be adopted

Mixed strategy :- When a player use combination of strategies then it is called mixed strategy.

Q3 What do you mean by pay off?

→ Each game has an outcome i.e. gain or loss or draw that is called pay-off. And when the player select their strategies it can be ~~termed~~ represent in form of matrix, that is called matrix.

Q.4

Module-III — GAME THEORY — (Quantitative Techniques)

(1)

A Game is a physical activity which involves two or more than two players, with certain set of rules, during the play of game, players add different types of strategies, there are two types strategies (1) Pure strategy and (2) Mixed strategies.

Pure strategy – Are the strategy which is known to both the players, during the play of game.

Mixed strategy – Are the strategies which is unknown to both the players during the game, they only can predict the chances of different strategies during the game.

Pay off Matrix is represented as in a ~~rect~~ react ~~ungular~~ ways with rows and columns and the entries of pay-off matrix are the outcomes of different strategies adopted by the players during the game.

Game theory divided into two parts one ~~is~~ based on (1) pure strategy (2) mixed strategy

Pure Strategy

- Maximin and Minmax method.
- Saddle point
- value of the Game

Mixed strategy

→ 2x2 game
→ 2xn game, mx2 game
→ mxn game

Q. solve the following game, whose pay-off matrix is

Player A \ Player B	B_1	B_2	B_3
A_1	1	3	1
A_2	0	-4	-3
A_3	1	5	-1

Player A \ Player B	B_1	B_2	B_3	Row minimum
A_1	1 (*) (+)	3	1 (+) (*)	(1)
A_2	0	-4 (*)	-3	(-4)
A_3	1 (+)	5 (+)	-1 (*)	(-1)
Column max	(1)	(5)	(1)	

maximin — (Player A) — minimax

Max (Row minimum) = max {1, -4, -1} = 1

Min (column maximum) = Min {1, 5, 1} = 1

value of the game = 1

Since max (min) = min (max) = 1, and optimal strategy is $(A_1 B_1)$ & $(A_1 B_3)$

Saddle point is a position in the pay off matrix where maximum of row minimum intersects with minimum of column maximum, and the saddle point is called as the value of game and max min is denoted by $\underline{v}$ and

The game having a saddle point is known as pure

Q.

	B_1	B_2	B_3	B_4	B_5	Row minimum
A_1	-2	0	0	5	3	(-2)
A_2	3	2	1	2	2	(1)
A_3	-4	-3	0	-2	6	(-4)
A_4	5	3	-4	2	-6	(-6)
Column max	(5)	(2)	(1)	(5)	(6)	

Max (Row min) = Max $\{-2, 1, -4, -6\} = 1$

Min (Column max) = Min $\{5, 2, 1, 5, 6\} = 1$

$\underline{v} = 1, \quad \bar{v} = 1$

the value of the game or saddle point = 1 and this is corresponding to $(A_2 B_3)$

Q. Determine the optimal

	B_1	B_2	B_3	B_4	(row column)
A_1	-5	2	0	7	(-5)
A_2	5	6	4	8	(4)
A_3	4	0	2	-3	(-3)
(Column max)	(5)	(6)	(4)	(8)	

Max (row min) = max $\{-5, 4, -3\} = 4$

Min (column max) = Min $\{5, 6, 4, 8\} = 4$

Ignoring the value of "λ"

Q.

	B_1	B_2	B_3	R-min
A_1	λ	6 (†)	2 (†)*	(2) ⟶ $\underline{v}$ (maximin)
A_2	-1 (†)	λ	-1 *	(-7)
A_3	-2 *	4	λ	(-2)
c-Max	(-1)	(6)	(2) ⟶ $\overline{v}$ (minimax)	

Here the value of the game = 2 and this is corresponding to $(A_1 B_3)$.

Q.

(a)

	B_1	B_2	R-min
A_1	5 (†)	0 *	(0)
A_2	0 *	2 (†)	(0)
c-Max	(5)	(2)	

$\overline{v}$ (minimax)

The ~~saddle~~ game has no saddle point as $\underline{v} \neq \overline{v}$.

(b)

	B_1	B_2	R-min
A_1	0 (†)(*)	2	(0) ⟶ $\underline{v}$ (maximin)
A_2	-1 (*)	4 (†)	(-1)
c-Max	(0)	(4)	

$\overline{v}$ (minimax)

The saddle point is zero. So the game is called fair.

Q.

	~~B_1~~	B_2	B_3	R-min
A_1	-1	2	-2 †*	(-2) ⟶ $\underline{v}$ (maximin)
A_2	6 †	4 †	-6 *	(-6)
c-Max	(6)	(4)	(-2)	

$\overline{v}$ (minimax)

The value of the game = (-2) and game is strictly determinable corresponding to (A_1, B_3)

Q.

	B_1	B_2	B_3	B_4	R-min
A_1	-5^*	2	0	7	(-5)
A_2	$5^\dagger$	$6^\dagger$	$4^{*\dagger}$	$8^\dagger$	(4) — $\underline{v}$ (maximin)
A_3	4	0	2	-3^*	(-3)
C-Max	(5)	(6)	(4)	(8)	

$\bar{v}$ (minimax)

The saddle point = 4, and it is corresponding to (A_2, B_3)

Q. A game having 2 players is known as 2 person game.

A game having more than 2 players is known as 'n' person game.

A game is said to be zero sum game, when the gain of one player is equal to the loss of another player.

Mixed Strategy (Game without saddle point)

→ 2x2 game (Algebric Method)
→ 2xn or mx2 game (Graphical & Algebric)
→ mxn game (Dominance principle & Algebric)

$$P_1 = \frac{a_{22} - a_{21}}{(a_{11} + a_{22}) - (a_{12} + a_{21})}, \quad q_1 = \frac{a_{22} - a_{12}}{(a_{11} + a_{22}) - (a_{12} + a_{21})}$$

$$v = \frac{a_{11}a_{22} - a_{12}a_{21}}{(a_{22} + a_{11}) - (a_{12} + a_{21})}$$ (Value of the game)

$$\begin{bmatrix} 5 & 0 \\ 0 & 2 \end{bmatrix}$$

$$P_1 = \frac{2}{7}$$

$$P_2 = \frac{5}{7}$$

$$q_1 = \frac{2}{7}, \quad q_2 = \frac{5}{7}$$

$$v = \frac{10 - 0}{7 - 0} = \frac{10}{7}$$

$$\begin{bmatrix} 5 & 1 \\ 3 & 4 \end{bmatrix}$$

(1) (3) (5) (4)

$a_{11}=5,\ a_{12}=1$

$a_{21}=3,\ a_{22}=4$

$$P_1 = \frac{4-3}{(5+4)-(1+3)} = \frac{1}{9-4} = \frac{1}{5}$$

$$P_2 = \frac{4}{5}$$

$$q_1 = \frac{4-1}{(5+4)-(1+3)} \qquad q_2 = \frac{2}{5}$$

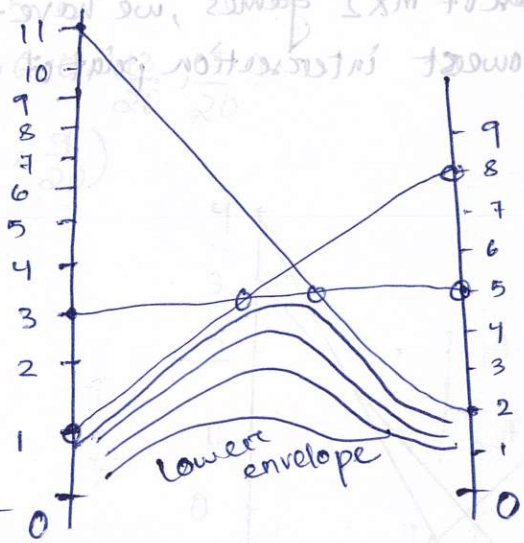
$$= \frac{3}{9-4}$$

$$= \frac{3}{5}$$

$$v = \frac{20-3}{(5+4)-(1+3)} = \frac{17}{5}$$

2xn or mx2 (Graphical & Algebraic)

$$\begin{bmatrix} 1 & 3 & 11 \\ 8 & 5 & 2 \end{bmatrix}$$



When a game is a (2xn) game, we have to use graphical technique to reduce the matrix into a (2x2) game.

In a 2xn game, we have to consider the highest intersection points of the lower envelope from the graph.

$a_{11}=3,\ a_{12}=11$

$a_{21}=5,\ a_{22}=2$

$$\begin{bmatrix} 3 & 11 \\ 5 & 2 \end{bmatrix} \quad \begin{matrix} 3 \\ 8 \end{matrix}$$

$$P_1 = \frac{2-5}{(2+3)-(11+5)} = \frac{-3}{5-16} = \frac{3}{11}$$

$$P_2 = \frac{8}{11}$$

$$q_1 = \frac{a_{22}-a_{12}}{(a_{11}+a_{22})-(a_{12}+a_{21})}$$

$$= \frac{2-11}{(3+2)-(11+5)}$$

$$= \frac{-9}{-11} = \frac{9}{11}$$

$$q_2 = \frac{2}{11}$$

$$v = \frac{9}{11} + \frac{40}{11} = \frac{49}{11}$$

$$S_A = \begin{pmatrix} SA_1 & SA_2 \\ 3/11 & 8/11 \end{pmatrix}$$

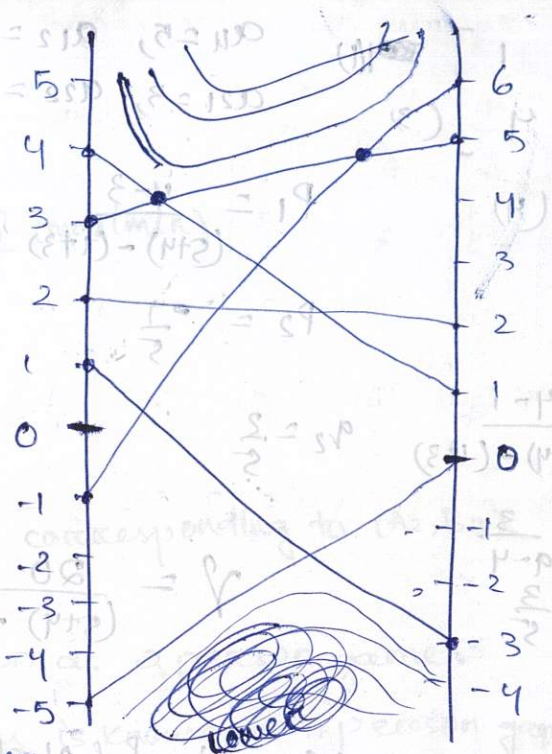
$$S_B = \quad SB_1 \quad SB_2 \quad SP_3$$

$$\begin{bmatrix} 1 & -3 \\ 3 & 5 \\ -1 & 6 \\ 4 & 1 \\ 2 & 2 \\ -5 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 4 & 1 \\ -1 & 6 \end{bmatrix}$$

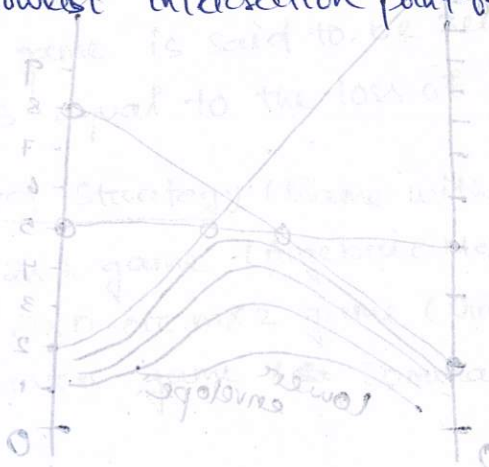
$$\begin{bmatrix} 3 & 5 \\ 4 & 1 \end{bmatrix} \quad \begin{matrix} 3 & \frac{3}{13} \\ 2 & \frac{2}{13} \end{matrix}$$

$$\begin{matrix} \frac{4}{13} & \frac{1}{13} \end{matrix}$$

$$v = \frac{9}{13} + \frac{8}{13} = \frac{17}{13}$$



In case of mx2 games, we have to consider the lowest intersection point of upper envelope from the graph.

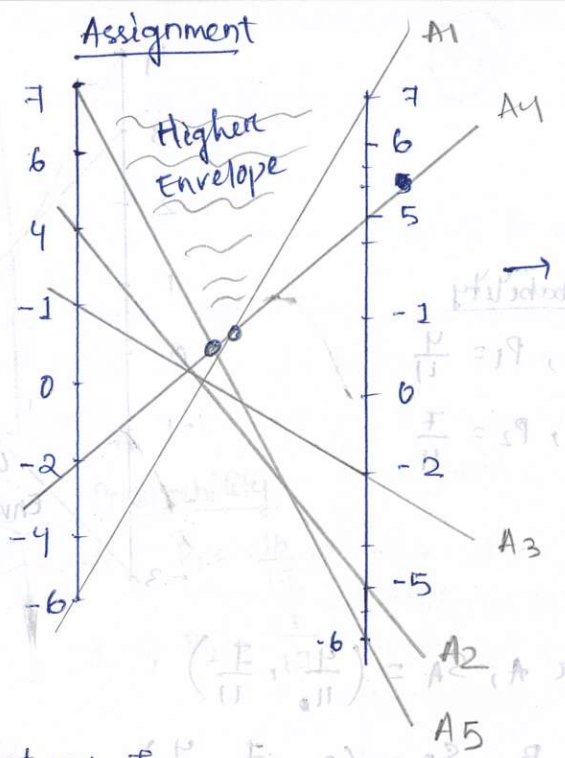


Assignment

(7)

Q.

	B_1	B_2
A_1	-6	7
A_2	4	-5
A_3	-1	-2
A_4	-2	5
A_5	7	-6



	B_1	B_2	P_1
A_4	-2	5	$\frac{13}{20}$
A_5	7	-6	$\frac{7}{20}$ (P_2)
	11	9	

$q_1 = \frac{11}{20}$ $\quad \frac{9}{20} = q_2$

$v = \frac{-13}{20}(-2) + \frac{7}{20}(7)$

$= \frac{-26+49}{20} = \frac{23}{20}$

∴ optimal strategy for A, $S_A = (0, 0, 0, \frac{13}{20}, \frac{7}{20})$

optimal strategy for B, $S_B = (\frac{11}{20}, \frac{9}{20})$

Q.

	B_1	B_2	B_3
A_1	4	-3	3
A_2	-3	1	-1

→

	B_1	B_2
A_1	4	-3
A_2	-3	1

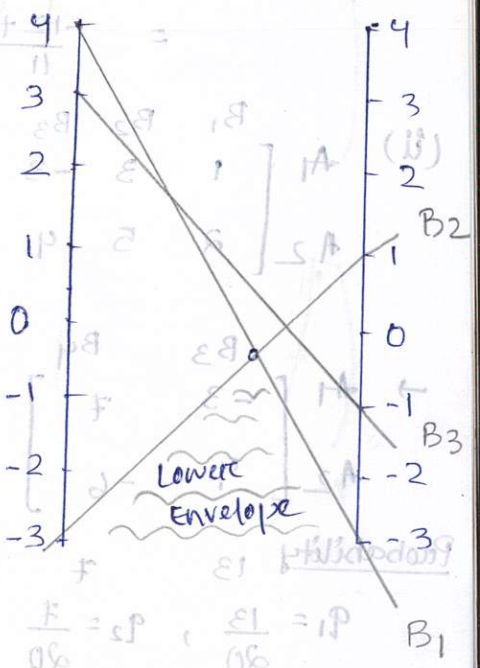
Probability

4 , $P_1 = \frac{4}{11}$

7 , $P_2 = \frac{7}{11}$

Probability 4 7

$q_1 = \frac{4}{11}$, $q_2 = \frac{7}{11}$



optimal strategy for A, $S_A = (\frac{4}{11}, \frac{7}{11})$

optimal strategy for B, $S_B = (\frac{4}{11}, \frac{7}{11}, 0)$

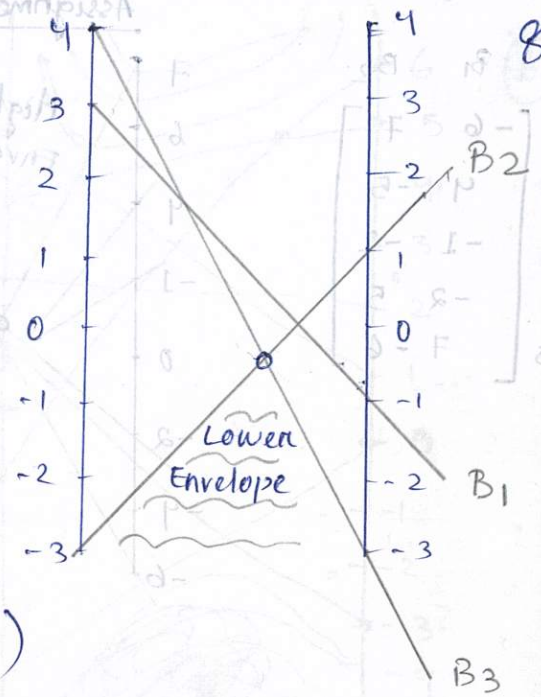
and the saddle point, $v = \frac{4}{11} \times 4 + \frac{7}{11} \times (-3)$

$= \frac{16}{11} - \frac{21}{11}$

$= -\frac{5}{11}$

Q.1

	B_1	B_2	B_3
A_1	3	-3	4
A_2	-1	1	-3



→

	B_2	B_3	Probability
A_1	-3	4	4 , $P_1 = \frac{4}{11}$
A_2	1	-3	7 , $P_2 = \frac{7}{11}$

Probability $\quad \frac{7}{11}, \quad \frac{4}{11}$

$q_1 = \frac{7}{11}, \quad q_2 = \frac{4}{11}$

∴ optimal strategy for A, $S_A = \left(\frac{4}{11}, \frac{7}{11}\right)$

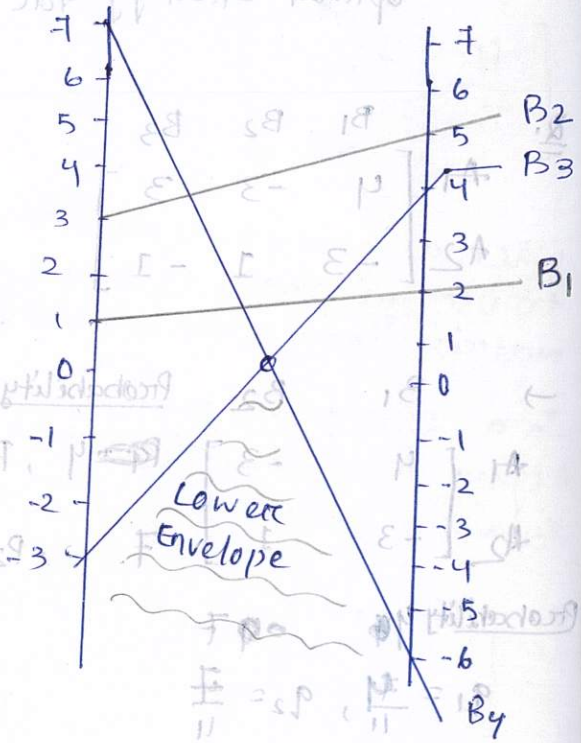
" " " B, $S_B = \left(0, \frac{7}{11}, \frac{4}{11}\right)$

$$v = \frac{4}{11}(-3) + \frac{7}{11}(1)$$

$$= \frac{-12+7}{11} = \frac{-5}{11}$$

(ii)

	B_1	B_2	B_3	B_4
A_1	1	3	-3	7
A_2	2	5	4	-6



→

	B_3	B_4	Probability
A_1	-3	7	10 , $P_1 = \frac{10}{20} = \frac{1}{2}$
A_2	4	-6	10 , $P_2 = \frac{10}{20} = \frac{1}{2}$

Probability $\quad 13 \quad 7$

$q_1 = \frac{13}{20}, \quad q_2 = \frac{7}{20}$

optimal strategy for, A, $S_A = \left(\frac{1}{2}, \frac{1}{2}\right)$

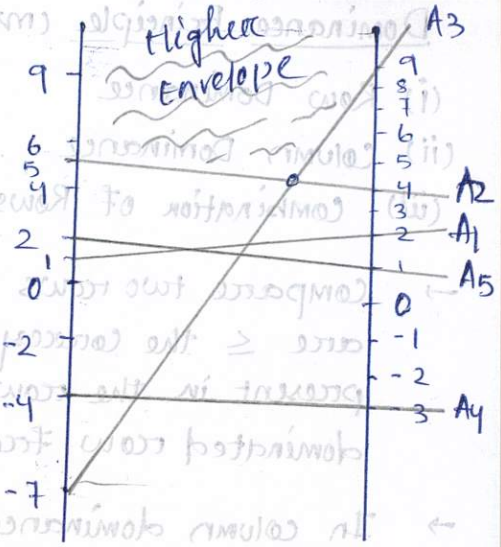
" " " B, $S_B = \left(0, 0, \frac{13}{20}, \frac{7}{20}\right)$

And $v = \frac{1}{2} \times (-3) + \frac{1}{2} \times (4)$

$$= \frac{-3+4}{2} = \frac{1}{2}$$

(iii)

	B_1	B_2
A_1	1	2
A_2	5	4
A_3	-7	9
A_4	-4	-3
A_5	2	1



→

	B_1	B_2	Probability
A_2	5	4	$16, \; P_1 = \frac{16}{17}$
A_3	-7	9	$1, \; P_2 = \frac{1}{17}$

Probability 5 12

$q_1 = \frac{5}{17}, \; q_2 = \frac{12}{17}$

Optimal strategy for A, $S_A = \left(0, \frac{16}{17}, \frac{1}{17}, 0, 0\right)$

" " " B, $S_B = \left(\frac{5}{17}, \frac{12}{17}\right)$

and $v = \frac{16}{17}(5) + \frac{1}{17}(-7)$

$= \frac{80}{17} - \frac{7}{17}$

$= \frac{73}{17}$

Dominance Principle (m×n matrix)

(i) Row Dominance

(ii) Column Dominance

(iii) Combination of Rows and Column Dominance

→ Compare two rows from the pay-off matrix if all the elements of a row are ≤ the corresponding element of another row then the smallest element present in the row dominated by largest element, Then remove the dominated row from matrix.

→ In column dominance, all the elements in a column ≤ the corresponding elements of another column then the largest element present in the column is dominated by the smallest element in the column then remove the dominated column from matrix.

Q.

$$\begin{pmatrix} 3 & 2 & 4 & 0 \\ 2 & 4 & 2 & 4 \\ 4 & 2 & 4 & 0 \\ 0 & 4 & 0 & 8 \end{pmatrix}$$

Row Dominance

$$\rightarrow \begin{pmatrix} \cancel{3} & \cancel{2} & \cancel{4} & \cancel{0} \\ 2 & 4 & 2 & 4 \\ 4 & 2 & 4 & 0 \\ 0 & 4 & 0 & 8 \end{pmatrix}$$

As $r_4 < r_3$

Column Dominance

$$\begin{pmatrix} \cancel{2} & 4 & 2 & 4 \\ 4 & 2 & 4 & 0 \\ 0 & 4 & 0 & 8 \end{pmatrix}$$

As $c_4 = c_3$

$$\begin{pmatrix} \cancel{4} & 2 & 4 \\ \cancel{2} & 4 & 0 \\ \cancel{4} & 0 & 8 \end{pmatrix} \begin{matrix} 3 \\ 2 \\ 4 \end{matrix}$$

As $c_1 > \frac{c_2 + c_3}{2}$

$$\rightarrow \begin{pmatrix} \cancel{2} & \cancel{4} \\ 4 & 0 \\ 0 & 8 \end{pmatrix} \rightarrow$$

As $r_1 = \frac{r_2 + r_3}{2}$

$$\underline{P} \quad \begin{matrix} 4 & 0 \\ 0 & 8 \end{matrix} \quad \begin{matrix} 8 \\ 4 \end{matrix}$$

$$\underline{Q} \quad \begin{matrix} 8 & 4 \end{matrix}$$

Probability

$P_1 = \frac{8}{12}$, $P_2 = \frac{4}{12}$

$q_1 = \frac{8}{12}$, $q_2 = \frac{4}{12}$

Saddle point, $v = 4 \times \frac{8}{12} + 0 \times \frac{4}{12} = \frac{8}{3}$

Optimal strategy for A, $S_A = \left(0, 0, \frac{8}{12}, \frac{4}{12}\right)$

$S_B = \left(0, 0, \frac{8}{12}, \frac{4}{12}\right)$

Q.1

(a) Quartiles divides the data set into 4 equal parts. By comparing Q_3, Q_2 & Q_1 we can measure the dispersion.

(b) The correlation coefficient ranges from -1 to 1.

(d) Coefficient of variation helps us to compare the consistency between two or more distributions.

(e) → P.E = 0.2 & n = 9

As $P.E = 0.6745\left(\frac{1-r^2}{\sqrt{n}}\right)$

$\Rightarrow 0.2 = 0.6745\left(\frac{1-r^2}{\sqrt{9}}\right)$

$\Rightarrow 0.2 \times 3 = (1-r^2)\,0.6745$

$\Rightarrow \frac{0.6}{0.6745} = 1-r^2$

$\Rightarrow r^2 = 1 - \frac{0.6}{0.6745}$

$\Rightarrow r^2 = 1 - 0.8895$

$\Rightarrow r = \sqrt{0.110}$

$\Rightarrow r = 0.331$

(f) Max $Z = 3x_1 - 2x_2$

s.t. $x_1 + x_2 \leq 1$

$2x_1 + 2x_2 \geq 4$

$\forall x \geq 0$

(g) → First we need to convert the maximization table/matrix to minimization matrix by subtracting all the elements of that matrix from the largest element of matrix, and then we can follow the hungarian technique.

(h) A T.P is said to be unbalanced, when sum of demand is not equal to sum of supply. In this situation we have to make it balance by adding a dummy row or column as per the requirement, with proper supply and demand.

Q.2

(a)

No. of Accidents	Frequency	fx
0	46	
1	f_1	
2	f_2	
3	25	
4	10	
5	5	
	f_1+f_2+86	

Given $\bar{X} = 1.46$

$N = 200$

$\Rightarrow f_1+f_2+86 = 200$

$\Rightarrow f_1+f_2 = 114$ —— (1)

(a)

X	f	fx
0	46	0
1	f_1	f_1
2	f_2	$2f_2$
3	25	75
4	10	40
5	5	25
	$140+f_1+2f_2$	

Given $\bar{X} = 1.46$

$N = 200$

$\Rightarrow f_1+f_2+86 = 200$

$\Rightarrow f_1+f_2 = 114$ —— (1)

Now $\bar{X} = 1.46$

$\Rightarrow \frac{\Sigma fx}{N} = 1.46$

$\Rightarrow \frac{140+f_1+f_2}{200} = 1.46$

$\Rightarrow f_1+2f_2+140 = 292$

$\Rightarrow f_1+2f_2 = 152$ —— (11)

Substracting eq^n(11) from eq^n(1), we get -

$$f_1 + 2f_2 = 152$$
$$-f_1 - f_2 = -114$$
$$f_2 = 38$$

Putting f_2 value on eq^n(1), we get -

$f_1 + 38 = 114$

$\Rightarrow f_1 = 76$

$\therefore (f_1, f_2) = (38, 76)$

(b) Given $\bar{x} = 43$

$\sigma x = 5$

and $n = 9$

→ So $\Sigma x = n \cdot \bar{x}$

$= 9 \times 43$

$= 387$

For addition of new item 63

$\Sigma x = 387 + 63$

$= 450$

Hence $\bar{x} = \frac{\Sigma x}{n} = \frac{450}{10} = 45$

Again $\Sigma x^2 = n(\sigma x^2 + (\bar{x})^2)$

$= 9(5^2 + 43^2)$

$= 9(25 + 1849)$

$= 9 \times 1874$

$= 16866$

For addition of new item 63,

$\Sigma x^2 = 16866 + 63^2 = 16866 + 3969$

$= 20835$

So $\sigma x = \sqrt{\frac{\Sigma x^2}{n} - (\bar{x})^2}$

$= \sqrt{\frac{20835}{10} - (45)^2}$

$= \sqrt{2083.5 - 2025}$

$= \sqrt{58.5} = 7.64$

∴ Mean = 45, & S.D = 7.64.

(c)

X	Y	Rx	Ry	D = Rx − Ry	D^2
48	13	3	5.5	−2.5	6.25
33	13	5	5.5	−0.5	0.25
40	24	4	1	3	9
9	6	10	8.5	1.5	2.25
16	15	8	4	4	16
16	4	8	10	−2	4
65	20	1	2	−1	1
24	9	6	7	−1	1
16	6	8	8.5	−0.5	0.25
57	19	2	3	−1	1
					$\Sigma = 41$

Here $m_1(16) = 3$, $m_2(13) = 2$,

& $m_3(6) = 2$

Hence $r = 1 - \frac{6\left[\Sigma d^2 + \frac{m_i(m_i^2 - 1)}{12}\right]}{n(n^2 - 1)}$

$= 1 - \frac{6\left[41 + \frac{3(3^2-1)}{12} + \frac{2(2^2-1)}{12} + \frac{2(2^2-1)}{12}\right]}{10(100 - 1)}$

$= 1 - \frac{6(41 + 2 + 0.5 + 0.5)}{990}$

$= 1 - \frac{264}{990} = 1 - 0.266 = 0.73$

(f) Max $Z = 5x_1 + 3x_2$

s.t. $3x_1 + 5x_2 \leq 15$

$5x_1 + 2x_2 \leq 10$

$\forall x \geq 0$

→ Max $Z = 5x_1 + 3x_2 + 0s_1 + 0s_2$

s.t. $3x_1 + 5x_2 + s_1 + 0s_2 = 15$

$5x_1 + 2x_2 + 0s_1 + s_2 = 10$

$x_1, x_2, s_1, s_2 \geq 0$

Simplex Table

		c_j	5	3	0	0	XB/EC
CB	BV	XB	x_1	x_2	s_1	s_2	
0	s_1	15	3	5	1	0	$\frac{15}{3} = 5$
0	s_2	10	5	2	0	1	$\frac{10}{5} = 2$ ←
		z_j	0	0	0	0	
		$z_j - c_j$	-5↑	-3	0	0	

		c_j	5	3	0	0	XB/EC
CB	BV	XB	x_1	x_2	s_1	s_2	
0	s_1	9	0	19/5	1	-3/5	$\frac{45}{19} = 0.78$ ←
5	x_1	2	1	2/5	0	1/5	$\frac{20}{2} = 10$
		z_j	5	2	0	1	
		$z_j - c_j$	0	-1↑	0	1	

		c_j	5	3	0	0	
CB	BV	XB	x_1	x_2	s_1	s_2	
3	x_2	$\frac{45}{19}$	0	1	$\frac{5}{19}$	$\frac{-3}{19}$	
5	x_1	20/19	1	0	$\frac{2}{19}$	5/19	
		z_j	5	3	[illegible]	[illegible]	5/19 16/19
		$z_j - c_j$	0	0	[illegible]	[illegible]	0 0

Since all $z_j - c_j \geq 0$, the soln has reached its optimal

~~and Max Z~~ = [illegible]

$x_1 = \frac{20}{19}$, $x_2 = \frac{45}{19}$, Max $Z = \frac{235}{19}$

(h) Machines — Row Reduction — Column Reduction

Jobs	A	B	C	D
1	18	24	28	32
2	8	13	17	18
3	10	15	19	22

$\rightarrow$

	0	6	10	14
	0	5	9	10
	0	5	9	12

$\rightarrow$

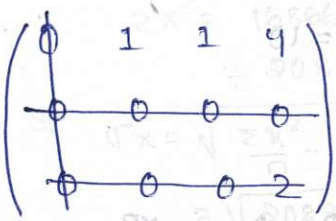
	0	1	1	4
	0	0	0	0
	0	0	0	2

$\rightarrow$

	[0]	1	1	4
	~~0~~	~~0~~	~~0~~	[0]
	~~0~~	[0]	~~0~~	2

(Assignment Process)

$\rightarrow$



(Drawing Lines)

$\rightarrow$

	1	0	0	3
	1	0	0	0
	1	0	0	2

(New Table)

	1	[0]	~~0~~	3
	1	~~0~~	~~0~~	[0]
	1	~~0~~	[0]	2

(h)

Jobs	A	B	C	D
1	18	24	28	32
2	8	13	17	18
3	10	15	19	22

$\rightarrow$

	18	24	28	32
	8	13	17	18
	10	15	19	22
	0	0	0	0

(Balanced Matrix)

$\rightarrow$

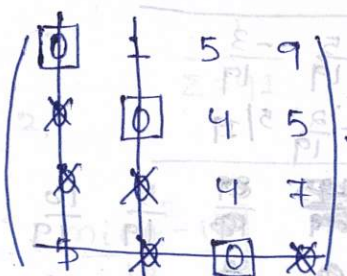
	0	6	10	14
	0	5	9	10
	0	5	9	12
	0	0	0	0

(Row Reduction & Column Reduction)

	[0]	6	10	14
	~~0~~	5	9	10
	~~0~~	5	9	12
	~~0~~	[0]	~~0~~	~~0~~

(Assignment process & Drawing lines)

$\rightarrow$



(New table, Assignment process & Drawing lines)

$\rightarrow$

	[0]	1	1	5
	~~0~~	[0]	~~0~~	1
	~~0~~	~~0~~	[0]	3
	9	4	~~0~~	[0]

Ans is

Jobs	Machines	Cost
1	A	18
2	B	13
3	C	19
		50

Q.4 A retired person's broker recommends him to invest his money in two bonds A & B

Let x_1 rupees invest in bond A

& x_2 rupees " " " B.

Bond A is yielding 7% and Bond B is yielding 10%.

Thus Max $Z = \frac{7}{100} x_1 + \frac{10}{100} x_2$

Also the retired person wants to invest up to an amount of Rs. 30,000

Hence $x_1 + x_2 \leq 30{,}000$

He decides to invest at most Rs. 12000 in Bond B

So $x_2 \leq 12000$

and at least Rs. 6000 in Bond A

So $x_1 \geq 6000$

He also wants the amount invested in bond A (x_1) to be at least equal to the amount invested in Bond B (x_2).

that means $x_1 \geq x_2$

$\Rightarrow x_1 - x_2 \geq 0$

Hence the LPP is Max $Z = \frac{7}{100} x_1 + \frac{10}{100} x_2$

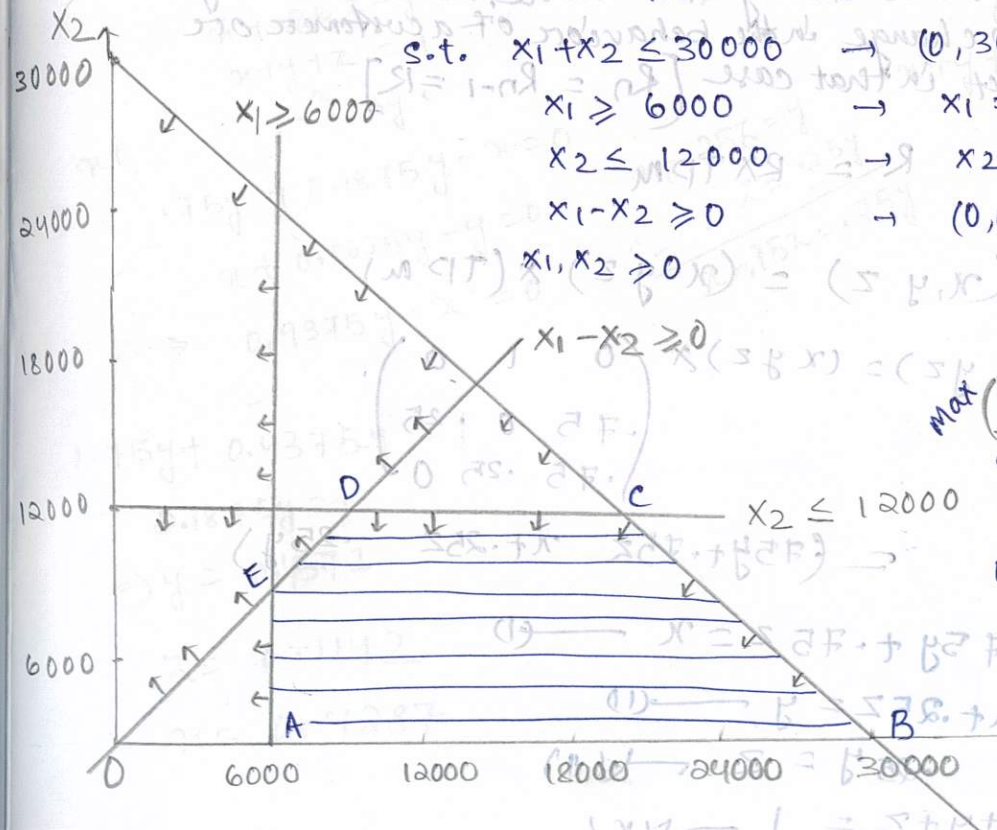
s.t. $x_1 + x_2 \leq 30000$ → (0, 30000) & (30000, 0)

$x_1 \geq 6000$ → $x_1 = 6000$

$x_2 \leq 12000$ → $x_2 = 12000$

$x_1 - x_2 \geq 0$ → (0,0)

$x_1, x_2 \geq 0$



Points	Max $Z = \frac{7}{100} x_1 + \frac{10}{100} x_2$
A (6000, 0)	$Z = \frac{42000}{100} = 420$
B (0, 30000)	$Z = 0 + 3000 = 3000$
C (18K, 12K)	$Z = 1260 + 1200 = 2460$
D (12K, 12K)	$Z = 840 + 1200 = 2040$
E (6K, 6K)	$Z = 420 + 600 = 1020$

Max

Here ABCDE is the feasible region

Markov process are analysis based on probability. In Markov process we will collect the

Q.

Today \ Next week	A	B	C
A	1	0	0
B	0	1	0
C	0	1	0

6 days

Next Week

	A	B	C
A	0	1	0
B	.75	0	.25
C	.75	.25	0



	A	B	C
A	90%	5%	5%
B	5%	85%	10%
C	10%	7%	8.3%

Row wise

In a TPM, the row wise movement depicted as retention & loss and more column wise it is described as retention & gain and the diagonal elements of a TPM described retention only.

In a long run the behavior of the customer on the market share of any product will be in a steady state mode, means in a long run there will be no change in the behavior of a customer or market share of product in that case $[R_n = R_{n-1} = R]$

$$R = R \times TPM$$

Q

	A	B	C
A	0	1	0
B	.75	0	.25
C	.75	.25	0

$$\Rightarrow (x, y, z) = (x\ y\ z) \times (TPM)$$

$$\Rightarrow (x\ y\ z) = (x\ y\ z) \times \begin{pmatrix} 0 & 1 & 0 \\ .75 & 0 & .25 \\ .75 & .25 & 0 \end{pmatrix}$$

$$= (.75y + .75z \quad x + .25z \quad .25y)$$

$$\Rightarrow .75y + .75z = x \quad \text{---(I)}$$

$$x + .25z = y \quad \text{---(II)}$$

$$.25y = z \quad \text{---(III)}$$

$$x + y + z = 1 \quad \text{---(IV)}$$

→ $.75y + .25y = x \quad (.25y = z)$

⇒ $x = y$

∴

$$.75y + .75z = x$$

Putting eq^n(3) in eq^n(1), we get

$$.75y + .75(.25y) = x$$

$$\Rightarrow .75y + 0.1875y = x$$

$$\Rightarrow x = 0.9375y \quad \text{—— (V)}$$

$$x + y + z = 1$$
$$-x + .75y + .75z = 0$$
$$1.75y + 1.75z = 1$$
$$-.25y + z = 0$$
$$1.5y + 2.75z = 1$$

$$.75z - x = -.75y$$
$$.25z + x = y$$
$$z = 0.25y$$

$$-.75y + .75z - x = 0$$
$$-y + .25z + x = 0$$
$$z - .25y = 1.75y$$

$$x + y + .25z = 0$$
$$2x + 1.25z = 1$$
$$z - .25$$

$$z - .25y = 0$$

$$.75y + .75z - x = 0$$
$$x + .25z - y = 0$$
$$x + y + z = 1$$
$$z = .25y$$

$$0.75y + 0.75z = x$$
$$x + 0.25z = y$$
$$0.25y = z$$
$$x + y + z = 1$$

$$.75y + 0.1875y - x = 0$$
$$x + 0.0625y - y = 0$$
$$\Rightarrow 0.9375y - x = 0$$

$$x + .25z = y$$
$$-z = .25y$$
$$x - .75z = .75y$$

$$.75y + .75z - x = 0$$
$$-y + .25z + x = 0$$
$$-0.25y + z = 0$$

$$1.75y + 0.4375y = 1$$
$$2.1875y = 1$$
$$\Rightarrow y = 0.4571$$
$$z = 0.1142$$
$$x = 0.4287$$

SIMULATION

Simulation is a process where experiment is carried out in an Artificial environment, where all the parameters are in control and through simulation we can gather the knowledge which helps us in real environment, for making decision.

The examples are :- Pilot training, car driving, Military training

Monte-Carlo simulation method / Technique

1) Objective
2) Data collection
3) Probability, cumulative probability
4) Tag number
5) Random number
6) Analysis
7) Interpretation of Result / Then Decision

Q. A manufacturing company keeps a stock of a special product, previous experience indicates the daily demand as follows

Daily Demand	Probability
5	~~0.01~~ .01
10	~~0.0101~~ 0.2
15	~~0.2~~ .15
20	~~0.2~~ .5
25	~~0.15~~ .12
30	~~0.5~~ .02

Simulate the demand for the next 10 days also find the daily average demand for that product on the basis of ~~simulate~~ simulated data.

→ Random Number

82
96
18
76
20
84
56
11
52

Daily Demand	P	cP	Tag number
5	0.01	0.01	0-0
10	0.2	0.21	1-20
15	0.15	0.36	21-35
20	0.5	0.86	36-85
25	0.12	0.98	86-97
30	0.02	1	98-99
	Σ=1		

R.N	Demand
82	20
96	25
18	10
96	25
20	10
84	20
56	20
11	10
52	20
03	10

Avg. Daily Demand = Total no. of unit / 10 = 170 / 10 = 17

Q.

Production	Probability	cP	Tag No
95	0.03	0.03	0-02
96	0.05	0.08	3-9
97	0.07	0.15	10-14
98	0.10	0.25	15-24
99	0.15	0.40	25-39
100	0.20	0.60	40-59
101	0.15	0.75	60-74
102	0.10	0.85	75-84
103	0.07	0.92	85-91
104	0.05	0.97	92-96
105	0.03	1	97-99

R.N	Tag No
97	0-2
2	3-7
80	8-14
66	15-24
	25-

R.N →	Production	Cars Waiting	Empty space
97	1050	105	95
02	95*	102	101
80	102°	104	100
66	101*	103	100
96	104*		99
55	100*	Σ=414	100
50	100*	Avg = 414/4	100
29	99*	=103.5	96
58	100*		98
51	100*		99
04	96*		100
86	103°		Σ=1088
24	98*		Avg = 98.90
39	99*		
47	100*		

~~RCA~~

Production	cores Waiting	Empty space
105	4	0
95	0	6
102	1	0
101	0	0
104	3	0
100	0	1
100	0	1
99	0	2
100	0	1
100	0	1
96	0	5
103	2	0
98	0	3
99	0	2
100	0	1
	$\Sigma = 10$	$\Sigma = 23$

~~Avg = $\frac{10}{4}$ = 2.5~~

Avg = $\frac{10}{15}$ = 0.66

~~Avg = $\frac{23}{10}$ = 2.3~~

Avg = $\frac{23}{15}$ = 1.53

Q.e

Time bet^n arrival (min)	Frequency	Probability	CP	Tag No.
1.0	18	18/100 = 0.18	0.18	0 - 17
1.5	15	15/100 = 0.15	0.33	18 - 32
2.0	36	36/100 = 0.36	0.69	33 - 68
2.5	19	19/100 = 0.19	0.88	69 - 87
3.0	12	12/100 = 0.12	1	88 - 99

Random Arrivals	Arrival (min)
25	1.5
39	2.0
65	2.0
76	2.5
12	1.0
05	1.0
73	2.5
89	3.0
19	1.5
49	2.0
	$\Sigma = 19$

Average Arrival Time = $\frac{19}{10} = 1.9$

Q.5

→ Let X_1 & X_2 be the number of ~~products~~ chemicals A and B.

The total production of the chemicals A and B should be at least 350kg.

$$X_1 + X_2 \geq 350$$

A major customer orders for 125kg of product A must also be supplied.

$$X_1 \geq 125$$

Product A requires 2 hrs of processing time/kg

and " B " 1 hrs " " " " .

But for the coming month, 600 hrs of processing time are available

$$2X_1 + X_2 \leq 600.$$

The company wants to meet the requirements at minimum total production cost, where A's ~~prodn~~ and B's production costs are ~~Rs~~

Rs. 2/kg & Rs. 3/kg respectively.

Q.

Time bet^n arrival (min)	frequency
0.5	2
1.0	6
1.5	10
2.0	25
2.5	20
3.0	14
3.5	10
4.0	7
4.5	4
5.0	2

Arrival R.N.
93
22
53
64
39
07
10
63
76
35

Time bet^n service	Frequency
0.5	12
1.0	21
1.5	36
2.0	19
2.5	7
3.0	5

Service R.N.
78
76
58
54
74
92
38
70
96
92

→

Inter Arrival	P	CP	Tag No
0.5	0.02	0.02	0–1
1.0	0.06	0.08	2–7
1.5	0.1	0.18	8–17
2.0	0.25	0.43	18–42
2.5	0.20	0.63	43–62
3.0	0.14	0.77	63–76
3.5	0.10	0.87	77–86
4.0	0.07	0.94	87–93
4.5	0.04	0.98	94–97
5.0	0.02	1	98–99

Inter Arrival (time)	Arrival time
4.0	4.0
2.0	6.0
2.5	8.5
3.0	11.5
2.0	13.5
1.0	14.5
1.5	16.0
3.0	19.0
3.0	22.0
2.0	24.0

Time bet^n service (min)	P	C.P	Tag No
0.5	0.12	0.12	0–11
1.0	0.21	0.33	12–32
1.5	0.36	0.69	33–68
2.0	0.19	0.88	69–87
2.5	0.07	0.95	88–94
3.0	0.05	1	95–99

Service time	Service start Intime	service end outtime	C.W	Server
2	4	6	—	4
2	6	8	—	—
1.5	8.5	10	—	0.5
1.5	11.5	13	—	1.5
2	13.5	15.5	—	0.5
2.5	15.5	18	1	0
1.5	18.0	19.5	2	—
2	19.5	21.5	0.5	—
3	22.0	25	0	0.5
2.5	25	27.5	1	—
			Σ = 4.5	Σ = 7

Customer waiting time = 4.5, Avg = $\frac{4.5}{10}$ = 0.45

Server idle time = 7, Avg = $\frac{7}{10}$ = 0.7

Q. A tourist car operator

1. What do you mean by simulation? What are the advantages and limitation of simulation.

2. What do you mean by Markov process? In what area of management markov process applied successfully.

3. A sample of 100 arrival of customer in a departmental store is acc. to the following. Simulate the for the next 8. R.N. 25, 39, 65, 76, 12, 5, 73, 89

Time bet^n arrival	Frequency
0.5	12
1	21
1.5	36
2	19
2.5	7
3	5

→

Time bet^n Arrival	Frequency	P	CP	Taq No	R.N	Arrival
0.5	12	0.12	0.12	0 – 11	25	1
1	21	0.21	0.33	12 – 32	39	1.5
1.5	36	0.36	0.69	33 – 68	65	1.5
2	19	0.19	0.88	69 – 87	76	2
2.5	7	0.07	0.95	88 – 94	12	1
3	5	0.05	1	95 – 99	5	0.5
					73	2
					89	2.5
						Σ = 12

Avg = $\frac{12}{8}$ = 1.5

Q.1 What do you mean by simulation? What are the Advantages & Limitations of simulation? (25)

→ Simulation is a process where experiment is carried out in an artificial environment, where all the parameter are in control and through simulation, we can gather the knowledge which helps us in real environment for making decision.

Simulation is one of the most widely used and popularly accepted tool in operation research. It helps us in deciding the best policy with prior assurances that it implementation will certainly prove to be beneficial to the organization.

Some examples where simulation is being used are :- Military Training, Car Training, Weather forecasting etc.

Advantages :-

(i) It can be used to study existing systems without disrupting the ongoing operations.

(ii) It allows us to control times.

(iii) It is best suited to analyze complex and large practical problems when it is not possible to solve them through a mathematical method.

(iv) It helps us to forecast or analyze the scenario for longer period.

Limitations :-

(i) It does not generate optimal solutions.

(ii) In certain cases simulation model can be very expensive.

(iii) It may take a long time to develope a good simulation model.

(iv) It needs all the information about the constraint for analyzing, as simulation does not give the answer by itself.

Q.2 What do you mean by Markov Process? In what area of management, Markov process applied successfully?

→ Markov process is a mathematical model that looks at an array of events and analyzes the tendencies of each event. It was named after the Russian mathematician Andrey Markov. Markov process have been used to analyze inventory problems, equipment replacement process, plant location problems, brand-switching problems and dynamic systems.

In the following area of management, Markov process can be applied successfully :-

1. Inventory Management :- It can be applied to model inventory systems where the availability of items fluctuates over time.

2. Quality control :- In Manufacturing, Markov process can model the state transitions of a system's quality. This is useful for improving product quality over time.

3. Financial Management :- Markov processes find applications in financial modelling, particularly in analyzing market states, portfolio management, and predicting changes in economic conditions.

4. Customer Relationship Management :- Markov processes are used to model customer behavior in areas such as retention and purchasing patterns. This helps businesses make informed decisions regarding marketing and customer engagement strategies.

Queueing Theory

A Queue is a waiting line where customers are waiting to get their service and the line sometime finite & sometime infinite and generally A queue arises when no. of customers are more than service facility.

Queueing System

There are 4 different system in queueing system

1) The Input
2) Queue Discipline
3) Service Mechanism
4) customer's Behavior

specific terms which describe the behavior's customer.

1) Managing
2) Balking
3) Jockeying

→ customer, who jumps from one Queue to another Queue by observing which queue is moving fast, those types of customer are known as Jockey type.

→ This type of customer left the queue due to impatience are Managing Balk

→ A customer, who do not join the queue because the queue is too long, are Balking type.

Queue Discipline

There are 4 type of Discipline

1) FIFO (First in first out)
2) LIFO (Last in first out)
3) SIRO (Service in Random order)
4) Priority (VIP, senior citizen)

→ FIFO is a usual discipline in a queueing system, also known as FCFS.

Service Mechanism / Service Pattern

1) Single Queue – single server
2) ,, ,, – Multiple server
3) Multiple queue – single server
4) Multiple queue – Multiple server

The most important parameter

λ is the parameter
μ (MU)

λ is defined as the arrival rate per unit time, per
μ is the parameter, which is defined as service rate/unit time.

Kendall's Notation (To represent Queueing Model)

Generally, the queueing models are described by the help of Kendall's notation, **a/b/c/:d/e**,

where a is the arrival pattern
b described as service pattern
c described as no. of channels / no. of service station
d ,, as queue length
e ,, as queue discipline.

There are 4 different types of models in queueing system

1) M/M/1 : ∞/FCFS
2) M/M/S : ∞/FCFS
3) M/M/1 : N/FCFS
4) M/M/S : N/FCFS

Important Measures in Queueing Theory

1) Traffic Intensity (Server Busy) (ρ) ($1-\rho$ = server idle)
2) Waiting Time of a Customer in a Queue (W_q)
3) Waiting Time of a ~~Customer~~ System in a ~~Queue~~ System (W_s)
4) ~~Max~~ Length of the Queue (L_q)
5) Length of the System (L_s)

What is Queue?
Define customer.
Describe Queueing System.
Kandall's Notation.

TP
Assignment Problem

possible 2 mark Q.

1) Define Decision variable
2) What do you mean by LPP
3) Define slack & surplus variable
4) What is feasible region.
5) What is unbalanced TP.
6) What is unbalanced Assignment problem
7) what is degeneracy is in TP.
8) what is unbounded solution in simplex method?
9) Define LOOP
10) What is Alternative optimal sol^n in Simplex method?
11) Define Artificial variable.

Q.1 What is Queue?

→ Queues are a part of everyday life. We all wait in queues to buy a movie ticket, to make bank deposit, pay for groceries, to have ride in an amusement park and have become adjustment to wait but still get annoyed by unusually long waits.

However, It can be defined as "A flow of customers from finite or infinite population towards the service facility forms a waiting line otherwise known as a Queue.

Q.2 Define customer

→ The one who requires service is called a customer. The customer may be persons, machines, vehicles, etc. The customer is otherwised known as "Arriving unit".

Q.3 Describe Queueing System.

→ A queueing system can be described as customers arriving for service, waiting for service if it is not immediate and leaving the service after being served.

There are 4 elements of a Queueing System.

(1) The Input
(2) Queue Discipline
(3) Service Mechanism
(4) Customer's Behavior

* The Input (Arrival Pattern)

It describes the way in which the customers arrive and join the system. Generally, customers arrive in a ~~poission~~ poisson fashion, which is not possible to predict. Hence, the arrival pattern can be described in terms of probabilities or probabilities for inter arrival times. The Mean arrival rate is denoted by λ.

* Queue Discipline

It can be defined as the rule for selecting the next customer from the queue for service. The most common disciplines are :-

1) FIFO
2) LIFO
3) SIRO
4) PRIORITY

* FIFO (First In First Out)

FIFO is a queueing discipline where the earliest item to join the queue is the first to be served.

ex:- checkout line at Grocery store. The first person to join the line is the first one to complete their purchase and leave.

* LIFO (Last In First Out)

LIFO is a queueing discipline, where the most recently added item or the last item that enters the queue is the first to be served.

ex:- Stack of Plates. The last plate added to the stack is the first plate you pick up to use.

* SIRO (Service In Random Order)

SIRO is a queueing discipline, where items are served in a random sequence without following a specific order.

ex:- Lottery ticket processing, where winners are randomly selected.

* PRIORITY

PRIORITY discipline is a queueing model, where items are assigned priority values, and the item with the highest priority is served first.

ex:- VIP's, Senior citizens, Emergency Patient.

SERVICE MECHANISM

This means, the arrangement of service facility to serve customers. A service station may have one or several servers arranged in parallel.

Basic elements of Service Mechanism/ Service Pattern

1) Single Queue - Single Server
   → A single cashier serving customers in a grocery store.
2) Single Queue - Multiple server
   → Airport security checkpoints with multiple screening lanes.
3) Multiple Queue - Single Server
   → Customer service desks at a large retail store.
4) Multiple Queue - Multiple Server
   → Fast-food restaurant with multiple order counters.

The most impotant parameter of service mechanism are "λ" and "μ"

λ is defined as the arrival rate per unit time.

μ is defined as the service rate per unit time.

CUSTOMER'S BEHAVIOR

It refers to how customers join the queue, their expectations, and their reaction to the queueing process. Understanding customer behavior is crucial for designing effective queueing systems and ensuring a positive experience for those waiting in line.

Customers generally behave in following ways.

1. Balking :- A customer who leaves the queue because the queue is too long, and he has no time to wait or does not have sufficient waiting space.

2. Reneging :- It refers to a situation, where a customer joins the queue but decides to leave before reaching the service point due to impatience or dissatisfaction with the wait.

3. Jockeying :- customer, who jumps from one queue to another queue by observing which queue is moving fast. This is most common in a supermarket.

8. In a supermarket, the avg. arrival rate of customer is 10 in every 30 min and the avg time taken by the server is 2.5m/customer. Find out server busy time and expected time spent by the customer in the system. (33)

Here λ is 10 in every 30 min $\lambda = \frac{10}{30}$ min

As avg time taken by the server $\mu = \frac{1}{2.5}$ min

$$\rho = \frac{\lambda}{\mu} = \frac{0.33}{0.4} = 0.825$$

$$W_s = \frac{1}{\mu - \lambda} \qquad \frac{1}{\mu - \lambda} = \frac{1}{0.4 - 0.33}$$

$$= \frac{1}{0.4} - 0.33 \qquad = \frac{1}{0.07}$$

$$= 2.5 - 0.33$$

$$= 2.17$$

People arrive at a cinema hall with a rate of 25 per hour and service time is per customer 2 mins. calculate the avg. no. of customer in the queue (L_q), W_s and utilization factor (ρ)

$$\lambda = \frac{25}{60} = 0.41$$

$$\mu = \frac{1}{2} = 0.5$$

$$\rho = \frac{\lambda}{\mu} = \frac{0.41}{0.5} = 0.82$$

$$L_q = \frac{\rho^2}{1 - \rho} = \frac{0.6724}{1 - 0.82} = \frac{0.6724}{0.18} = 3.735$$

$$W_s = \frac{1}{\mu - \lambda} = \frac{1}{0.5 - 0.41} = \frac{1}{0.09} = 11.11$$

$\lambda = 25$

$\mu = 30$

$\rho = \frac{25}{30} = 0.83$

$L_q = \frac{\rho^2}{1 - \rho} = \frac{0.6889}{1 - 0.83} = \frac{0.6889}{0.17}$

(1) Traffic Intensity / Server Busy / Utilization Factory (ρ) rho

$$\rho = \frac{\lambda}{\mu}$$

(2) Server Idle $= (1 - \rho)$

(3) Avg no. of customer in the system $L_s = \frac{\rho}{1-\rho}$

(4) Avg no. of customer in the queue $L_q = \frac{\rho^2}{1-\rho}$

(5) Waiting time of a customer in the system $W_s = \frac{1}{\mu - \lambda}$

(6) Waiting time of a customer in the queue $W_q = \frac{\lambda}{\mu(\mu-\lambda)}$

$$L_s = \frac{\rho}{1-\rho}$$

$$L_q = \frac{\rho^2}{1-\rho}$$

$$W_s = \frac{1}{\mu} - \lambda$$

$$W_q = \frac{\lambda}{\mu(\mu-\lambda)}$$

Q. A T.V repairemen repair a T.V with a mean rate of 30 mins per Tv. and the arrival of T.V to the repairman 10 per shift in a day. Find out the traffic intensity, Server idle, length of the system, length of the queue and W_s, W_q.

→ Here λ is 10

As 30min is the repair time per T.V,

Thus $\mu = 16$

Now Traffic intensity, $\rho = \frac{10}{16}$

Server Idle, $\rho = 1 - \frac{10}{16}$

Length of the System $L_s = \frac{\rho}{1-\rho} = \frac{0.625}{0.375} = 1.66$

Length of the queue, $L_q = \frac{\rho^2}{1-\rho} = \frac{0.390625}{0.375} = 1.041$

$$W_s = \frac{1}{\mu} - \lambda = \frac{1}{16} - 10 = 0.16$$

$$W_q = \frac{\lambda}{\mu(\mu-\lambda)} = \frac{10}{16 \times 6} = \frac{10}{96} = 0.104$$

Q. Cars arrive at a petrol pump with a mean rate of half min. The attendent take $\frac{1}{5}$ min to supply the petrol. calculate the avg. no waiting to be serve? The avg no. of cars in the queue and calculate the petrol pump attendent idle time. (35)

→ ~~$\frac{1}{2}$~~ Cars are

$\lambda = 2$

$\mu = \frac{1}{5}$ $\quad \mu = \frac{1}{1/5} = 5$

$\rho = \frac{1}{2} \times \frac{5}{1}$ $\quad \rho = 0.4$

$L_s = \frac{\rho}{1-\rho} = \frac{0.4}{1-0.4} = \frac{0.4}{0.6} = 0.66$

$L_q = \frac{\rho^2}{1-\rho} = \frac{0.16}{0.6} = 0.26$

idle time $1 - \rho = 0.6$

24 hr — 30 min

1 — $\frac{30}{24}$ min

1 min $\frac{30}{24 \times 60}$

$\frac{1}{5}$ hours $\quad \frac{60}{5} = 12$

Q.

→ Train arrive at a rate of 30 ~~mins~~ trains per day.

means 24 hr — 30 trains

1 hr → $\frac{30}{24}$ trains per hr

and 1 min → $\frac{\frac{30}{24}}{60} = \frac{30}{24 \times 60} = \frac{1}{48}$ trains/min

Also Service rate per unit, $\mu = \frac{1}{30}$ min

(i) Mean queue size, $\rho = \frac{\lambda}{\mu} = \frac{1}{48} \times 30 = 0.625$

(ii)

Q.

100	200	300
140	100	500
80	120	60

			Supply
100	200	300	100
140	100	500	110
Demand 80	120	60	

Supply

→ 100	80	200 20	300	~~100~~ 20 ~~0~~
140		100 100	500 10	~~110~~ 10
0		0	0 50	50
Demand	~~80~~ 0	~~120~~ ~~100~~ 0	60	

T.P cost = 100×80 + 200×20 + 100×100 + 500×10 + 0×50

= 8000 + 4000 + 10000 + 5000

= 27000

Using MODI

Allocated cell ($U_1 = 0$)

$C_{11} = U_1 + V_1 = 100$, $V_1 = 100$

$C_{12} = U_1 + V_2 = 200$, $V_2 = 200$

$C_{22} = U_2 + V_2 = 100$, $U_2 = -100$

$C_{23} = U_2 + V_3 = 500$, $V_3 = 600$

$C_{33} = U_3 + V_3 = 0$, $U_3 = -600$

Non Allocated

$\Delta_{13} = C_{13} - (U_1 + V_3) = 300 - (0 + 600) = -300 < 0$

$\Delta_{21} = C_{21} - (U_2 + V_1) = 140 - (-100 + 100) = 140 > 0$

$\Delta_{31} = C_{31} - (U_3 + V_1) = 0 - (-600 + 100) = 500 > 0$

$\Delta_{32} = C_{32} - (U_3 + V_2) = 0 - (-600 + 200) = 400 > 0$

As $\Delta_{13} < 0$, so we need to start a loop from C_{13}

100	80	200 20 (-)	300 (+)
140		100 (+) 100	500 10 (-)
0		0	0 50

$\theta = \min\{20, 10\} = 10$

→

100	80	200 10	300 10
140		100 110	500
0		0	0 50

It is a non-degeneracy.

Allocated cell ($U_1 = 0$)

$C_{11} = U_1 + V_1 = 100$, $V_1 = 100$

$C_{12} = U_1 + V_2 = 200$, $V_2 = 200$

$C_{13} = U_1 + V_3 = 300$, $V_3 = 300$

$C_{22} = U_2 + V_2 = 100$, $U_2 = -100$

$C_{33} = U_3 + V_3 = 0$, $U_3 = -300$

Net evalution

$\Delta_{21} = C_{21} - (U_2 + V_1) = 140 - (-100 + 100) = 140 > 0$

$\Delta_{23} = C_{23} - (U_2 + V_3) = 500 - (-100 + 300) = 300 > 0$

$\Delta_{31} = C_{31} - (U_3 + V_1) = 0 - (-300 + 100) = 200 > 0$

$\Delta_{32} = C_{32} - (U_3 + V_2) = 0 - (-300 + 200) = 100 > 0$

As all $\Delta_{ij} > 0$, so our Soln reached optimal.

And total cost = 100×80 + 200×10 + 300×10 + 100×110 + 0×50

= 8000 + 2000 + 3000 + 11000

= 24000

Q. Mean = 40

S.D = 5.1

n = 100

→ As Mean = 40

$\Rightarrow \frac{\Sigma X}{n} = 40$

$\Rightarrow \Sigma X = 400$

Thus correct sum = wrong sum − wrong obs^n + correct obs^n

$\Sigma X = 4000 - 50 + 40$

$= 3900$

Thus correct Mean $= \frac{3900}{100} = 39.9$

Now we know $\sigma_x^2 = \frac{\Sigma x^2}{n} - (\bar{x})^2$

$\Rightarrow (5.1)^2 = \frac{\Sigma x^2}{n} - (40)^2$

$\Rightarrow \frac{(26.01 + 1600)}{100} = \Sigma x^2$

$\Rightarrow \Sigma x^2 = (26.01 + 1600) \times 100$

$\Rightarrow \Sigma x^2 = 162601$

Hence correct sum square $= \Sigma x^2 - (50)^2 + (40)^2$

$\Sigma x^2 = 162601 - 2500 + 1600$

$\Rightarrow \Sigma x^2 = 161701$

$\therefore \sigma_x = \sqrt{\frac{\Sigma x^2}{n} - (\bar{x})^2}$

$= \sqrt{\frac{161701}{100} - (39.9)^2}$

$= \sqrt{1617.01 - 1592.01}$

$= 5$

Q.2

Price of A	Price of B	$(x-\bar{x})$	$(x-\bar{x})^2$	$(y-\bar{y})$	$(y-\bar{y})^2$
55	108	2	4	3	9
54	107	1	1	2	4
52	105	-1	1	0	0
53	105	0	0	0	0
56	106	3	9	1	1
58	107	5	25	2	4
52	104	-1	1	-1	1
50	103	-3	9	-2	4
51	104	-2	4	-1	1
49	101	-4	16	-4	16
530	1050		70		40

$\bar{x} = \frac{530}{10} = 53$ $\quad \bar{y} = \frac{1050}{10} = 105$

Now standard deviation of x,

$$\sigma_x = \sqrt{\frac{(x-\bar{x})^2}{N}} = \sqrt{\frac{70}{10}} = 2.64$$

and $$\sigma_y = \sqrt{\frac{(y-\bar{y})^2}{N}} = \sqrt{\frac{40}{10}} = 2$$

Q.8 Find out mean, median & mode

Marks (x)	Frequency f	M	fM	C.F
10-20	15	15	225	15
20-30	20	25	500	35
30-40	45	35	1575	80
40-50	15	45	675	95
50-60	5	55	275	100
	100		3250	

Mean $= \frac{\Sigma fM}{N} = \frac{3250}{100} = 32.5$

Median class $\frac{N}{2} = \frac{100}{2} = 50$,

So it lies betⁿ class (30-40) having frequency 45

So Median $= L_1 + \frac{L_2 - L_1}{f_1}(M - c)$

$= 30 + \frac{40-30}{45}(50-25)$

$= 30 + \frac{10}{45} \times 25$

$= 30 + 5.55 = 35.55$

and Modal class is 30-40 45

Hence Mode $= L_1 + \frac{f_1 - f_0}{2f_1 - f_2 - f_0}(L_2 - L_1)$

$= 30 + \frac{45-20}{90-15-20}(40-30)$

$= 30 + \frac{25}{55} \times 10$

$= 30 + 4.54$

$= 34.54$

Mode = 3Median - 2Mean

Emperical relation betⁿ Mean, Me, Mo

Q.3

Test scores	weekly sales (in 000)	$dx = x - \bar{x}$	$dy = y - \bar{y}$	dx^2	dy^2	$dx\,dy$
50	30	-10	-13	100	169	130
60	60	0	17	0	289	0
50	40	-10	-3	100	9	30
60	50	0	7	0	49	0
80	60	20	17	400	289	340
50	30	-10	-13	100	169	130
80	7	20	-36	400	1296	-720
40	50	-20	7	400	49	-140
70	60	10	17	100	289	170
540	387	$\Sigma = 0$	0	1600	2608	-60

$b_{yx} = \frac{\Sigma dx\,dy}{\Sigma dx^2} = \frac{-60}{1600} = -0.0375$

(i) regression eq yonx $\Rightarrow (y - \bar{y}) = b_{yx}(x - \bar{x})$

$(y - 43) = -0.0375(x - 54)$

$\Rightarrow y + 0.0375x + 45.025 = 0$

(ii) If $x = 65$

$\rightarrow y + 0.0375(65) + 45.025 = 0$

$\Rightarrow y + 2.4375 - 40.975$

$\Rightarrow y = 38.5375$

Q.7 Derive regression lines from the following data. (40)

$\Sigma X = 30$, $\Sigma X^2 = 190$, $\Sigma XY = 192$

$\Sigma Y = 30$, $\Sigma Y^2 = 190$, $n = 5$

$$b_{yx} = \frac{n\Sigma XY - \Sigma X \cdot \Sigma Y}{n\Sigma X^2 - (\Sigma X)^2}$$

$$b_{xy} = \frac{n\Sigma XY - \Sigma X \cdot \Sigma Y}{n\Sigma Y^2 - (\Sigma Y)^2}$$

→ According to direct method,

$$b_{xy} = \frac{n\Sigma XY - \Sigma X \cdot \Sigma Y}{n\Sigma Y^2 - (\Sigma Y)^2}$$

$$= \frac{5 \times 192 - (30 \times 30)}{5 \times 190 - (30)^2} = \frac{960 - 900}{950 - 900} = \frac{60}{50} = 1.2$$

And $b_{yx} = \frac{n\Sigma XY - \Sigma X \cdot \Sigma Y}{n\Sigma X^2 - (\Sigma X)^2} = \frac{5 \times 192 - (30 \times 30)}{5 \times 190 - (30)^2} = 1.2$

We know the regression eq^n X on Y is

~~$(x - \bar{x}) = b_{yx}(y - \bar{y})$~~

$$(X - \bar{X}) = b_{xy}(Y - \bar{Y})$$

$$\Rightarrow (X - 6) = 1.2(Y - 6) \quad \left[\because \text{As } \bar{X} = \frac{\Sigma X}{n} = \frac{30}{5} = 6, \ \bar{Y} = \frac{\Sigma Y}{n} = \frac{30}{5} = 6\right]$$

$$\Rightarrow X - 6 = 1.2Y - 7.2$$

$$\Rightarrow X - 1.2Y + 1.2 = 0$$

and Y on X is

$$(Y - \bar{Y}) = b_{yx}(X - \bar{X})$$

$$\Rightarrow (Y - 6) = 1.2(X - 6)$$

$$\Rightarrow Y - 6 = 1.2X - 7.2$$

$$\Rightarrow Y - 1.2X + 1.2 = 0$$

(F)

	B_1	B_2	B_3	B_4
A_1	3	2	4	0
A_2	3	4	2	4
A_3	4	2	4	0
A_4	0	4	0	8

Using Dominance

~~3 2 4 0~~

3 4 2 4

4 2 4 0

0 4 0 8

(As $\pi_1 < \pi_3$)

Using Dominance

3 4 2 4

4 2 4 0

0 4 0 8

(As $c_3 < c_4$)

Using Dominance

~~3 4 4~~

4 2 0

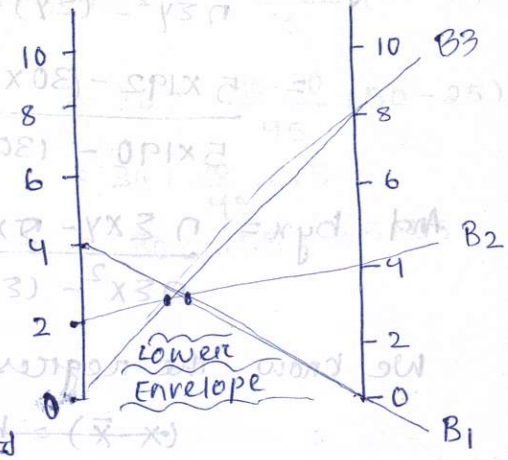
0 4 8

Avg of (π_2, π_3) (2) (3) (4)

(As $\pi_4 < \frac{\pi_2 + \pi_3}{2}$)

4 2 0

0 4 8



~~The new 2×2 game is~~

~~Probability~~

~~2 0~~ ~~4~~, $P_1 = \frac{4}{6}$

~~4 8~~ ~~2~~, $P_2 = \frac{2}{6}$

~~Probability~~ ~~2~~

The new 2×2 ~~order~~ game is

Probability

4 2 4, $P_1 = \frac{4}{6}$

0 4 2, $P_2 = \frac{2}{6}$

Probability 2 4

$q_1 = \frac{2}{6}$, $q_2 = \frac{4}{6}$

$\therefore$ Saddle point, $v = 4 \times \frac{4}{6} + 0 \times \frac{2}{6}$

$= \frac{16}{6} = 2.66$

And strategy for player A, $S_A = (0, 0, \frac{4}{6}, \frac{2}{6})$

" " " " B, $S_B = (\frac{2}{6}, \frac{4}{6}, 0, 0)$

Q6 Calculate co-relation co-efficient.

X	Y	U = X − A	V = Y − A	U^2	V^2	UV
1	25	−4	13	16	169	−52
2	23	−3	11	9	121	−33
3	(20)	−2	8	4	64	−16
4	16	−1	4	1	16	−4
(5)	12	0	0	0	0	0
6	(12)	1	0	0	0	0
7	10	2	−2	4	4	−4
8	8	3	−4	9	16	−12
9	5	4	−7	16	49	−28
10	4	5	−8	25	64	−40
55	135	5	15	85	439	189

$$r = \frac{n\Sigma UV - (\Sigma U)(\Sigma V)}{\sqrt{\left[n\Sigma U^2 - (\Sigma U)^2\right]\left[n\Sigma V^2 - (\Sigma V)^2\right]}}$$

$$= \frac{10 \times 189 - 5 \times 15}{\sqrt{\left[10 \times 85 - (5)^2\right]\left[10 \times 439 - (15)^2\right]}}$$

$$= \frac{1890 - 75}{\sqrt{(850 - 25)(4390 - 225)}}$$

$$= \frac{1815}{\sqrt{(825 \times 4165)}}$$

$$= \frac{1815}{28.72 \times 64.53}$$

$$= \frac{1815}{1853.49}$$

$$= 0.979$$

Q.5 (a) The mean and S.D of a set of 100 obs^n are 40 & 5 respectively. Later it was found, the computers by mistake took value 50 in place of 40 for one observation. Find Correct mean and S.D.

→ Given $n=100$,

$\bar{X} = 40$

and $\sigma_x = 5$

From mean we have,

$\frac{\Sigma X}{n} = 40$

$\Rightarrow \frac{\Sigma X}{100} = 40 \Rightarrow \Sigma X = 4000$

Again, we know $\sigma_x^2 = \frac{\Sigma X^2}{n} - (\bar{X})^2$

$\Rightarrow (5)^2 = \frac{\Sigma X^2}{100} - (40)^2 \Rightarrow \frac{\Sigma X^2}{100} = 25 + 1600$

$\Rightarrow \Sigma X^2 = 162500$ (Wrong sum square)

Correct sum square = (Wrong sum)^2 − (Wrong obs^n)^2 + (Correct sum)^2

$\Rightarrow \Sigma X^2 = 162500 - (50)^2 + (40)^2$

$\Sigma X^2 = (162500 - 2500 + 1600) = 161600$

Now Correct sum = Wrong Sum − Wrong obs^n + Correct obs^n

$\Rightarrow \Sigma X = 4000 - 50 + 40$

$= 3990$

Hence correct mean, $\bar{X} = \frac{\Sigma X}{n} = \frac{3990}{100} = 39.90$

Hence Correct S.D

$\sigma_x = \sqrt{\frac{\Sigma X^2}{n} - (\bar{X})^2}$

$= \sqrt{\frac{161600}{100} - (39.90)^2}$

$= \sqrt{1616 - 1592.01}$

$= \sqrt{23.99}$

$= 4.89$

(b)

No. of Goals (X)	f_1 Team (A)	f_2 Team (B)	(For Team A) f_1X	f_1X^2	(For Team B) f_2X	f_2X^2
0	27	17	0	0	0	0
1	9	9	9	9	9	9
2	8	6	16	36	12	24
3	5	5	15	45	15	45
4	4	3	16	64	12	48
5	1	2	5	25	10	50
	54	42	61	179	58	176

So $\bar{X}_A = \frac{\Sigma f_1 X}{N} = \frac{61}{54} = 1.13$

Now $\sigma_A = \sqrt{\frac{\Sigma f_1 X^2}{N} - (\bar{X}_A)^2}$

$= \sqrt{\frac{179}{54} - (1.13)^2}$

$= \sqrt{(3.31 - 1.27)}$

$= \sqrt{(2.04)}$

$= 1.42$

$\bar{X}_B = \frac{\Sigma f_2 X}{N} = \frac{58}{42} = 1.38$

$\sigma_B = \sqrt{\frac{\Sigma f_2 X^2}{N} - (\bar{X}_B)^2}$

$\sigma_B = \sqrt{\frac{176}{42} - (1.38)^2}$

$= \sqrt{4.2 - 1.90}$

$= \sqrt{2.3}$

$= 1.51$

Hence $CV_A = \frac{\sigma_A}{\bar{X}_A} \times 100$ and $CV_B = \frac{\sigma_B}{\bar{X}_B} \times 100$

$= \frac{1.42}{1.13} \times 100$ $= \frac{1.51}{1.38} \times 100$

$= 160.46$ $= 109.42$

$\therefore CV_B < CV_A$.

So Team B is more consistent.

$AM \geq GM \geq HM$

$G^2 = A \times H$

$(GM^2 = AM \times HM)$ For obs^n

Qy If the AM of two obs^n is 127.5 and their GM is 60. Find HM? and two obs^n?

→ Let the two observation are x and y

So $\frac{x+y}{2} = 127.5 \quad (AM = 127.5)$

$\Rightarrow x+y = 255$ —— (1)

Again $GM = 60$

So $\sqrt{xy} = 60$

On squaring both side, we get –

$xy = 3600$ —— (2)

and $HM = \frac{n}{\sum(\frac{1}{x})} = \frac{2}{(\frac{1}{x}+\frac{1}{y})}$

$= \frac{2xy}{x+y}$

$= \frac{2(3600)}{255}$ [$\because$ From eq^n (1) & (2)]

$= 28.23$

Now we know

$GM^2 = AM \times HM$

~~Now from eq^n (1), we have $y = (127.5 - x)$~~

~~Again, we know $(a+b)^2 = a^2 + b^2 + 2ab$ / $(x+y)^2 = x^2 + y^2 + 2xy$~~

~~$\Rightarrow (255)^2 = x^2 + y^2 + 2(3600)$~~

~~$\Rightarrow 65025 = x^2 + (127.5 - x)^2 + 7200$~~

~~$\Rightarrow x^2$~~

we know $(a-b)^2 = (a+b)^2 - 4ab$

$= (x-y)^2 = (x+y)^2 - 4xy$

$\Rightarrow (x-y)^2 = (255)^2 - 4 \times 3600$

$\Rightarrow x-y = \sqrt{65025 - 14400}$

$\Rightarrow x-y = 225$ —— (3)

From eq^n (2) and (3), we have

$x+y = 255$

$x-y = 225$

$2x = 480$

$\Rightarrow x = 240$

Hence $y = 15$

$\therefore x = 240$

$y = 15$

Q. (Q_1, Q_2, Q_3)

Heights in CM	No. of Person	Cummilative Frequency	
145–150	2	2	→
150–155	4	6	
155–160	12	18	
160–165	22	40	Q_1
165–170	30	70	Q_2
170–175	25	95	Q_3
175–180	10	105	

Q_1 class $= \frac{N}{4} = \frac{105}{4} = 26.25$

So $Q_1 = L_1 + \frac{L_2 - L_1}{f_1}\left(\frac{N}{4} - C\right)$

$= 160 + \frac{165-160}{22} + (26.25 - 18)$

$= 160 + \frac{5}{22} \times 8.25$

$= 160 + \frac{41.25}{22}$

$= 160 + 1.875$

$= 161.875$ (Lower Quartile)

Now Q_2 class $= \frac{N}{2} = \frac{105}{2} = 52.5$

Thus $Q_2 = L_1 + \frac{L_2 - L_1}{f_1}\left(\frac{N}{2} - C\right)$

$= 165 + \frac{170-165}{30}(52.5 - 40)$

$= 165 + \frac{5}{30}(12.5)$

$= 165 + \frac{62.5}{30}$

$= 165 + 2.083$

$= 167.083$ (Median)

Again Q_3 class $= \frac{3N}{4} = 3 \times \frac{105}{4} = 78.75$

Hence $Q_3 = L_1 + \frac{L_2 - L_1}{f_1}\left(\frac{3N}{4} - C\right)$

$= 170 + \frac{175-170}{25}(78.75 - 70)$

$= 170 + \frac{5}{25} \times 8.75$

$= 170 + \frac{8.75}{5}$

$= 170 + 1.75$

$= 171.75$ (Upper Quartile)

Module - I (Quantitative Techniques) (1)

There are 2 types of data

1) Primary data - The person is directly collected the data.
2) Secondary data - The data is already collected by someone and we are following it.

examples of primary data :- Interview, mail, survey
" of secondary data :- Journal, Newspaper, Internet, social media, Govt notification.

Data Classification

There are 4 types of classification

1) Qualitative
2) Quantitative
3) Geographical
4) Chronological

- There are two different variables used in statistics for representation of data.
  1) Discrete Variables (It is easily countable and finite in nature)
  2) Continuous Variables (It is not easily " and infinite in nature)

- There are three different series in statistic to represent the data.
  1) Individual Series - It has no repetation
  2) Discrete series - It has repetation with whole no. and frequency
  3) Continuous Series - Class boundaries (0-10, 10-20, 9-19..)

→ Continuous series are of 2 types.

1) Exclusive type
2) Inclusive type

In exclusive class, lower limit is included and upper limit is excluded.

For ex:-

C.I	f
0-10	3
10-20	4
20-30	5

Similarly in Inclusive types both lower limit and upper limit are included & through this

ex:-

C.I	f
0-9	3
10-19	4
20-29	2
29-39	4

- **Measure of Central Tendency**

There are 5 different of measures of central tendency.

→ Arithmetic Mean
→ Median
→ Mode
→ Geometric mean
→ Harmonic mean

⇔ Arithmetic Mean

- **Mean of Individual Series :-**

$$M = \frac{\Sigma x}{n}$$

- **Mean of Discrete Series :-**

$$M = \frac{\Sigma fx}{N}, \text{ where } N = \Sigma f$$

ex-

x	f	fx
20	2	40
10	3	30
30	4	120
40	6	240
	$N=15$	$\Sigma fx = 430$

$$M = \frac{\Sigma fx}{N} = \frac{430}{15} = 28.6$$

- **Mean of Continuous Series :-**

$$M = \frac{\Sigma f(m)}{N}, \text{ where } N = \Sigma f$$

[Midvalue is sum of upper limit and lower limit, divided by 2]

ex:-

C.I	f	midvalue	f(m)
0-10	2	5	10
10-20	3	15	45
20-30	4	25	100
	$N=9$		$\Sigma fm = 155$

So here $M = \frac{\Sigma fm}{N} = \frac{155}{9} = 17.2$

H.W

Q.1

C.I	f	(m) Midvalue	Fm
0-5	7	2.5	17.5
5-15	3	10	30
15-45	8	30	240
45-55	2	50	100
55-95	5	75	375
	$N=25$		$\Sigma f(m) = 762.5$

So $M = \frac{\Sigma fm}{N} = \frac{762.5}{25} = 30.5$

Q.2

C.I	f	Fx
5	1	5
10	7	70
13	2	26
20	3	60
25	5	125
30	4	120
35	8	280
	$N=30$	$\Sigma fx = 690$

So $M = \frac{\Sigma fx}{N} = \frac{690}{30} = 23$

Q.3

Class	f	midvalue	fm
-80-(-70)	1	-75	-75
-70-(-60)	2	-65	-130
-60-(-50)	3	-55	-165
-50-(-0)	4	-25	-100
	10		-470

So $M = \frac{\Sigma fm}{N} = \frac{-470}{10} = -47$

Q. The Nos 3.2, 5.8, 7.9 & 4.5 have frequencies x, $x+2$, $x-3$ & $x+6$ respectively. (3) whose arithmetic mean is 4.876. Find the value of x.

X	F	F(x)
3.2	x	$3.2x$
5.8	$x+2$	$5.8x+11.6$
7.9	$x-3$	$7.9x-23.7$
4.5	$x+6$	$4.5x+27$
	$N=4x+5$	$\Sigma f(x) = 21.4x+14.9$

So $M = \frac{\Sigma f(x)}{N}$

$\Rightarrow 4.876 = \frac{21.4x+14.9}{4x+5}$

$\Rightarrow 19.504x + 24.38 = 21.4x + 14.9$

$\Rightarrow 1.896x = 9.48$

$\Rightarrow x = \frac{9.48}{1.896} = 5.01 \simeq 5$

Q. Find out the missing frequency from the following distribution. Given $N=200$ & mean $= 1.46$

→

C.I.	F	F(x)
0	46	0
1	f_1	f_1
2	f_2	$2f_2$
3	25	75
4	10	40
5	5	25
	$N=200$	$\Sigma fx = 140 + f_1 + 2f_2$

→ $M = \frac{\Sigma f(x)}{N}$

~~$\Rightarrow 1.46 =$~~

As $N = 200$

$\Rightarrow 86 + f_1 + f_2 = 200$

$\Rightarrow f_1 + f_2 = 114$ —①

Now $M = \frac{\Sigma fx}{N}$

$\Rightarrow 1.46 = \frac{f_1 + 2f_2 + 140}{200}$

$\Rightarrow 292 = f_1 + 2f_2 + 140 = 0$

$\Rightarrow f_1 + 2f_2 = 152$ —②

Now calculating eq^n(1) & (2), we get

$f_1 + f_2 = 114$

$f_1 + 2f_2 = 152$

Subtracting eq^n(1) from eq^n(2)

$\Rightarrow f_1 + f_2 = 114$

$\underline{f_1 + 2f_2 = 152}$

$-f_2 = -38$

$\Rightarrow f_2 = 38$

Putting f_2 value on eq^n(1), we get -

$f_1 + 38 = 114$

$\Rightarrow f_1 = 76$

$\therefore f_1 = 76, f_2 = 38$

Continous : Combined Mean :-

Let $x_1, x_2 \ldots\ldots x_k$ are K no. of series with $n_1, n_2 \ldots\ldots n_k$ frequencies.

Combined Mean is denoted by $\boxed{\bar{X} = \dfrac{n_1\bar{x}_1 + n_2\bar{x}_2 + \ldots\ldots + n_k\bar{x}_k}{n_1 + n_2 + \ldots\ldots + n_k}}$

Q. There are 120 students in a classroom, where 70 are boys and 50 students are girls. So in a mathematics examination, the average mark of boys and girls are 55 & 65 respectively. So find out the average marks of both boys and girls in maths.

→ Lets take boys $\bar{x}_1$ & girls $\bar{x}_2$

Here $N = 120$, $n_1 = 70$ & $n_2 = 50$

$\bar{x}_1 = 55$, $\bar{x}_2 = 65$

So $\bar{X} = \dfrac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$

$= \dfrac{70 \times 55 + 50 \times 65}{120}$

$= \dfrac{3850 + 3250}{120}$

$= \dfrac{7100}{120}$

$= 59.16$

Q. The mean marks of statistic in a class was 72, where the mean marks of boys are 75 and there are 70 boys in the class room and 30 girls in the class room. Find the mean marks of girls.

→ Lets take Boys as $\bar{x}_1$ & Girls as $\bar{x}_2$

So $\bar{x}_1 = 75$, $\bar{x}_2 = ?$

Given $\bar{X} = 72$, $n_1 = 70$ & $n_2 = 30$

So $\bar{X} = \dfrac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$

$72 = \dfrac{75 \times 70 + x_2 \times 30}{100}$

$7200 = 5250 + 30x_2$

$\Rightarrow 30x_2 = 7200 - 5250 = 1950$

$\Rightarrow x_2 = 1950/30 = 65$

Q. The mean mark of 75 students in a class room is 65, later it was found that one observation wrongly taken as 45 instead of 54. Find out the combined mean. (5)

→ Here $\bar{x} = 65$ & $N = 75$

$$\bar{x} = \frac{\Sigma x}{n} \Rightarrow \Sigma x = n \cdot \bar{x}$$

Correct sum = (Wrong sum − Wrongly taken + correct)

$$\bar{x} = \frac{\Sigma x}{n} \Rightarrow 65 = \frac{\Sigma x}{75}$$

$\Rightarrow \Sigma x = 65 \times 75$

$= 4875$ (this is wrong sum)

Now correct sum $= 4875 - 45 + 54$

$= 4884$

Now mean $= \frac{\Sigma x}{n} = \frac{4884}{75} = 65.12$

Q. The avg. salary of 20 employees in an organization is 10,000 later it was found that 2 employee salary 3,000 & 4000 wrongly taken as 9000 & 7000. Find out the correct sum.

$\Rightarrow \bar{x} = 10,000$

$N = 20$

Wrong sum $= n \cdot \bar{x}$

$= 20 \times 10,000$

$= 200,000$

Correct sum = Wrong sum − (9000 + 7000) + 3000 + 4000

$= 200,000 - 9000$

$= 191,000$

Mean $= \frac{\text{Correct sum}}{N}$

$= \frac{191,000}{20}$

$= 9550$

Assignments

(6)

Q.1 Write 3 merits and demerits of arithematic mean.

Q.2 What is exclusive and inclusive type of class with example.

Q.3 What is the difference between Qualitative and Quantitative with examples.

Q.4 " " " " " Primary and Secondary data. 5 different source of Collecting primary and secondary data.

Q.5 What is the difference between discrete and continuous Variables. Explain with examples.

Q.1 Merits : (1) Arithmetic mean is easy to calculate and understand
(2) Arithmetic mean is suitable for various mathematical and ~~statist~~ statistical operations.
(3) It is the most precise estimate of the central value.

Demerits : (1) Arithmetic mean can not be used in qualitative characteristics.
(2) Arithmetic mean can not determine if a single observation is missing or lost.
(3)

Q.2 Exclusive Type :- The exclusive type of class means, in the given data, the lower limit is included and the upper limit is excluded.
ex:- In a exclusive class (20 - 30), all the greater value than 20 are included like 23, 24, 26 and 29 but not 20 or 30.

Inclusive Type :- The inclusive type of class means in the given data both the lower limit and upper limit is included.
ex:- In a inclusive class (20 - 30) all the values greater than or equal to 20 and less than or equal to 30 are included.

Q.3 Qualitative Data :- It is a classification of characteristics or qualities. We can not count Qualitative data.
ex:- She is a beautiful girl.

Quantitative Data :- It is a classification of numbers and values. These data are measurables and countables.
ex:- Her income is 7 lakh per annum.

Q.4 The meaning of primary data is a data which is directly collected by individuals or sources for researches or analysis.
The 5 sources are :- Interviews, Surveys, Experiments, Mail & Observation.

The meaning of secondary data is the data which is already collected by someone for their purposes and we are following it.
The 5 sources are :- Journal, Newspaper, Internet, social media & Govt notification

Q.15 (b)

Size	Frequency	fx
10	3	30
12	7	84
14	?	$14f_1$
16	20	320
18	8	144
20	5	100

→ Let's put the unknown frequency as f_1

Now $\Sigma f = 43 + f_1$ and $\Sigma fx = 678 + 14f_1$

Again, Given $M = 15.38$

So $M = \frac{\Sigma fx}{\Sigma f}$

$\Rightarrow 15.38 = \frac{678 + 14f_1}{43 + f_1}$

$\Rightarrow 15.38 f_1 + 661.34 = 678 + 14f_1$

$\Rightarrow 15.38 f_1 - 14f_1 = 678 - 661.34$

$\Rightarrow 1.38 f_1 = 16.66$

$\Rightarrow f_1 = \frac{16.66}{1.38} = 12.07 \simeq 12$

Q.16

Class	f	M	f(M)
0-20	17	10	170
20-40	f_1=?	30	$30f_1$
40-60	32	50	1600
60-80	f_2=?	70	$70f_2$
80-100	19	90	1710

Now $\Sigma f(M) = 30f_1 + 70f_2 + 3480$

$\Sigma f = f_1 + f_2 + 68$

$\Rightarrow 120 = f_1 + f_2 + 68$

$\Rightarrow f_1 + f_2 = 52$ —(1)

So $M = \frac{\Sigma f(M)}{N}$

$\Rightarrow 50 = \frac{30f_1 + 70f_2 + 3480}{120}$

$\Rightarrow 30f_1 + 70f_2 + 3480 - 6000 = 0$

$\Rightarrow 30f_1 + 70f_2 = 2520$

$\Rightarrow 3f_1 + 7f_2 = 252$ —(2)

Q.16(b)

Marks	No of students	M	FM
0-10	3	5	15
10-20	8	15	120
20-30	11	25	275
30-40	15	35	525
40-50	f_1	45	$45f_1$
50-60	25	55	1375
60-70	40	65	2600
70-80	f_2	75	$75f_2$
80-90	2	85	170
90-100	1	95	95
	$f_1 + f_2 + 105$		$45f_1 + 75f_2 + 5175$

Given $M = 65$ & $N = 150$

As $N = 150 \Rightarrow f_1 + f_2 + 105 = 150 \Rightarrow f_1 + f_2 = 45$ —(1)

Again $M = \frac{\Sigma fM}{N}$

$\Rightarrow 65 = \frac{45f_1 + 75f_2 + 5175}{150}$

$\Rightarrow 9750 = 45f_1 + 75f_2 + 5175$

$\Rightarrow 45f_1 + 75f_2 = 4575$

$\Rightarrow 3f_1 + 5f_2 = 305$ —(2)

Multiplying 3 on eq^n(1) & and substracting with eq^n(2), we get -

$3f_1 + 3f_2 = 135$

$3f_1 + 5f_2 = 305$

$-2f_2 = -170$

$\Rightarrow f_2 = 85$

Putting f_2 value on eq^n(1), we get

$f_1 + 85 = 45$

$\Rightarrow f_1 = -45$

$\therefore (f_1, f_2) = (-45, 85)$

multiplying 3 on eq^n(1) & substracting on eq^n(2)

3 × eq^n(1) $= 3f_1 + 3f_2 = 156$

$3f_1 + 7f_2 = 252$

$-4f_2 = -96$

$\Rightarrow f_2 = \frac{96}{4} = 24$

As $f_1 + f_2 = 52$

$f_1 + 24 = 52$

$\Rightarrow f_1 = 28$

$\therefore f_1 = 28, f_2 = 24$

17(a) The average salary of 49 out of 50 employees in a firm is Rs. 100. The salary of the 50th employee is Rs. 97.50 more than the average salary of all the 50 workers. Find the mean salary of all the employees in the firm. (8)

→ There are 50 employees in the firm.

49 employees mean salary is 100

Let mean salary 50 employees is x

The salary of 50th employer is $x + 97.50$

Total salary of 49 employee is

$$\frac{\Sigma x}{49} = 100 \Rightarrow \Sigma x = 4900.$$

$$x = \frac{\text{Total salary of all 50 employee}}{50}$$

$$= \frac{4900 + x + 97.50}{50}$$

$$\Rightarrow 50x - x = 4997.50 \Rightarrow 49x = 4997.50 \Rightarrow x = \frac{4997.5}{49} = 101.98$$

- **Step Deviation Method**

Assume mean method along with step deviation method.

$$\bar{X} = A + \frac{\Sigma f(d)}{N}, \text{ where } d = X - A$$

$$\bar{X} = A + \frac{\Sigma f(d)}{N} \cdot h \text{ where } d = \frac{X-A}{h}$$

where h = length

A = Assumed mean

x = Mid value

Q.

C.I	f	x	$d = \frac{x-A}{h}$	f(d)
0-20	17	10	-2	-34
20-40	24	30	-1	-24
40-60	32	50	0	0
60-80	28	70	1	28
80-100	19	90	2	38
	N=120			$\Sigma f(d) = 8$

$A = 50$

$h = 20$

$A + \frac{\Sigma f(d)}{N} \cdot h$

→ $\bar{X} = A + \frac{\Sigma f(d)}{N}$

$= 50 + \frac{8}{120}$

$= 50 + 1.3$

$= 51.3$

• Median (Positional Approach)

It is defined as the mid value of perticular series. Median divides the whole series in two equal parts, where one part is greater than the median value and another part is less than the median.

→ Median for Individual Series

(i) Arrange the series either in ascending order or descending order

(ii) Check wheather it is odd or even.

(iii) If it is odd, the median is $(\frac{n+1}{2})$ th term, if it is even, the median is $\frac{(\frac{n}{2})^{th} + (\frac{n}{2}+1)^{th}}{2}$ term

where n is no. of observation present in the series.

Q.

x	f
6	19
7	5
13	4
14	3
15	2
	1

→ Median for x is $(\frac{n+1}{2})$ th term

$(\frac{5+1}{2})$ term

3rd term i.e. 13

Median for f is $\frac{\frac{n}{2} + (\frac{n}{2}+1)}{2}$ term

$= \frac{6/2 + (6/2+1)}{2}$

$= \frac{3+4}{2} = \frac{7}{2} = 3.5$ median

H.W

Q.

- **Median of Discrete Series**

X	f	cf (successive addition of frequency)
20	2	2
30	3	5
31	4	9
33	9	18
39	6	24
65	7	31
66	8	39
67	4	43
	N=43	

39 is the median

In discrete series, first arrange the series on ascending or descending order, then find out the cumilative frequency column. Calculate the $\left(\frac{N+1}{2}\right)$th term, where n is sum of F and the value of $\left(\frac{N+1}{2}\right)$ in CF column, corcresponding to the observation is known as Median.

How to find the median of continuous series

C.I.	f	CF
0-10	4	4
10-20	6	10
20-30	8	18
30-40	9	27
40-50	10	37
50-60	12	49
	N=49	

Here $L_1 = 30$, $L_2 = 40$
$f = 9$, $M = 24.5$
$c = 18$

$\therefore$ Median class is $\frac{N}{2} = \frac{49}{2} = 24.5$

and Median $= L_1 + \frac{L_2 - L_1}{f_1}(M - c)$

where L_1 is lower limit of median
L_2 is upper " " "
f is the frequency of "
M is $\frac{N}{2}$
c is the preceeding CF of median class
Preceeding - Previous
Successive - After

$\rightarrow$ So $Md = L_1 + \frac{L_2 - L_1}{f}(M - c)$

$= 30 + \frac{40-30}{9}(24.5 - 18)$

$= 30 + \frac{10}{9} \times 6.5$

$= 30 + 7.5$

$= 37.5$

Q. Find the missing frequency from the following data.

Q. Find the missing frequency from the following data. Given N = 100 & M = 50

C.I	f	CF
0-20	?	14
20-40	f_1	$14+f_1$
40-60	27	$41+f_1+f_2$
60-80	f_2	$41+f_1+f_2$
80-100	15	$56+f_1+f_2$

→ Let f_1 & f_2 for missing frequency

As we know,

$$Md = \frac{N}{2} = \frac{100}{2} = 50$$

Now $N = 56+f_1+f_2$

$\Rightarrow f_1+f_2+56 = 100$

$\Rightarrow f_1+f_2 = 44$ —①

Again, $Md = L_1 + \frac{L_2-L_1}{f}(M-C)$

$$= 40 + \frac{60-40}{27}(50-14-f_1)$$

$$\Rightarrow 50 = 40 + \frac{20}{27}(36-f_1)$$

$$\Rightarrow 50 = 40 + \frac{720-20f_1}{27}$$

$\Rightarrow 1350 = 1080 + 720 - 20f_1$

$\Rightarrow 1350 = 1800 - 20f_1$

$\Rightarrow 450 - 20f_1 = 0$

$\Rightarrow f_1 = \frac{450}{20} = 22.5$

As, from eq(1), we know

$f_1+f_2 = 44$

So $22.5 + f_2 = 44$

$\Rightarrow f_2 = 44 - 22.5$

$\Rightarrow f_2 = 21.5$

Assignments

42//

Weight	No. of Boys	CF
10-19	8	8
20-29	15	23
30-39	21	44
40-49	34	78
50-59	45	123
60-69	26	149
70-79	20	169
80-89	15	184
90-99	10	194
100-109	6	200
	N=200	

→ Median class, $M = \frac{N}{2} = \frac{200}{2} = 100$

Here $l_1 = 50, l_2 = 59, f = 45, C = 78$

Now $Md = L_1 + \frac{L_2-L_1}{f_1}(M-C)$

$$= 50 + \frac{59-50}{45}(100-78)$$

$$= 50 + \frac{9}{45} \times 22$$

$$= 50 + 44$$

$$= 54.4$$

43//

Marcks	No. of students	C.F
0-10	2	2
10-20	5	7
20-30	8	15
30-40	10	25
40-50	25	50
50-60	16	66
60-70	12	78
70-80	8	86
80-90	4	90
90-100	5	95
	N=95	

→ Median class, $M = \frac{N}{2} = \frac{95}{2} = 47.5$

So $Md = L_1 + \frac{L_2-L_1}{f_1}(M-C)$

$$= 40 + \frac{50-40}{25}(47.5-25)$$

$$= 40 + \frac{10}{25} \times 22.5$$

44

Midvalue	f	C.I.	CF
5	1	2.5-7.5	1
10	2	7.5-12.5	3
15	3	12.5-17.5	6
20	4	17.5-22.5	10
25	5	22.5-27.5	15
30	6	27.5-32.5	21
35	7	32.5-37.5	28
40	8	37.5-42.5	36
45	9	42.5-47.5	45
	N=45		

Median class, $m = \frac{N}{2}, \frac{45}{2} = 22.5$

$L_1 = 32.5, L_2 = 37.5, f_1 = 7,$

$c = 21$

So $Md = L_1 + \frac{L_2 - L_1}{f_1}(m - c)$

$= 32.5 + \frac{37.5 - 32.5}{7}(22.5 - 21)$

$= 32.5 + \frac{5}{7} \times 1.5$

$= 32.5 + \frac{7.5}{7}$

$= 32.5 + 1.07$

$= 33.57$

45

C.I	F	CF
30-32	3	3
32-34	8	11
34-36	24	35
36-38	31	66
38-40	50	116
40-42	61	177
42-44	38	215
44-46	21	236
46-48	12	248
48-50	2	250
	N=250	

Here, $M = \frac{N}{2} = \frac{250}{2} = 125$

So $L_1 = 40, L_2 = 42, f_1 = 61, c = 116$

So $Md = L_1 + \frac{L_2 - L_1}{f_1}(m - c)$

$= 40 + \frac{42 - 40}{61}(125 - 116)$

$= 40 + \frac{2}{61} \times 9 = 40 + \frac{18}{61} = 40 + 0.295$

$= 40.295$

46

C.I.	F	CF
3-5	6	6
5-7	f_1	$6 + f_1$
7-9	18	$24 + f_1$
9-11	20	$44 + f_1$
11-13	f_2	$44 + f_1 + f_2$
13-15	10	$54 + f_1 + f_2$
	$f_1 + f_2 + 54$	

→ Given $N = 80, Md = 9.20$

Here $f_1 + f_2 + 54 = 80 \Rightarrow f_1 + f_2 = 26$ —(1)

So Mode class $= \frac{N}{2} = \frac{80}{2} = 40$

Here $l_1 = 9, l_2 = 11, f_1 = 20$ & $c = 24 + f_1$

Thus $Md = L_1 + \frac{L_2 - L_1}{f_1}(m - c)$

$\Rightarrow 9.20 = 9 + \frac{11 - 9}{20} \times (40 - 24 - f_1)$

$\Rightarrow 9.20 = 9 + \frac{2}{20} \times 16 - f_1 = 9 + \frac{16 - f_1}{10}$

$\Rightarrow 9.20 = 9 + \frac{32 - f_1}{20} = \frac{90 + 16 - f_1}{10}$

~~$\Rightarrow 184 = 180 + 32 - f_1$~~ $\Rightarrow 92 = 90 + 16 - f_1$

~~$\Rightarrow -f_1 = 184 - 180 - 32$~~ $\Rightarrow f_1 = 106 - 92$

~~$\Rightarrow -f_1 = -28$~~ $\Rightarrow f_1 = 14$

~~$\Rightarrow f_1 = 28$~~

Putting f_1 value on eq(1), we get -

$14 + f_2 = 26 \Rightarrow f_2 = 26 - 14 = 12$

$\therefore f_1 = 14, f_2 = 12$

47

x	F	CF
10	6	6
11	9	15
12	25	40
13	15	55
14	20	75
15	16	91
16	11	102
17	10	112
18	8	120
	N=120	

So Median $= \frac{N+1}{2} = \frac{120+1}{2} = \frac{121}{2} = 60.5$

Mode

It is defined as most frequently observed value in a perticular series.

There are 3 different types of mode.

1) Unimodal (1, 2, 2, 3, 4, 5)
2) Bimodal (1, 2, 2, 3, 3, 4, 5)
3) Multimodal (1, 2, 2, 3, 3, 4, 4, 5)

A series having single mode is known as unimodal series.

A series having two mode then it is bimodal series.

A series having more than two mode is multimodal series.

There is no chance of mode in Individual series.

In a discrete series mode is the observation which have highest frequency.

Continuous series

C.I.	f
0-10	2
10-20	5
20-30	8
30-40	10
40-50	25
50-60	16
60-70	12

$$\text{Mode} = 3\,\text{median} - 2\,\text{mean}$$

Empirical relation between mean, median and mode

[If it is unable to find mode of a continuous series with the help of usual technique, then in that case we need to follow the impercical relation]

In a continuous series first find out the modal class, modal class is defined as the class having highest frequencies.

Here modal class is 40 - 50 16

$$M_0 = L_1 + \frac{f_1 - f_0}{2f_1 - f_2 - f_0}(L_2 - L_1)$$

$L_1 = 40$, $L_2 = 50$

f_0 = preceeding frequency of modal class.

f_1 = corrcesponding " " " "

f_2 = succeeding " " " " .

$$M_0 = 40 + \frac{25 - 10}{50 - 16 - 10}(50 - 40)$$

$$= 40 + \frac{15}{24} \times 10$$

$$= 40 + \frac{150}{24}$$

$$= 40 + 6.25$$

$$= 46.25$$

Assignments

① Write 3 merits and demerits of ~~median~~ mode.

→ Merits :- (i) It is easy to calculate

(ii) It is very much useful for both

Q.2

X	f	C.F
0-10	4	4
10-20	16	20
20-30	f_1	f_1+20
30-40	f_2	f_1+f_2+20
40-50	f_3	$f_1+f_2+f_3+20$
50-60	6	$f_1+f_2+f_3+26$
60-70	4	$f_1+f_2+f_3+30$

Given $N = 230$, $M_0 = 34$, $M_d = 33.5$

As $M_0 = 34$, it lies in class (30-40) and having frequency f_2.

So here $l_1 = 30$, $l_2 = 40$, $f_0 = f_1$, $f_1 = f_2$

& $f_2 = f_3$

$$\Rightarrow M_0 = l_1 + \frac{f_1 - f_0}{2f_1 - f_2 - f_0}(l_2 - l_1)$$

$$\Rightarrow 34 = 30 + \frac{f_2 - f_1}{2f_2 - f_3 - f_1}(40-30)$$

$$\Rightarrow 34 = \frac{60f_2 - 30f_3 - 30f_1 + 10f_2 - 10f_1}{2f_2 - f_3 - f_1}$$

$$\Rightarrow 68f_2 - 34f_3 - 34f_1 - 60f_2 + 30f_3 + 30f_1 - 10f_2 + 10f_1 = 0$$

$$\Rightarrow -2f_2 - 4f_3 + 6f_1 = 0$$

$$\Rightarrow 3f_1 - f_2 - 2f_3 = 0 \quad \text{—— ③}$$

Now Given $N = 230$

$$\Rightarrow f_1 + f_2 + f_3 + 30 = 230$$

$$\Rightarrow f_1 + f_2 + f_3 = 200 \quad \text{—— ②}$$

Again, Median = 33.5

this implies the median class is (30-40) and having frequency f_2.

So as we know median $= l_1 + \frac{l_2 - l_1}{f_1}(m - c)$

(Here $m = \frac{N}{2} = \frac{230}{2} = 115$)

$$\Rightarrow 33.5 = 30 + \frac{40-30}{f_1}(115 - f_1 - 20)$$

$$\Rightarrow 33.5 = 30 + \frac{10}{f_2} \times 95 - f_1$$

$$\Rightarrow 33.5 = \frac{30f_2 + 950 - 10f_1}{f_2}$$

$$\Rightarrow 33.5f_2 + 10f_1 = 950 \quad \text{—— (3)}$$

$$\Rightarrow 3.5f_2 + 10f_1 = 950 \quad \text{—— (3)}$$

Now Solving 3 eq'n using Substitution method

$$f_1 + f_2 + f_3 = 200 \quad \text{—— (2)}$$

$$3f_1 - f_2 - 2f_3 = 0 \quad \text{—— (1)}$$

$$3f_1 - f_2 - 2f_3 = 0 \quad \text{—— (1)}$$

$$f_1 + f_2 + f_3 = 200 \quad \text{—— (2)}$$

$$10f_1 + 3.5f_2 = 950 \quad \text{—— (3)}$$

Now multiplying eq 2 in eq'n (2) & adding with eq'n (1), we get -

$$3f_1 - f_2 - 2f_3 = 0$$

$$2 \times \text{eq(2)} \quad 2f_1 + 2f_2 + 2f_3 = 400$$

$$5f_1 + f_2 = 400 \quad \text{—— (4)}$$

Now, multiplying 2 in eq'n (4) & substracting with eq'n (3)

$$2 \times \text{eq'n (4)} = 10f_1 + 2f_2 = 800$$

$$10f_1 + 3.5f_2 = 950$$

$$-1.5f_2 = -150$$

$$\Rightarrow f_2 = \frac{150}{1.5} = 100$$

Putting f_2 value in eq'n (4), we get -

$$5f_1 + 100 = 400 \Rightarrow f_1 = \frac{300}{5} = 60$$

Now putting f_1 & f_2 value in eq'n (2), we get -

$$60 + 100 + f_3 = 200$$

$$\Rightarrow f_3 = 40$$

$\therefore (f_1, f_2, f_3) = (60, 100, 40)$

Q.

C.I (X)	F	C.F	M	$d = M - A$	$d' = \frac{d}{c}$	fd'
100 – 110	6	6	105	-10	-1	-6
110 – 120	4	10	115	0	0	0
120 – 130	20	30	125	10	1	20
130 – 140	32	62	135	20	2	64
140 – 150	39	101	145	30	3	117
150 – 160	17	118	155	40	4	68
160 – 170	8	126	165	50	5	40
170 – 180	2	128	175	60	6	12
	N=128					$\Sigma fd' = 315$

$\rightarrow \bar{X} = A + \frac{\Sigma fd'}{N} \times c$

$= 115 + \frac{315}{128} \times 10$

$= 115 + \frac{3150}{128}$

$= 115 + 24.60$

$= 139.60$

$M = (\frac{N}{2})^{th}$ term $= \frac{128}{2} = 64^{th}$

Median class in between (140 – 150)

$M = L_1 + \frac{L_2 - L_1}{f}(m - c)$

$= 140 + \frac{150 - 140}{39}(64 - 62)$

$= 140 + \frac{10}{39} \times 2$

$= 140 + \frac{20}{39}$

$= 140 + 0.51$

$= 140.51$

Now Mode $= L_1 + \frac{f_1 - f_0}{2f_1 - f_0 - f_2}(L_2 - L_1)$

$= 140 + \frac{39 - 32}{2 \times 39 - 32 - 17}(150 - 140)$

$= 140 + \frac{7}{78 - 32 - 17} \times 10$

$= 140 + \frac{70}{29}$

$= 140 + 2.41$

$= 142.41$

- **Geometric Mean** (For Individual Series)

If $x_1, x_2, x_3 \ldots\ldots x_n$ are 'n' observation in a series, then the Geometric mean is defined as n^{th} root of product of all observation.

$$G = \sqrt[n]{x_1 \cdot x_2 \cdot \ldots \cdot x_n} \quad \text{or} \quad G.M = (x_1 \cdot x_2 \cdots x_n)^{1/n}$$

ex:- G.M of 5 & 6

$$G.M = (x_1 \cdot x_2)^{1/2}$$
$$= (5 \times 6)^{1/2}$$
$$= (30)^{1/2}$$
$$= 5.47$$

Q. Mean of 2 observation is 25 and GM is 20, then find out both the observation

→ Here $G.M = 20$

Lets take 2 observation i.e. x & y

$$M = \frac{x+y}{2} = 25 \quad \text{—(1)}$$

$$G.M = \sqrt{xy} = 20 \quad \text{—(2)}$$

from eq^n(1), we can get - $x+y = 50$ —(1)

" eq^n(2), " " " - $\sqrt{(xy)^2} = 400$

$\Rightarrow xy = 400$

Now we know $(x-y)^2 = (x+y)^2 - 4xy$

$\Rightarrow (x-y)^2 = (50)^2 - 4 \times 400$

$= 2500 - 1600$

$= 900$

$\Rightarrow (x-y)^2 = 900$

$\Rightarrow (x-y) = 30$

$\Rightarrow x-y = 30 \quad \text{—(2)}$

Adding eq^n(1) & (2), we get -

$$x+y = 50$$
$$x-y = 30$$
$$\overline{2x = 80}$$
$$x = 40.$$

Putting x value in eq^n(1), we get

$\rightarrow 40+y = 50$

$\Rightarrow y = 10$

→ The application of Geometric mean is helpful in calculating the annual percentage growth of any activities, mostly applied.

→ When the frequency of any observation is very large at the same time the observation is small and vice versa. In that case GM is mostly applicable than AM.

ex:-

X	f
2	8
4	10
30	2
40	4

→

$$G.M = (X_1 \cdot X_2 \cdots X_n)^{1/n}$$

$$\log G.M = \log (X_1 \cdot X_2 \cdots X_n)^{1/n}$$

$$= \frac{1}{n} \log(X_1 \cdot \cdots X_n)$$

$$= \frac{1}{n} (\log X_1 + \log X_2 + \cdots \log X_n)$$

$$= \frac{1}{n} \left(\sum_{i=1}^{n} \log X_i\right)$$

$$\log GM = \frac{\sum_{i=1}^{n} \log X_i}{n}$$

$$GM = AL\left(\frac{\sum_{i=1}^{n} \log X_i}{n}\right)$$

$$\left[\begin{array}{l} \log m^n = n \log m \\ \log(mn) = \log m + \log n \end{array}\right]$$

Assignments

Q. Find G.M of (2, 3, 7, 9)

→

X	log X
2	0.30
3	0.47
7	0.84
9	0.95
	$\Sigma \log X = 2.56$

$$G.M = AL\left(\frac{\Sigma \log x_i}{n}\right)$$

$$= AL\left(\frac{2.56}{4}\right)$$

$$= AL(0.64)$$

$$= 4.40$$

Q. Find GM of (7, 7, 3, 3, 3, 2, 2, 4, 4, 4)

X	f	log X	f log X
7	2	0.84	1.69
3	3	0.47	1.41
2	2	0.30	0.60
4	3	0.60	2.4
	N=11		$\Sigma = 6.13$

$$G.M = AL\left(\frac{\Sigma f \log x}{n}\right)$$

$$= AL\left(\frac{6.13}{11}\right)$$

$$= AL(0.55)$$

$$= 3.54$$

Q Find out AM & GM of following series.

X	f	log x	f log x	f(x)
2	18	0.30	5.4	36
4	10	0.60	6	40
30	2	1.47	2.94	60
40	4	1.60	6.4	160
	N=34		Σ=20.74	Σ=296

$$AM = \frac{\Sigma f(x)}{N} = \frac{296}{34} = 8.70$$

$$GM = AL\left(\frac{\Sigma f \log x}{N}\right) = AL\left(\frac{20.74}{34}\right)$$

$$= AL(0.61) = 4.07$$

For this particular series the arithematic mean is 8.75 and geometric mean is 4.07 and this shows that 4.07 is representing the whole series thats why geometric mean is more applicable, where observation is small corresponding to large frequency and observation is large corresponding to small frequency.

Assignments

If the GM of the following series 70, then find out the missing frequency of the series. where N is 40.

C.I	f	log x	f log x
10	5	1	5
20	7	1.30	9.1
30	a	1.47	1.47a
40	b	1.60	1.60b
50	9	1.69	15.21
	a+b+21		1.47a+1.60b+29.31

↦ Here Given N = 40 , G.M=70

If N=40 ⇒ a+b+21 = 40

⇒ a+b = 19 —(1)

Now $GM = AL\left(\frac{\Sigma f \log x}{N}\right)$

$$\Rightarrow 70 = AL\left(\frac{1.47a + 1.60b + 29.31}{40}\right)$$

$$\Rightarrow \log 70 \times 40 = 1.47a + 1.60b + 29.31$$

$$\Rightarrow 1.84 \times 40 = 1.47a + 1.60b + 29.31$$

$$\Rightarrow 73.8 = 1.47a + 1.60b + 29.31$$

$$\Rightarrow 1.47a + 1.60b = 73.8 - 29.31$$

$$\Rightarrow 1.47a + 1.60b = 44.49 \quad —(2)$$

Now multiplying 1.47 on eq'(1) & substracting with eq'(2), we get -

$$1.47 \times \text{eq'}(1) = 1.47a + 1.47b = 27.93$$

$$\underline{-\ 1.47a + 1.60b = 44.49}$$

$$-0.13b = -16.56$$

$$\Rightarrow b = \frac{16.56}{0.13}$$

$$= 127.38$$

On putting the value of b, on eq'(1), we get -

$$a+b = 19$$

$$\Rightarrow a + 127.38 = 19$$

$$\Rightarrow a = 19 - 127.38$$

$$\Rightarrow a = -108.38$$

Q. Find the GM of the following series.

X	f	log x	f log x
2	4	0.30	1.2
4	3	0.60	1.8
8	2	0.90	1.8
16	1	1.20	1.2
	10		6

→ $GM = AL\left(\frac{\Sigma f \log x}{N}\right)$

$= AL\left(\frac{6}{10}\right)$

$= AL(0.6)$

$= 3.98 \simeq 4$

Q. Find the GM of the following series (M9)

X	f	M	log M	f log M
0-10	5	5	0.69	3.45
10-20	7	15	1.17	8.19
20-30	15	25	1.39	20.85
30-40	25	35	1.54	38.5
40-50	8	45	1.65	13.2
	60			84.19

$GM = AL\left(\frac{\Sigma f \log x}{N}\right)$

$= AL\left(\frac{84.19}{60}\right)$

$= AL(1.40)$

$= 25.11$

Combined Mean (VVIP 4 Exam)

If the mean series 1 is 30 with observation 9 and the mean of series 2 is 20 with observation 5. Find the combined mean.

→ Here $\bar{X}_1 = 30$, $\bar{X}_2 = 20$

$n_1 = 9$, $n_2 = 5$

$$\text{Combined Mean} = \frac{n_1\bar{X}_1 + n_2\bar{X}_2}{n_1 + n_2}$$

$$= \frac{270 + 100}{14}$$

$$= \frac{370}{14}$$

$$= 26.4$$

(short Q - 2 mark - VIP)

Q. The mean mark of 75 students in a class room is 65, later it was found that one observation wrongly taken as 45 instead of 54. Find the correct mean.

→ Here $\bar{X} = 65$ & $N = 75$

We know $\bar{X} = \frac{\Sigma X}{n}$

$\Rightarrow 65 = \frac{\Sigma X}{75}$

$\Rightarrow \Sigma X = 4875$

Correct sum = wrong sum − wrongly taken ob + right ob

$\Rightarrow$ Correct sum = $4875 - 45 + 54 = 4884$

Now mean = $\frac{\Sigma X}{n} = \frac{4884}{75} = 65.12$

Combined Geometric mean

Let G_1, G_2, G_3 are Geometric mean of 3 different Series with n_1, n_2 & n_3 observation. Then the combined Geometric mean is

$$G = AL\left(\frac{n_1 \log G_1 + n_2 \log G_2 + n_3 \log G_3}{n_1 + n_2 + n_3}\right)$$

→ If $G_1 = 10$, $G_2 = 7$, $G_3 = 8$

$n_1 = 10$, $n_2 = 8$, $n_3 = 6$

So combined Geometric mean $= AL\left(\frac{n_1 \log G_1 + n_2 \log G_2 + n_3 \log G_3}{n_1 + n_2 + n_3}\right)$

$= AL\left(\frac{10\times 10 + 8\times 7 + 6\times 8}{10+8+6}\right)$

$= AL\left(\frac{10\times 1 + 8\times 0.84 + 6\times 0.90}{10+8+6}\right)$

$= AL\left(\frac{10 + 6.72 + 5.4}{24}\right)$

$= AL\left(\frac{22.12}{24}\right) = AL(0.92) = 8.317$

Q. Given $n_1 = 8$, $n_2 = 7$, $n_3 = 5$

$G_1 = 8$, $G_2 = 10.12$ $G_3 = 7.75$

→ $CGM = AL\left(\frac{n_1 \log G_1 + n_2 \log G_2 + n_3 \log G_3}{n_1 + n_2 + n_3}\right)$

$= AL\left(\frac{8\times \log 8 + 7 \log(10.12) + 5 \log(7.75)}{8+7+5}\right)$

$= AL\left(\frac{8\times 0.90 + 7\times 1 + 5\times 0.88}{20}\right)$

$= AL\left(\frac{7.2 + 7 + 4.4}{20}\right)$

$= AL\left(\frac{18.6}{20}\right)$

$= AL(0.93)$

$= 8.51$

Assignments

	A	B	C	Combined
Number	10	8	?	24
Mean	20	?	6	15
GM	10	7	?	8.397

Here Given A=10, B=8 and the combined value of 3 no = 24

So $A+B+C=24$

$\Rightarrow 10+8+C=24$

$\Rightarrow C=24-18=6$

Now, we have Mean of A=20, Mean of C=6 and the combined mean of ABC=15

Thus combined mean of ABC $= \dfrac{\bar{X}_1 A + \bar{X}_2 B + \bar{X}_3 C}{A+B+C}$, where $\bar{X}_1$ = Mean of A

$\bar{X}_2$ = Mean of B

$\bar{X}_3$ = Mean of C

$$\Rightarrow 15 = \frac{20\times 10 + \bar{X}_2 \times 8 + 6\times 6}{10+8+6}$$

$$\Rightarrow 15 = \frac{200 + 8\bar{X}_2 + 36}{24}$$

$$\Rightarrow 360 = 8\bar{X}_2 + 236$$

$$\Rightarrow 8\bar{X}_2 = 124$$

$$\Rightarrow \bar{X}_2 = \frac{124}{8} = 15.5$$

$\therefore$ So Mean of B = 15.5

Again, we have GM of A = 10, GM of B = 7 and the combined GM of ABC = 8.397

Thus combined GM of ABC $= AL\left(\dfrac{A\log G_1 + B\log G_2 + C\log G_3}{A+B+C}\right)$ $\{\because G_1$ = GM of A, G_2 = GM of B, G_3 = GM of C$\}$

$$\Rightarrow 8.397 = AL\left(\frac{10\times\log 10 + 8\log 7 + 6\times\log G_3}{10+8+6}\right)$$

$$\Rightarrow 8.397 = AL\left(\frac{10\times 1 + 8\times 0.84 + 6\log G_3}{24}\right)$$

$$\Rightarrow \log(8.397)\times 24 = 10 + 6.72 + 6\log G_3$$

$$\Rightarrow 0.92\times 24 = 16.72 + 6\log G_3$$

$$\Rightarrow 22.08 = 16.72 + 6\log G_3$$

$$\Rightarrow 6\log G_3 = 22.08 - 16.72 = 5.36$$

$$\Rightarrow \log G_3 = \frac{5.36}{6} = 0.89 \Rightarrow G_3 = AL(0.89) = 7.76$$

• Harmonic Mean

Let $x_1, x_2 \dots x_n$ are n no. of observation, then the harmonic mean is defined as the reciprocal of arithmetic mean of reciprocal of observation.

Individual

	reciprocal
x_1	$\frac{1}{x_1}$
x_2	$\frac{1}{x_2}$
x_3	$\frac{1}{x_3}$
⋮	⋮
x_n	$\frac{1}{x_n}$

So AM = $\frac{\frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3} + \dots + \frac{1}{x_n}}{n}$

Thus Harmonic mean is $\frac{n}{\frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3} + \dots + \frac{1}{x_n}}$

Discrete

$$H.M = \frac{N}{\frac{f_1}{x_1} + \frac{f_2}{x_2} + \dots \frac{f_n}{x_n}} = \sum_{i=1}^{n} \frac{N}{f_i/x_i}, (N = \Sigma n)$$

continuous

$$H.M = \frac{N}{f_1/m_1 + f_2/m_2 + \dots f_n/m_n} = \sum_{i=1}^{n} \frac{N}{f_i/m_i}$$

where $N = \Sigma n$
m = midvalue of class

Q. Find the Harmonic mean of 2, 4, 5

Here $n = 3$

So $H.M = \frac{n}{\frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3}}$

$= \frac{3}{0.5 + 0.25 + 0.2}$

$= \frac{3}{0.95}$

$= 3.15$

Q. Find H.M of 10, 20, 40, 60, 120

So ~~H.M~~, $n = 5$

$H.M = \frac{5}{0.1 + 0.2 + \frac{0.4}{0.25} + 0.6 + 1.2}$ $\frac{5}{0.1 + 0.5 + 0.25 + 0.16 + 0.008}$

$= \frac{5}{2.5}$ $= \frac{5}{1.993}$

$= 2$ $= 0.$

Q. Find the $\frac{N}{\sum f_i/x_i}$

C.I	f
10	5
12	18
14	20
16	10
18	6
20	1
	60

$$H.M = \sum_{i=1}^{n} \frac{N}{\sum f_i/x_i}$$

$$= \frac{60}{0.5+1.5+1.42+0.62+0.33+0.05}$$

$$= \frac{60}{4.42}$$

$$= 13.57$$

Q. From the following, find H.M

X	f
5	5
15	10
25	7
35	3
45	2

$$H.M = \frac{27}{1+0.66+0.28+0.08+0.04}$$

$$= \frac{27}{2.06}$$

$$= 13.10 \simeq 13$$

Q.

class	f	m
0 – 20	11	10
20 – 40	13	30
40 – 60	17	50
60 – 80	21	70
80 – 100	23	90
	85	

$$N = \sum_{i=1}^{n} \frac{N}{f_i/m_i}$$

$$= \frac{85}{1.1+0.65+0.56+0.52+0.46+0.43+0.34+0.3+0.25}$$

$$= \frac{85}{2.42}$$

$$= 25.83$$

Q. Find H.M of

wage in rupees	no of f	m
5 – 10	1	7.5
10 – 15	2	12.5
15 – 20	3	17.5
20 – 25	4	22.5
25 – 30	3	27.5
30 – 35	2	32.5
	15	

$$H.M = \frac{15}{0.13+0.16+0.17+0.17+0.10+0.06}$$

$$= \frac{15}{0.79}$$

$$= 18.57 \simeq 18$$

Relation between AM, GM & HM

Find the AM, GM & HM of 2,4,8 and tell me

$$AM = \frac{\Sigma x}{n} = \frac{14}{3} = 4.66$$

$$GM = \sqrt[3]{2*4*8} = (64)^{1/3} = 3.9 \cong 4$$

$$HM = \frac{n}{\Sigma f} = \frac{3}{\frac{1}{2}+\frac{1}{4}+\frac{1}{8}} = \frac{3}{0.5+0.25+0.125}$$

$$= \frac{3}{0.875} = 3.42$$

So the relation between AM, GM & HM is $\boxed{AM \geq GM \geq HM}$

Assignments

(1) Write 3 advantages and disadvantages of H.M.

(2) Write 3 uses of HM

(3) Find out the missing frequency of the following series where mode is 34 & N is 60 (Ans = 10, 5)

C.I	f	C.F
0-10	8	8
10-20	7	15
20-30	x	x+15
30-40	20	x+35
40-50	y	x+y35
50-60	6	x+y+41
60-70	4	x+y+45
	x+y+45	

Given N = 60

$\Rightarrow x+y+45 = 60$

$\Rightarrow x+y = 15$ —(1)

Now, the mode is 34 that means it lies in class (30-40) having ft frequency 20.

So here $l_1 = 30$, $l_2 = 40$, $f_0 = x$, $f_1 = 20$, $f_2 = y$

Thus $M_0 = l_1 + \frac{f_1 - f_0}{2f_1 - f_2 - f_0}(l_2 - l_1)$

$\Rightarrow 34 = 30 + \frac{20-x}{40-y-x}(40-30)$

$\Rightarrow 34 = \frac{1200-30y-30x+20-x}{40-y-x}(10)$

$\Rightarrow 34(40-y-x) = 1200-30y-30x+200-10x$

$\Rightarrow 1360-34y-30x = 1200-30y-30x+200-10x$

$\Rightarrow 1360-1400-34y-34x = 1400-30y-40x$

$\Rightarrow -40-4y+6x = 0$

$\Rightarrow 6x-4y = 40$ —(2)

Now multiplying 4 on eq'(1) and adding with eq'(2), we get -

4 × eq'(1) $4x+4y = 60$

$6x-4y = 40$

$10x = 100$

$\Rightarrow x = 10$

Now putting x value on eq'(1), we get -

$x+y = 15$

$\Rightarrow 10+y = 15$

$\Rightarrow y = 5$

$\therefore (x, y) = (10, 5)$

• Measures of Dispersion

1. Range
2. Quantile
3. Mean Deviation
4. Standard Deviation
5. Variance.

*Standard Deviation

Standard deviation is the most appropriate measure in measures of dispersion. If the standard deviation is less then there is less dispersion in data. If the S.D is more then there is highest dispersion in the particular series of data.

If the S.D is zero then there is no dispersion i.e. the data means are same.

For Individual Series

$$\sigma x = \sqrt{\frac{(x-\bar{x})^2}{n}}$$

Q. Find the standard deviation of the series

→ The mean of the data

$$\bar{X} = \frac{20}{2}$$

	$(x-\bar{x})$	$(x-\bar{x})^2$
5	−15	225
10	−10	100
15	−5	25
20	0	0
25	5	25
30	10	100
35	15	225
		700

$$\therefore \sigma x = \sqrt{\frac{\Sigma (x-\bar{x})^2}{n}} = \sqrt{\frac{700}{20}}$$

Q. Find the value of x, from the following data where S.D is 3.

C.I
3
5
9
x
10

Q.2

Marcks	No. of students	fx	$(x-\bar{x})$	$(x-\bar{x})^2$	$f(x-\bar{x})^2$
10	8	80	-20.83	433.88	3471.04
20	12	240	-10.83	117.28	1407.36
30	20	600	-0.83	0.68	13.6
40	10	400	9.17	84.08	840.8
50	7	350	19.17	367.48	2572.36
60	3	180	29.17	850.88	2552.64
	N=60	$\Sigma fx = 1850$			10857.8

→ Here Mean $\bar{x} = \frac{\Sigma fx}{N} = \frac{1850}{60} = 30.83$

So $\sigma_x = \sqrt{\frac{\Sigma f(x-\bar{x})^2}{N}} = \sqrt{\frac{10857.8}{60}} = \sqrt{180.96} = 13.45$

Q.4

C.I	f	m	fm	$(m-\bar{x})$	$(m-\bar{x})^2$	$f(m-\bar{x})^2$
10-19	3	14.5	43.5	-27.52	757.35	2272.05
20-29	61	24.5	1494.5	-17.52	306.95	18723.95
30-39	233	34.5	8038.5	-7.52	56.55	13176.15
40-49	137	44.5	6096.5	2.48	6.15	842.55
50-59	53	54.5	2888.5	12.48	155.75	8254.75
60-69	79	64.5	5095.5	22.48	505.35	39922.65
70-79	4	74.5	298	32.48	1054.95	4219.8
	N=570		$\Sigma fm = 23955$			$\Sigma = 87411.9$

→ Here Mean, $\bar{x} = \frac{\Sigma fm}{N} = \frac{23955}{570} = 42.02$

Thus $\sigma_x = \sqrt{\frac{\Sigma f(m-\bar{x})^2}{N}} = \sqrt{\frac{87411.9}{570}} = \sqrt{153.35} = 12.38$

Q.5

Wages	No. of Persons
Less than 10	12
" " 20	30
" " 30	65
" " 40	107
" " 50	157
" " 60	202
" " 70	222
" " 80	230

→ The meaning of this data is 12 no. of persons having wages less than 10, and 30 persons having wages less than 20, 65 persons having wages less than 40 and so on.

Thus, the no. of persons are cumilative frequency.

So we need find out frequency of the data first.

→

C.I	CF	f	m	fm	$(m-\bar{x})$	$(m-\bar{x})^2$	$f(m-\bar{x})^2$
0-10	12	12	5	60	-34.34	1179.23	14150.76
10-20	30	(30-12)=18	15	270	-24.34	592.43	10663.74
20-30	65	(65-30)=35	25	625	-14.34	205.63	7197.05
30-40	107	(107-65)=42	35	1470	-4.34	18.83	790.86
40-50	157	(157-107)=50	45	2250	5.66	32.03	1601.5
50-60	202	(202-157)=45	55	2475	15.66	245.23	11035.35
60-70	222	(222-202)=20	65	1300	25.66	658.43	13168.6
70-80	230	(230-222)=8	75	600	35.66	1271.63	10173.04
		N=230		Σfm=9050			Σ=68780.9

→ Here, Mean, $\bar{x} = \frac{\Sigma fm}{N} = \frac{9050}{230} = 39.34$

Thus $\sigma x = \sqrt{\frac{\Sigma f(m-\bar{x})^2}{N}} = \sqrt{\frac{68780.9}{230}} = \sqrt{299.04} = 17.29$

• Step Deviation Method

$$\sigma x = \sqrt{\frac{\Sigma fd^2}{N} - \left(\frac{\Sigma fd}{N}\right)^2}$$

$$\sigma x = h\sqrt{\frac{\Sigma fd^2}{N} - \left(\frac{\Sigma fd}{N}\right)^2}$$

* $A = 64.5, h = 9$

Class	f	x	$d = \frac{x-A}{h}$	d^2	fd	fd^2
90-99	2	94.5	3.33	11.08	6.66	22.16
80-89	12	84.5	2.22	4.92	26.64	59.04
70-79	22	74.5	1.11	1.23	24.42	27.06
60-69	20	64.5	0	0	0	0
50-59	14	54.5	-1.11	1.23	-15.54	17.22
40-49	4	44.5	-2.22	4.92	-8.88	19.68
30-39	1	34.5	-3.33	11.08	-3.33	11.08
	75				29.97	156.24

$$\sigma_x = h \sqrt{\frac{\Sigma fd^2}{N} - \left(\frac{\Sigma fd}{N}\right)^2}$$

$$= 9 \sqrt{\frac{156.24}{75} - \left(\frac{29.97}{75}\right)^2}$$

$$= 9 \sqrt{2.08 - (0.39)^2}$$

$$= 9 \sqrt{2.08 - 0.15}$$

$$= 9 \sqrt{1.93} = 9 \times 1.38 = 12.50$$

Q.

Class	f	x	$d = \frac{x-A}{h}$	d^2	fd	fd^2
11-12	5	11.5	-4	16	-20	80
13-14	426	13.5	-3	9	-1278	3834
15-16	720	15.5	-2	4	-1440	2880
17-18	741	17.5	-1	1	-741	741
19-20	665	(19.5)	0	0	0	0
21-22	395	21.5	1	1	395	395
23-24	38	23.5	2	4	76	152
25-26	8	25.5	3	9	24	72
27-28	5	27.5	4	16	20	80
29-30	7	29.5	5	25	350	175
	3010				-2929	8409

$$\sigma_x = h \sqrt{\frac{\Sigma fd^2}{N} - \left(\frac{\Sigma fd}{N}\right)^2} = 2 \sqrt{\frac{8409}{3010} - \left(\frac{-2929}{3010}\right)^2} = 2 \sqrt{2.79 - 0.94}$$

$$= 2 \sqrt{1.84}$$

$$= 2 \times 1.356$$

$$= 2.71$$

Q.

Class	f	m	$d=\frac{x-A}{h}$	d^2	fd	fd^2
1-3	9	2	-2	4	-18	36
4-6	6	5	-1	1	-6	6
7-9	2	(8)	0	0	0	0
10-12	2	11	1	1	2	2
13-15	1	14	2	4	4	4
	20				-18	$\Sigma fd^2=48$

$$\rightarrow \sigma_x = h\cdot\sqrt{\frac{\Sigma fd^2}{N}-\left(\frac{\Sigma fd}{N}\right)^2}$$

$$= 3\sqrt{\frac{48}{20}-\left(\frac{-18}{20}\right)^2}$$

$$= 3\sqrt{2.4-0.81}$$

$$= 3\sqrt{1.59}$$

$$= 3.126$$

$$= 3.78$$

H.W

14.(a)

Class	f	x	$d=\frac{x-A}{h}$	d^2	fd^2	fd
0-10	5	5	-3	9	45	-15
10-20	7	15	-2	4	28	-14
20-30	14	25	-1	1	14	-14
30-40	12	(35)	0	0	0	0
40-50	9	45	1	1	9	9
50-60	6	55	2	4	24	12
60-70	2	65	3	9	18	6
	55				138	$\Sigma fd=-16$

$$\rightarrow \sigma_x = h\cdot\sqrt{\frac{\Sigma fd^2}{N}-\left(\frac{\Sigma fd}{N}\right)^2}$$

$$= 10\sqrt{\frac{138}{55}-\left(\frac{-16}{55}\right)^2}$$

$$= 10\sqrt{2.50-0.084}$$

$$= 10\sqrt{2.416}$$

$$= 10\times 1.55$$

$$= 15.54$$

(b)

C.I	f	x	$d=\frac{x-A}{h}$	d^2	fd	fd^2
100-105	200	102.5	-5	25	-1000	5000
105-110	210	107.5	-4	16	-840	3360
110-115	230	112.5	-3	9	-690	2070
115-120	320	117.5	-2	4	-640	1280
120-125	350	122.5	-1	1	-350	350
125-130	520	127.5	0	0	0	0
130-135	410	132.5	1	1	410	410
135-140	320	137.5	2	4	640	1280
140-145	280	142.5	3	9	840	2520
145-150	210	147.5	4	16	840	3360
150-155	160	152.5	5	25	800	4000
155-160	90	157.5	6	36	540	3240
	3360				550	26870

$$\sigma_x = h \cdot \sqrt{\frac{\sum fd^2}{N} - \left(\frac{\sum fd}{N}\right)^2}$$

$$= 5\sqrt{\frac{2680}{3360} - \left(\frac{550}{3360}\right)^2}$$

$$= 5\sqrt{7.99 - 0.026}$$

$$= 5\sqrt{7.96}$$

$$= 5 \times 2.82$$

$$= 14.10$$

Q. 15(a)

C.I	f	x	$d=\frac{x-A}{h}$	d^2	fd	fd^2
20-29	5	24.5	-3.33	11.08	-16.65	55.4
30-39	12	34.5	-2.22	4.92	-26.64	59.04
40-49	15	44.5	-1.11	1.23	-16.65	18.45
50-59	20	(54.5)	0	0	0	0
60-69	18	64.5	1.11	1.23	19.98	22.14
70-79	10	74.5	2.22	4.92	22.2	49.2
80-89	6	84.5	3.33	11.08	19.98	66.48
90-99	4	94.5	4.44	19.71	17.76	78.84
	90				19.98	349.55

$$\sigma_x = 9 \times \sqrt{\frac{349.55}{90} - \left(\frac{19.98}{90}\right)^2} = 9\sqrt{3.88 - 0.049} = 9 \times 1.95 = 17.61$$

$$\text{Mean } \bar{x} = A + \frac{\sum fd}{N} \cdot h = 54.5 + \frac{19.98}{90} \times 9 = 54.5 + 1.998 = 56.49$$

(b)

Age under (Years)	No. of persons dying
10	15
20	30
30	53
40	75
50	100
60	110
70	115
80	125

The data says 15 persons dying under age of 10, 30 persons dying under age of 20 and so on. So the no. of persons dying column is the C.F. Hence in order to find AM and SD, we have to find frequency from CF first.

C.I	f	x	$d = \frac{x-A}{h}$	d^2	fd	fd^2
0-10	15	5	-3	9	-45	135
10-20	15	15	-2	4	-30	60
20-30	23	25	-1	1	-23	23
30-40	22	(35)	0	0	0	0
40-50	25	45	1	1	25	25
50-60	10	55	2	4	20	40
60-70	5	65	3	9	15	45
70-80	10	75	4	16	40	160
	125				2	488

Now Mean, $\bar{x} = A + \frac{\Sigma fd}{N} \cdot h$

$= 35 + \frac{2}{125} \times 10$

$= 35 + 0.16$

$= 35.16$

and $\sigma_x = h \sqrt{\frac{\Sigma fd^2}{N} - \left(\frac{\Sigma fd}{N}\right)^2}$

$= 10 \sqrt{\frac{488}{125} - \left(\frac{2}{125}\right)^2}$

$= 10 \sqrt{3.904 - 0.00256}$

$= 10 \sqrt{3.90}$

$= 10 \times 1.974$

$= 19.74$

Q. Find the standard deviation of n natural numbers.

→ S.D of 1nd $= \sqrt{\frac{\Sigma (x-\bar{x})^2}{n}}$

$= \sqrt{\frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2}$

$\sigma_x = \sqrt{\frac{\Sigma x^2}{n} - (\bar{x})^2}$

$\Sigma x^2 = 1^2 + 2^2 + 3^2 + \cdots + n^2$

$= \frac{n(n+1)(2n+1)}{6}$

$\Rightarrow$ ~~$n(n+1)$~~

Now $\sigma_x = \sqrt{\frac{\Sigma x^2}{n} - (\bar{x})^2}$

$= \sqrt{\frac{\frac{n(n+1)(2n+1)}{6}}{n} - \left(\frac{n+1}{2}\right)^2}$

$= \sqrt{\frac{(n+1)(2n+1)}{6} - \left(\frac{n+1}{4}\right)^2}$

$= \sqrt{\left(\frac{n+1}{2}\right)\left(\frac{2n+1}{3} - \frac{n+1}{2}\right)}$

$= \sqrt{\frac{n+1}{2} \times \frac{4n+2-3n-3}{6}}$

$= \sqrt{\frac{n+1}{2} \times \frac{n-1}{6}}$

$= \sqrt{\frac{n^2-1}{12}}$

$\sigma_x^2 = \frac{\Sigma x^2}{n} - (\bar{x})^2$

$\frac{\Sigma x^2}{n} = \sigma_x^2 + (\bar{x})^2$

$\Rightarrow \Sigma x^2 = n\left(\sigma_x^2 + (\bar{x})^2\right)$

Q. The AM and the standard deviation of 9 items are 43 and 5 respectively. If an item 63 is added to the set then what is the new mean and new S.D.

→ Here $n = 9$, $\bar{x} = 43$ & $\sigma_x = 5$

So $\Sigma x = n \cdot \bar{x}$

$\Rightarrow \Sigma x = 9 \times 43$

$\Rightarrow \Sigma x = 387$

For addition of new item 63,

$\Sigma x = 387 + 63$

$= 450$

Thus $\bar{x} = \frac{\Sigma x}{10}$ ($\because$ 9 items + 63, $n = 10$)

$= \frac{450}{10} = 45$

Now in order to find the ~~SD~~ new S.D, first we need Σx^2

Thus $\Sigma x^2 = n\left(\sigma_x^2 + (\bar{x})^2\right)$

$= 9\left(5^2 + 43^2\right) = 9(25 + 1849)$

$= 16866$

For addition of new item 63,

$\Sigma x^2 = 16866 + 63^2 = 20835$

Hence $\sigma_x = \sqrt{\frac{20835}{10} - (45)^2}$

$= \sqrt{2083.5 - 2025}$

$= \sqrt{58.5} = 7.64$

* Relation between Variance and standard deviation

Variance defined as σ_x^2

Q. The variance of a series 2, 3, 11, x is 12.25. Find the value of x.

$\sigma_x^2 = 12.25$, $n = 4$

The Mean, $\bar{x} = \frac{2+3+11+x}{4} = \frac{16+x}{4}$

x	x^2
2	4
3	9
11	121
x	x^2

So $\Sigma x^2 = 134 + x^2$

$\sigma_x^2 = \frac{\Sigma x^2}{n} - (\bar{x})^2$

$\Rightarrow 12.25 = \frac{134+x^2}{4} - \left(\frac{16+x}{4}\right)^2$

$\Rightarrow 12.25 = \frac{134+x^2}{4} - \frac{256+x^2+32x}{16}$

$\Rightarrow 16(12.25) = 536 + 4x^2 - 256 - x^2 - 32x$

$\Rightarrow 196 = 536\ 4x^2 - x^2 - 32x + 280$

$\Rightarrow 196 - 280 = 4x^2\ 3x^2 - 32x$

$\Rightarrow 3x^2 - 32x = -84$

$\Rightarrow 3x^2 - 32x + 84 = 0$

Now $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$ $(a=3, b=-32, c=84)$

$= \frac{32 \pm \sqrt{32^2 - 4\times3\times84}}{6}$

$= \frac{32 \pm \sqrt{1024-1008}}{6}$

$= \frac{32 \pm \sqrt{16}}{6}$

$= \frac{32 \pm 4}{6}$

$= \frac{32+4}{6}$ or $\frac{32-4}{6}$

$= \frac{36}{6}$ or $\frac{28}{6}$

$= 6$ or 4.6

HW

Q. The mean of 5 observation is 4.4 and variance is 8.24. If 3 observation 1, 2 & 6. Find the other two.

→

C.I	x^2
1	1
2	4
6	36
x	x^2
y	y^2
x+y+9	x^2+y^2+41

Here given AM of 5 observation, $\bar{X} = 4.4$

and $\sigma x^2 = 8.24$

Thus $\sigma x^2 = \frac{\Sigma X^2}{n} - (\bar{X})^2$

$\Rightarrow 8.24 = \frac{x^2+y^2+41}{5} - (4.4)^2$

$\Rightarrow 8.24 = \frac{x^2+y^2+41}{5} - 19.36$

$\Rightarrow 5(8.24 - 19.36) = x^2+y^2+41$

$\Rightarrow 138 = x^2+y^2+41$

$\Rightarrow x^2+y^2 = 97$ ——— (1)

As $\bar{X} = 4.4$

$\Rightarrow \bar{X} = \frac{\Sigma x}{n}$

$\Rightarrow 4.4 = \frac{x+y+9}{5}$

$\Rightarrow 22 = x+y+9$

$\Rightarrow x+y = 13$ ——— (2)

$\Rightarrow y = (13-x)$

Now putting the value of y on eq^n(1), we get -

$x^2+(13-x)^2 = 97$

$\Rightarrow x^2+169+x^2-26x-97 = 0$

$\Rightarrow 2x^2-26x+72 = 0$

$\Rightarrow x^2-13x+36 = 0$

So $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$ $(a=1, b=-13, c=36)$

$= \frac{13 \pm \sqrt{169-4\times1\times36}}{2}$

$= \frac{13 \pm \sqrt{169-144}}{2}$

$= \frac{13 \pm \sqrt{25}}{2}$

$= \frac{13 \pm 5}{2} = \frac{13+5}{2}$ or $\frac{13-5}{2} = (9 \text{ or } 4)$

Putting x value on eq^n(1), we get-

if $x=9$, then $9+y=13$

$\Rightarrow y = 4$

If $x=4$, then $4+y=13$

$\Rightarrow y = 9$

So $(x,y) = (4,9)$ or $(9,4)$

- **Combined Standard Deviation**

Let $\sigma_1, \sigma_2 \cdots \sigma_k$ are standard deviation of K no. of series, and $\bar{x}_1, \bar{x}_2 \cdots \bar{x}_k$ are no. of means of K series. Then the combined standard deviation is defined as

$$\sigma^2 = \frac{n_1(\sigma_1^2 + d_1^2) + n_2(\sigma_2^2 + d_2^2) + \cdots + n_k(\sigma_k^2 + d_k^2)}{n_1 + n_2 + \cdots + n_k}$$

where $d_k = \bar{x}_k - \bar{x}$

and $\bar{x}$ is combined mean.

$x_1 - \bar{x}$

$\sigma_1, n_2, \bar{x}_1, \bar{x}_2$

$d_1 = x_1 - \bar{x}$ or $\bar{x} = \frac{n_1 x_1 + n_2 x_2}{n_1 + n_2}$

The means of two sample size 50 & 100 are 54.1 and 50.3, ~~the mean~~ and standard deviations are 8 & 7. Find

→ Here $n_1 = 50, n_2 = 100$

$\bar{x}_1 = 54.1, \bar{x}_2 = 50.3$

$\sigma_1^2 = 8, \sigma_2^2 = 7$

$d_1 = \bar{x}_1 - \bar{x} =$

$= 54.1 - 51.56$

$= 2.53$

$d_2 = \bar{x}_2 - \bar{x}$

$= 50.3 - 51.56$

$= -1.26$

$\frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$

$\bar{x} = \frac{50 \times 54.1 + 100 \times 50.3}{150}$

$= \frac{2705 + 5030}{150} = \frac{7735}{150} = 51.56$

Now Combined standard deviation

$$\sigma^2 = \frac{n_1(\sigma_1^2 + d_1^2) + n_2(\sigma_2^2 + d_2^2)}{n_1 + n_2}$$

$$= \frac{50(64 + 6.40) + 100(49 + 1.58)}{150}$$

$$= \frac{50 \times 70.4 + 100 \times 50.58}{150}$$

$$\sigma^2 = \frac{3520 + 5058}{150} = \frac{8578}{150} = 57.18$$

$$\sigma = \sqrt{57.18} = 7.56$$

Q. The mean weight of 150 student 60 kg, where mean weight boys 70 kg and mean weight of girls 55 kg. and the S.D are 10 & 15 kg respectively. Find the no. of boys and the combined S.D. boys = 50, girls = 100

→ $\bar{X} = 60$kg, $\bar{x}_1 = 70$, $\bar{x}_2 = 55$

$\sigma_1 = 10$, $\sigma_2 = 15$

$n_1 = 50$, $n_2 = 100$

$d_1 = \bar{x}_1 - \bar{X} = 10$

$d_2 = \bar{x}_2 - \bar{X} = -5$

$\sigma^2 = n_1$

$$\sigma^2 = \frac{n_1(\sigma_1^2 + d_1^2) + n_2(\sigma_2^2 + d_2^2)}{n_1 + n_2}$$

$$= \frac{50(100+100) + 100(225+25)}{150}$$

$$= \frac{10000 + 25000}{150}$$

$$= \frac{35000}{150}$$

$$= 233.33$$

$$\sigma = \sqrt{233.33}$$

$$= 12.90 = 15.27$$

$$\bar{X} = \frac{n_1 + \bar{x}_1 + n_2 + \bar{x}_2}{n_1 + n_2}$$

$$60 = \frac{n_1 70 + 55 n_2}{150}$$

$$\Rightarrow 9000 = 70n_1 + 55n_2$$

$$\Rightarrow 70n_1 + 55n_2 = 9000 \quad (1)$$

$$70 n_1 + 70 n_2 = 10500$$

70 × eq (2),

$$70n_1 + 55n = 9000$$

$$- 70n_1 + 70n_2 = 10500$$

$$-15n_2 = -1500$$

$$\Rightarrow 15n_2 = 1500$$

$$\Rightarrow n_2 = 100$$

$$n_1 + n_2 = 150$$

$$\Rightarrow n_1 + 100 = 150$$

$$\Rightarrow n_1 = 50$$

$(n_1, n_2) = (50, 100)$

Q. Find the missing information from the following

	I	II	III	combined
Number	50	?	90	200
Standard deviation	6	7	?	7.746
Mean	113	?	115	116

→ Given combined of 3 no = 200

$\Rightarrow 50 + x + 90 = 200$

$\Rightarrow x = 200 - 140$

$\Rightarrow x = 60$

Now, combined mean, $\bar{X} = 116$,

$\bar{X}_1 = 113$

$\bar{X}_3 = 115$

we need to find $\bar{X}_2$.

As we know $\bar{X} = \dfrac{n_1\bar{X}_1 + n_2\bar{X}_2 + n_3\bar{X}_3}{n_1 + n_2 + n_3}$

$\Rightarrow 116 = \dfrac{50 \times 113 + 60\bar{X}_2 + 90 \times 115}{200}$

$\Rightarrow 23200 = 5650 + 60\bar{X}_2 + 10350$

$\Rightarrow 23200 = 60\bar{X}_2 + 16000$

$\Rightarrow 60\bar{X}_2 = 23200 - 16000 = 7200$

$\Rightarrow \bar{X}_2 = \dfrac{7200}{60} = 120$

Again $\sigma_1 = 6$, $\sigma_2 = 7$, & $\sigma = 7.746$, we need to find σ_3 value.

Here $n_1 = 50$, $n_2 = 60$, $n_3 = 90$

$\bar{X}_1 = 113$, $\bar{X}_2 = 120$, $\bar{X}_3 = 115$, $\bar{X} = 116$

$d_1 = \bar{X}_1 - \bar{X} = -3$, $d_2 = \bar{X}_2 - \bar{X} = 4$, $d_3 = \bar{X}_3 - X = -1$

$\sigma^2 = \dfrac{n_1(\sigma_1^2 + d_1^2) + n_2(\sigma_2^2 + d_2^2) + n_3(\sigma_3^2 + d_3^2)}{200}$

$$\sigma^2 = \frac{50(36+9)+60(49+16)+90(\sigma_3^2+1)}{200}$$

$$= \frac{50\times45+60\times65+90\sigma_3^2+90}{200}$$

$$= \frac{90\sigma_3^2+6240}{200}$$

$$(7.746)^2 = \frac{90\sigma_3^2+6240}{200}$$

$$\Rightarrow 60 = \frac{90\sigma_3^2+6240}{200}$$

$$\Rightarrow 12000 = 90\sigma_3^2+6240$$

$$\Rightarrow 90\sigma_3^2 = 12000-6240$$

$$\Rightarrow 90\sigma_3^2 = 5760$$

$$\Rightarrow \sigma_3^2 = \frac{5760}{90} = 64$$

$$\Rightarrow \sigma_3 = \sqrt{64} = 8$$

* Assignments

12.(a) From the following information, find the standard deviation of x & y variables

→ ~~Given~~ For Variable x

→ Given $\Sigma x = 235$, $\Sigma x^2 = 6750$, $N = 10$

So $\sigma_x = \sqrt{\frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2} = \sqrt{\frac{6750}{10} - \left(\frac{235}{10}\right)^2} = \sqrt{675 - (23.5)^2} = \sqrt{675 - 552.25}$

$= \sqrt{122.75} = 11.07$

For Variable y

Given $\Sigma y = 250$, $\Sigma y^2 = 6840$, $N = 10$

So $\sigma_y = \sqrt{\frac{\Sigma y^2}{N} - \left(\frac{\Sigma y}{N}\right)^2} = \sqrt{\frac{6840}{10} - \left(\frac{250}{10}\right)^2} = \sqrt{684 - 625} = \sqrt{59} = 7.68$

(b)

13.(a)

→ Let $X = 1, 2, 3, \dots n$

$$\Sigma X = \frac{n+1}{2}$$

So $X^2 = 1^2, 2^2, 3^2, \dots n^2$

$$\Sigma X^2 = 1^2 + 2^2 + 3^2 + \dots + n^2$$

$$= \frac{n(n+1)(2n+1)}{6}$$

Now standard deviation of n natural numbers,

$$\sigma_x = \sqrt{\frac{\Sigma X^2}{n} - \left(\frac{\Sigma X}{n}\right)^2}$$

$$= \sqrt{\frac{n(n+1)(2n+1)}{6} \cdot \frac{1}{n} - \left(\frac{n+1}{2}\right)^2}$$

$$= \frac{(n+1)}{2} \cancel{n}$$

$$= \sqrt{\frac{(n+1)(2n+1)}{6} - \frac{(n+1)^2}{4}}$$

$$= \sqrt{\left(\frac{n+1}{2}\right)\left(\frac{2n+1}{3} - \frac{(n+1)}{2}\right)}$$

$$= \sqrt{\frac{n+1}{2} \times \frac{4n+2-3n-3}{6}}$$

$$= \sqrt{\frac{n+1}{2} \times \frac{n-1}{6}}$$

$$= \sqrt{\frac{n^2-1}{12}}$$

(b) S.D of 1, 2, 3, 4, 5

Here $n = 5$,

So $\sigma_x = \sqrt{\frac{5^2-1}{12}}$

$$= \sqrt{\frac{25-1}{12}}$$

$$= \sqrt{\frac{24}{12}}$$

$$= \sqrt{2}$$

(c) Mean of 10 items is 50 and S.D is 14.

So $n = 10$, $\bar{X} = 50$, $\sigma_x = 14$

Thus $\sigma_x = \sqrt{\frac{\Sigma X^2}{n} - \left(\frac{\Sigma X}{n}\right)^2}$

$$\Rightarrow 14 = \sqrt{\frac{\Sigma X^2}{10} - (50)^2}$$

On squaring both sides, we get -

$$\Rightarrow 196 = \frac{\Sigma X^2}{10} - 2500$$

$$\Rightarrow 196 + 2500 = \frac{\Sigma X^2}{10}$$

$$\Rightarrow 2696 = \frac{\Sigma X^2}{10}$$

$$\Rightarrow \Sigma X^2 = 2696 \times 10 = 26960$$

Q.14

C.I	f	m	fm	$(m-\bar{X})$	$(m-\bar{X})^2$	$f(m-\bar{X})^2$
0-10	5	5	25	-27.09	733.86	3669.3
10-20	7	15	105	-17.09	292.06	2044.42
20-30	14	25	350	-7.09	50.26	703.64
30-40	12	35	420	2.91	8.46	101.52
40-50	9	45	405	12.91	166.66	1499.94
50-60	6	55	330	22.91	524.86	3149.16
60-70	2	65	130	32.91	1083.06	2166.12
						13334.1

Here Mean, $\bar{X} = \frac{\Sigma fm}{N} = \frac{1765}{55} = 32.09$

Thus $\sigma_x = \sqrt{\frac{\Sigma f(m-\bar{X})^2}{N}}$

$$= \sqrt{\frac{13334.1}{55}}$$

$$= \sqrt{242.43}$$

$$= 15.57$$

(b)

C.I	f	x	$d=\frac{x-A}{h}$	d^2	fd	fd^2
100-105	200	102.5	-5	25	-1000	5000
105-110	210	107.5	-4	16	-840	3360
110-115	230	112.5	-3	9	-690	2070
115-120	320	117.5	-2	4	-640	1280
120-125	350	122.5	-1	1	-350	350
125-130	520	(127.5)	0	0	0	0
130-135	410	132.5	1	1	410	410
135-140	320	137.5	2	4	640	1280
140-145	280	142.5	3	9	840	2520
145-150	210	147.5	4	16	840	3360
150-155	160	152.5	5	25	800	4000
155-160	90	157.5	6	36	540	3240
	3360				550	26870

$$\rightarrow \sigma_x = h \cdot \sqrt{\frac{\Sigma fd^2}{N} - \left(\frac{\Sigma fd}{N}\right)^2}$$

$$= 5\sqrt{\frac{26870}{3360} - \left(\frac{550}{3360}\right)^2}$$

$$= 5\sqrt{7.997 - 0.026}$$

$$= 5\sqrt{7.971}$$

$$= 5 \times 2.83$$

$$= 14.15$$

12.(b)

Given $\Sigma x = 240$, $\Sigma x^2 = 6400$

$\Sigma y = 250$, $\Sigma y^2 = 7060$

Also ten items are included in both the survey

So $n = 10$

Now, $\sigma_x = \sqrt{\frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2}$

$$= \sqrt{\frac{6400}{10} - \left(\frac{240}{10}\right)^2}$$

$$= \sqrt{640 - (24)^2}$$

$$= \sqrt{640 - 576}$$

$$= \sqrt{64}$$

$$= 8$$

$\sigma_y = \sqrt{\frac{\Sigma y^2}{n} - \left(\frac{\Sigma y}{n}\right)^2}$

$$= \sqrt{\frac{7060}{10} - \left(\frac{250}{10}\right)^2}$$

$$= \sqrt{706 - (25)^2}$$

$$= \sqrt{706 - 625}$$

$$= \sqrt{81}$$

$$= 9$$

$(\because \sigma_x, \sigma_y = 8, 9)$

Measures of Dispersion

There are two types of measures dispersion.

(I) Absolute measure

(II) Relative measure

~~Range, Quartile deviation, mean deviation and standard deviation~~

(a) Range
(b) Quartile deviation
(c) Mean deviation
(d) standard deviation
(e) Variance

Relative Measure

(a) Co-efficient range
(b) Co-efficient of Quartile deviation
(c) Co-efficient of M.D
(d) Co-efficient of Variation

Difference between absolute measure & Relative measure

In absolute measure, the results are dependent on particular unit. But in case of co-efficient of variation, the results are independent of unit or unit-3

In case of absolute measure, we can only compare with same unit data. But in case of Co-efficient of variation, we can compare any distribution.

The Co-efficient Variation is defined as $CV = \frac{\sigma}{\bar{x}}$, where σ is S.D & $\bar{x}$

$$\boxed{CV = \frac{\sigma}{\bar{x}} \times 100}$$

Q. The A.M of runs scored by 3 batsman in 10 innings are 50, 48, 12 and their standard deviation 15, 12 & 2. Now find out the who is more consistent among the 3 batsman. Calculate CV of all the 3 batsman.

→ Here $\bar{x}_1 = 50$, $\sigma_1 = 15$

$\bar{x}_2 = 48$, $\sigma_2 = 12$

$\bar{x}_3 = 12$, $\sigma_3 = 2$

So $CV_1 = \frac{\sigma_1}{\bar{x}} \times 100$, $CV_2 = \frac{\sigma_2}{\bar{x}_2} \times 100$, $CV_3 = \frac{\sigma_3}{\bar{x}_3} = \frac{2}{12} \times 100$

$= \frac{15}{50} \times 100$ $= \frac{12}{48} \times 100$ $= 16.6$

$= 30$ $= 25$

Here the lowest CV is belongs to batsman 3. So he is more consistent than other two.

Co-efficient of variance of 2 series are 75% & 90% & their S.D are 15 & 18 respectively. Find the mean.

$CV_1 = 75$, $CV_2 = 90$

$\sigma_1 = 15$, $\sigma_2 = 18$ $\frac{75}{100} \times$

So $CV_1 = \frac{\sigma_1}{\bar{x}}$ $CV_1 = \frac{\sigma_1}{\bar{x}_1} \times 100$

$\Rightarrow 75 = \frac{15}{\bar{x}} = \bar{x} = \frac{15}{75} = 0.2$ $\Rightarrow \frac{75}{100} = \frac{15}{\bar{x}_1} \times 100$

$CV_2 = \frac{\sigma_2}{\bar{x}_2} \Rightarrow \bar{x}_2 = \frac{\sigma_2}{CV_2}$ $\Rightarrow 75\bar{x}_1 = 15000$

$= \frac{18}{}$ $\Rightarrow \bar{x}_1 = \frac{15000}{75} = 20$

Now $CV_2 = \frac{\sigma_2}{\bar{x}_2} \times 100$

$\Rightarrow 90 = \frac{18}{x_2} \times 100$

$\Rightarrow x_2 = \frac{18000}{90} = 20$

Q.

→ Here $\bar{X}_1 = 78$, S.D = 8.0

$$CV_1 = \frac{\sigma_1}{\bar{X}_1} \times 100 \qquad CV_2 = \frac{\sigma_2}{\bar{X}_2} \times 100$$

$$= \frac{8.0}{78} \times 100 \qquad = \frac{7.6}{80} \times 100$$

$$= 10.25 \qquad = 9.5$$

c.

Here CV_1 is 10.25 and CV_2 is 9.5, this is clear that CV_1 is greater than CV_2 so the students of 1995 are more consistent in their result in comparing to the students of 1990.

It can be observe that over the time from 1990 to 1995, the avg performance of students has increased and they are more consistent in their performance.

Q. 6

	Company A		Company B	
	Males	Females	Males	Females
Mean	30	20	27	32
Variance	8	3	12	5
No. of emp	40	10	20	30

→ For Company A,

the Combine Mean = $\frac{40 \times 30 + 10 \times 20}{50}$

$= \frac{1200 + 200}{50}$

$= \frac{1400}{50}$

$= 28$

Again Combine variance for Combine S.D

$d_1 = \bar{X}_1 - \bar{X} = 30 - 28 = 2$

$d_2 = \bar{X}_2 - \bar{X} = 20 - 28 = -8$

$$\sigma = \left[\frac{n_1(\sigma_1^2 + d_1^2) + n_2(\sigma_2^2 + d_2^2)}{50}\right]^{1/2}$$

$$= \left(\frac{40(8+4) + 10(3+64)}{50}\right)^{1/2}$$

$$= \left(\frac{480 + 670}{50}\right)^{1/2} = \left(\frac{1150}{50}\right)^{1/2}$$

$= 4.79$

For Company B

the Combine Mean = $\frac{20 \times 27 + 30 \times 32}{50}$

$= \frac{540 + 960}{50}$

$= 30$

$d_1 = \bar{X}_1 - \bar{X} = -3$

$d_2 = \bar{X}_2 - \bar{X} = 2$

$$CV_2 = \left(\frac{20(12+9) + 30(5+4)}{50}\right)^{1/2}$$

$$= \left(\frac{3060 + 870}{50}\right)^{1/2}$$

$= (78.6)^{1/2} = 8.86$

$$CV_A = \frac{4.79}{28} \times 100 = 17.10$$

$$CV_2 = \left(\frac{20(12+9) + 30(5+4)}{50}\right)^{1/2}$$

$$= \left(\frac{420 + 270}{50}\right)^{1/2}$$

$$= \left(\frac{690}{50}\right)^{1/2}$$

$$= (13.5)^{1/2}$$

$$= 3.71$$

$$CV_2 = \frac{\sigma_B}{30} \times 100$$

$$= \frac{3.71}{30} \times 100$$

$$= 12.36$$

Here CV_B is less than CV_A, so productivity in company B is more consistent, whereas highest productivity observe in Comp A.

Assignments

Q.2

Co-efficient of variation of the first n natural numbers.

$n = 1, 2, 3, \ldots n$

As we know the standard deviation of n natural numbers is

$$\sqrt{\frac{n^2-1}{12}}$$

Also the mean of n natural numbers is

$$\left(\frac{n+1}{2}\right)$$

Thus C.V of n natural numbers $= \frac{\sigma_n}{\bar{x}_n} = \frac{\sqrt{\frac{n^2-1}{12}}}{\frac{n+1}{2}}$

$$\Rightarrow C.V_n = \sqrt{\frac{n^2-1}{12}} \times \frac{2}{n+1}$$

On squaring both side, we get -

$$(C.V_n)^2 = \frac{n^2-1}{12} \times \frac{4}{(n+1)^2}$$

$$= \frac{4(n^2-1^2)}{12(n+1)^2}$$

$$= \frac{4(n+1)(n-1)}{3 \cancel{12}(n+1)^2}$$

$$= \frac{(n-1)}{3(n+1)}$$

$$\Rightarrow C.V_n = \sqrt{\frac{n-1}{3(n+1)}}$$

Q.4

→ Here $\bar{x}_1 = 50, \sigma_1 = 15$

$\bar{x}_2 = 48, \sigma_2 = 12$

$\bar{x}_3 = 12, \sigma_3 = 2$

So $CV_1 = \frac{\sigma_1}{\bar{x}_1} \times 100$

$= \frac{15}{50} \times 100$

$= 30$

$CV_2 = \frac{\sigma_2}{\bar{x}_2} \times 100$

$= \frac{12}{48} \times 100$

$= 25$

$CV_3 = \frac{2}{12} \times 100$

$= 16.67$

Q.4 If two samples A & B having same standard deviation. But the mean of A is greater than B, then obviously, the CV of A is will be greater than B. Because if in two cases, the numerators are same, and the denominator of A is greater than the denominator B. Then the ~~data having~~ CV of A is greater than CV of B.

Q.5 (a) Here, $CV = 60\%$, $\sigma = 12$, $\bar{X} = ?$

As we know $CV = \frac{\sigma}{\bar{X}} \times 100$

$\Rightarrow 60 = \frac{12}{\bar{X}} \times 100$

$\Rightarrow \frac{60}{100} = \frac{12}{\bar{X}}$

$\Rightarrow 6\bar{X} = 120$

$\Rightarrow \bar{X} = \frac{120}{6} = 20$

(b) Here $\sigma^2 = 16$, $n = 20$ & $\Sigma n = 160$, $CV = ?$

Thus $\bar{X} = \frac{\Sigma n}{n} = \frac{160}{20} = 8$

As $\sigma^2 = 16$,

$\sigma = \sqrt{16} = 4$

So $CV = \frac{\sigma}{\bar{X}} \times 100$

$= \frac{4}{8} \times 100$

$= 50\%$.

Q.6 (a) Here $CV_1 = 60\%$, $CV_2 = 80\%$.

$\sigma_1 = 20$, $\sigma_2 = 16$

$\bar{X}_1 = ?$ $\bar{X}_2 = ?$

For CV_1,

$CV_1 = \frac{\sigma_1}{\bar{X}_1} \times 100$

$60 = \frac{20}{\bar{X}_1} \times 100$

$\Rightarrow \bar{X}_1 = \frac{2000}{60}$

$= 33.33$

For CV_2

$CV_2 = \frac{\sigma_2}{\bar{X}_2} \times 100$

$\Rightarrow 80 = \frac{16}{\bar{X}_2} \times 100$

$\Rightarrow \bar{X}_2 = \frac{1600}{80}$

$= 20$

(b) Here $CV_1 = 60\%$. $CV_2 = 80\%$.

$\sigma_1 = 24$ $\sigma_2 = 20$

$\bar{X}_1 = ?$ $\bar{X}_2 = ?$

For CV_1,

$CV_1 = \frac{\sigma_1}{\bar{X}_1} \times 100$

$\Rightarrow 60 = \frac{24}{\bar{X}_1} \times 100$

$\Rightarrow \bar{X}_1 = \frac{2400}{60}$

$= 40$

For CV_2,

$CV_2 = \frac{\sigma_2}{\bar{X}_2} \times 100$

$\Rightarrow 80 = \frac{20}{\bar{X}_2} \times 100$

$\Rightarrow \bar{X}_2 = \frac{2000}{80}$

$= 25$

Q.7

After settlement, the mean increased from 800 to 1200 & the standard deviation had increased from 200 to 250.

So $\bar{x}_1 = 800$ & $\bar{x}_2 = 1200$

$\sigma_1 = 200$ & $\sigma_2 = 250$

Thus $CV_1 = \frac{200}{800} \times 100$ & $CV_2 = \frac{250}{1200} \times 100$

$= 25$ $= 20.83$

Now we can say the wages have become ~~higher~~ more uniform after settlement.

Q.8

$\bar{x}_1 = 800$ (weekly avg. wages of workers)

$\bar{x}_2 = 1200$ (increase avg. wages ")

& their $\sigma_1 = 100$ to $\sigma_2 = 500$

Thus $CV_1 = \frac{100}{800} \times 100$ & $CV_2 = \frac{500}{1200} \times 100$

$= 12.5$ $= 41.66$

Hence the wages after increment become less uniform.

Q.9 Students ~~having~~ secured average marks 50 with a standard deviation of 2 in the first year.

So $\bar{x}_1 = 50$, $\sigma_1 = 2$

that implies $CV_1 = \frac{2}{50} \times 100 = 4\%$

After five years same students secured avg. marks 60 with an increased S.D of 3.

So $\bar{x}_2 = 60$, $\sigma_2 = 3$

$\Rightarrow CV_2 = \frac{3}{60} \times 100$

$= 5\%$

Hence we can say, the students showed improved performance.

Q.10

Here $C.V_m = 55\%$, $C.V_f = 70\%$

$\sigma_m = 22$, $\sigma_f = 15.40$

Calculate the combined average wages of all the workers, if 80% of the workers were males.

that means $n_m = 80$ and obviously $n_f = 20$

Now, $C.V_m = \frac{\sigma_m}{\bar{x}_m} \times 100$, and

$\Rightarrow 55 = \frac{22}{\bar{x}_m} \times 100$

$\Rightarrow \bar{x}_m = \frac{2200}{55} = 40$

again $C.V_f = \frac{\sigma_f}{\bar{x}_f} \times 100$

$\Rightarrow 70 = \frac{15.40}{\bar{x}_f} \times 100$

$\Rightarrow \bar{x}_f = \frac{1540}{70} = 22$

So the combined mean, $\bar{x} = \frac{n_m \bar{x}_m + n_f \bar{x}_f}{n_m + n_f}$

$= \frac{80 \times 40 + 20 \times 22}{100}$

$= \frac{3200 + 440}{100}$

$= \frac{3640}{100}$

$\bar{x} = 36.40$

Q.11

Q.11

	Brand I	Brand II
Means	800 hours	770 hours
S.D	100 hours	60 hours

→ Here $\bar{x}_1 = 800$, $\sigma_1 = 100$

$\bar{x}_2 = 770$, $\sigma_2 = 60$

So $CV_1 = \frac{\sigma_1}{\bar{x}_1} \times 100$

$= \frac{100}{800} \times 100$

$= 12.5$

$CV_2 = \frac{60}{770} \times 100$

$= \frac{600}{77}$

$= 7.79$

Brand II is more uniform.

Q.12 (b)

In factory A, Variance, $\sigma^2 = 16 \Rightarrow \sigma = 4$

Mean $\bar{x} = 120$

So $CV_A = \frac{\sigma}{\bar{x}} \times 100$

$= \frac{4}{120} \times 100$

$= 3.3$

In factory B, Variance, $\sigma^2 = 25 \Rightarrow \sigma = 5$

Mean, $\bar{x} = 200$

So $CV_B = \frac{\sigma}{\bar{x}} \times 100$

$= \frac{5}{200} \times 100$

$= 2.5$

Q.13

In factory A, $\sigma^2 = 9$

So $\sigma = 3$

and Mean, $\bar{x} = 120$

So $C.V_A = \frac{\sigma}{\bar{x}} \times 100$

$= \frac{3}{120} \times 100$

$= \frac{30}{12} = 2.5$

In factory B, $\sigma^2 = 16$

$\sigma = 4$

and $\bar{x} = 85$

So $CV_B = \frac{\sigma}{\bar{x}} \times 100$

$= \frac{4}{85} \times 100$

$= 4.70$

Q.14

In 1960, thousand students of B.A. Hons. Economics gave avg. marks secured as 50%. So $\bar{x}_1 = 50$ with a S.D of 3, $\sigma_1 = 3$.

Thus $CV_1 = \frac{\sigma_1}{\bar{x}_1} \times 100$

$= \frac{3}{50} \times 100$

$= 6$

In 1970, $\bar{x}_2 = 55\%$. $\sigma_2 = 5$

So $CV_2 = \frac{\sigma_2}{\bar{x}_2} \times 100$

$= \frac{5}{55} \times 100$

$= 9.09$

Thus the results have improved from 1960 to 1970.

Range

Range is the difference between largest observation and the smallest observation in the data set.

$$R = L - S$$

Absolute Range = Largest value - Smallest value.

Q. Find the range of the following distribution

16 - 20
20 - 24
24 - 28
28 - 32

→ Here Range = 32 - 16 = 16

In case of range of a continious series, if the series is exclusive, then

R = Upper limit of last class - Lower limit of first class.

but if it is a inclusive series, then convert it to exclusive and find range.

x	f
16 - 20	3
21 - 25	9
26 - 30	7

→

x
15.5 - 20.5
20.5 - 25.5
25.5 - 30.5

30.5 - 15.5 = 15

Co-efficient of Range

It is defined as $\frac{L-S}{L+S}$

Q. Find the range & co-efficient of range from the following observation

65
70
82
59
81
59
81
76
57
60
55
50

Range = 82 - 50 = 32

Co-efficient of range = $\frac{L-S}{L+S}$

$= \frac{32}{132} = 0.24$

Q. From the monthy income of 10 families, given below. Calculate No.1 median, GM, Co-Range

Family 1	F.1	log X
145	73	1.86
367	115	2.06
268	145	2.16
73	185	2.26
185	268	2.42
619	280	2.44
280	315	2.49
115	367	2.56
870	619	2.79
315	870	2.93
		23.97

$$\frac{\left(\frac{n}{2}\right)+\left(\frac{n}{2}+1\right)}{2} = \frac{5+6}{2} \quad \frac{11}{2} = 5.5$$

$$\frac{268+280}{2} =$$

$$A\left(\frac{\Sigma \log y}{n}\right)$$

$$\text{Med} = AL\left(\frac{\Sigma \log y}{n}\right)$$

$$= AL\left(\frac{23.97}{10}\right)$$

$$= AL(2.397)$$

$$= 249.45$$

$$\text{Co-range} = \frac{870-73}{870+73} = \frac{797}{943} = 0.84$$

Quartile

Quartile divides the whole data set into 4 equal parts.

Q Find the quartile of following series

94	69	
95	73	
96	79	
93	87	
87	93	
79	94	
73	95	
69	96	
	N=8	

$$\frac{n}{2} + \frac{n}{2}+1 \rightarrow \frac{4^{th}+5^{th}}{2} = \frac{87+93}{2} = \frac{180}{2} = 90$$

$Q_1 = 38$

$Q_2 = 90$

$Q_3 = 47.5$

$Q_2 = 90$

$$Q_1 = \frac{N}{4} = \frac{180}{4} = 45$$

$$Q_3 = \frac{3N}{4} = \frac{135}{4} =$$

$$Q_1 = \frac{\frac{N}{4}\text{th} + \left(\frac{N}{4}+1\right)\text{th}}{4}$$

$$= \frac{2^{th}+3^{rd}}{4}$$

$$= \frac{73+79}{4}$$

$$= 38$$

$$\frac{3N}{4} \quad \frac{57+46}{4} =$$

$$Q_3 = \frac{3N}{4} = \frac{3\frac{N}{4}\text{th} + 3\left(\frac{N}{4}+1\right)\text{th}}{4}$$

$$= \frac{6^{th}+7^{th}}{4}$$

$$= \frac{94+95}{4} = \frac{189}{4} = 47.5$$

Continuous Series

x	f	cf	
10-19	1	1	
20-29	6	7	
30-39	4	11	
40-49	3	14	Q_1
50-59	14	28	Q_2
60-69	19	47	Q_3

N=99
47

→ Median class $= \frac{N}{2} = \frac{47}{2} = 23.5$

$Q_1 \quad \frac{N}{4} = \frac{47}{4} = 11.75$

$Q_3 = \frac{3N}{4} = 3 \cdot \frac{47}{4} = 35.25$

So $Q_1 = L_1 + \frac{L_2 - L_1}{f_1}\left(\frac{N}{4} - c\right)$

$= 40 + \frac{49-40}{3}(11.75 - 11)$

$= 40 + 3(0.75)$

$= 40 + 2.25$

$= 42.25$

& $Q_3 = L_1 + \frac{L_2 - L_1}{f_1}\left(\frac{3N}{4} - c\right)$

$= 60 + \frac{69-60}{19}(35.25 - 28)$

$= 60 + \frac{9}{19} \times 7.25$

$= 60 + 0.47 \times 7.25$

$= 60 + 3.43$

$= 63.43$

Assignments

Find the quartile of ?

Quartile deviation: $\frac{Q_3 - Q_1}{2}$

& co-Q.D $= \frac{Q_3 - Q_1}{Q_3 + Q_1}$

Q.

H.I	f	cf	
50-53	2	2	
53-56	7	9	
56-59	24	33	Q_1
59-62	27	60	Q_2
62-65	13	73	
65-68	3	76	
	76		

Here Q_1 class $= \frac{N}{4} = \frac{76}{4} = 19$

So $Q_1 = L_1 + \frac{L_2 - L_1}{f_1}\left(\frac{N}{4} - c\right)$

$= 56 + \frac{59-56}{24}(19-9)$

$= 56 + \frac{3}{24} \times 10$

$= 56 + 0.125 \times 10$

$= 56 + 1.25$

$= 57.25$

Now Q_2 class $= \frac{N}{2} = \frac{76}{2} = 38$

So $Q_2 = L_1 + \frac{L_2 - L_1}{f_1}\left(\frac{N}{2} - c\right)$

$= 59 + \frac{62-59}{27}(38-33)$

$= 59 + \frac{3}{27} \times 5$

$= 59 + 0.55$

$= 59.55$

Now Q_3 class $= \frac{3N}{4} = \frac{3 \cdot 76}{4} = 57$

So $Q_3 = L_1 + \frac{L_2 - L_1}{f_1}\left(\frac{3N}{4} - c\right)$

$= 59 + \frac{62-59}{27}(57-33)$

$= 59 + \frac{3}{27} \times 24$

$= 59 + 2.66$

$= 61.66$

Quartile deviation $= \frac{Q_3 - Q_1}{2} = \frac{61.66 - 57.25}{2} = \frac{4.41}{2} = 2.205$

Q.2

C.I	f	CF	
20-29	306	306	Q_1
30-39	182	488	Q_3
40-49	144	632	
50-59	96	728	
60-69	42	770	
70-79	34	804	
	804		

Here Q_1 class $= \frac{N}{4} = \frac{804}{4} =$ ~~402~~ 201

So $Q_1 = L_1 + \frac{L_2 - L_1}{f_1}\left(\frac{N}{4} - c\right)$

$= 39 + \frac{39-30}{182}(402-306)$

$= 39 + \frac{9}{182}(96)(-105)$

$= 39 + 0.049 \times 96$

$= 39 + 4.74$

$= 43.74$

Q_2 class $= \frac{N}{2} = \frac{804}{2}$

Here Q_1 class $= \frac{N}{4} = \frac{804}{4} = 201$

So $Q_1 = L_1 + \frac{L_2 - L_1}{f_1}\left(\frac{N}{4} - c\right)$

$= 20 + \frac{29-20}{306}$

→

Midpoint	f	CF
24.5	306	306
34.5	182	488
44.5	144	632
54.5	96	728
64.5	42	770
74.5	34	804

~~$Q_1 = \frac{N}{4} = \frac{804}{4} = 201$~~

~~$Q_3 = \frac{3N}{4} =$~~

~~$Q_1 = \frac{\text{N}^{\text{th}} + \left(\frac{N}{2}+1\right)^{\text{th}}}{4}$~~

$=$

$Q_1 = \frac{N+1}{4} = \frac{804+1}{4} = \frac{805}{4} = 201.25$

C.I	f	cF	
20-29	306	306	Q_1
30-39	182	488	
40-49	144	632	Q_3
50-59	96	728	
60-69	42	770	
70-79	34	804	

Q_1 Class $= \frac{N}{4} = \frac{804}{4} = 201$

So $Q_1 = L_1 + \frac{L_2 - L_1}{f_1}\left(\frac{N}{4} - c\right)$

$= 20 + \frac{29-20}{306}(201 - 0)$

$= 20 + \frac{9}{306} \times 201$

$= 20 + 5.91$

$= 25.91$

$Q_3 = \frac{3N}{4} = \frac{3 \times 804}{4} = 603$

So $Q_3 = L_1 + \frac{L_2 - L_1}{f_1}\left(\frac{3N}{4} - c\right)$

$= 40 + \frac{49-40}{144}(603 - 488)$

$= 40 + \frac{9}{144} \times 115$

$= 40 + 7.18$

$= 47.18$

So Quartile deviation $= \frac{Q_3 - Q_1}{2} = \frac{47.18 - 25.91}{2}$

$= 10.63$

Mean Deviation

Mean deviation of a series is the arithmetic average of the deviation of various items from a measure of central tendency.

→ It is based on all the items of the series.

→ While calculating deviations from the selected average, the signs (+ or −) of deviations are ignored and all the deviations are taken as positive.

Individual Series

~~Mean Deviation from~~

- Calculate the specific average (Mean or Median) from which mean deviation is to be found out.
- Obtain absolute (positive) deviations of each observation from the specific average.
- Absolute deviations are totalled up to find out $\Sigma|D|$
- Apply the formula: Mean Deviation from Mean $(MD_x) = \frac{\Sigma|x-\bar{x}|}{n} = \frac{\Sigma|D|}{n}$
- Co-efficient of Mean Deviation: $\frac{\text{Mean Deviation}}{\text{Mean}}$ (for mean), $\frac{\text{Mean Deviation}}{\text{Mode}}$ (for mode)

Ex Find mean deviation for the data, 1, 2, 3, 4, 5, 6, 7.

→

x	$\lvert x-\bar{x}\rvert$
1	3
2	2
3	1
4	0
5	1
6	2
7	3
28	$\Sigma = 12$

Here Mean $= \frac{\Sigma x}{n} = \frac{28}{7} = 4$

So Mean Deviation $(MD_x) = \frac{\Sigma|x-\bar{x}|}{n} = \frac{12}{7} = 1.7$

& Co-efficient of $MD_x = \frac{MD_x}{\bar{x}} = \frac{1.7}{4} = 0.42$

Again for median,

Median of the data $= \left(\frac{n+1}{2}\right)$ th term

$= \left(\frac{7+1}{2}\right)^{th}$ term

$= 4^{th}$ term

$= 4$

As Median $= 4$, so $|x-\bar{x}|$ column will be same as for mean.

Thus Mean deviation from median $(MD_x) = \frac{\Sigma|x-\bar{x}|}{n} = \frac{12}{7} = 1.7$

and Co-efficient of $MD_x = \frac{MD_x}{\bar{x}} = \frac{1.7}{4} = 0.42$

For Discrete Series

M.D about mean = $\frac{\Sigma f|x-\bar{x}|}{N}$, here $\bar{x}$ is mean

M.D about median = $\frac{\Sigma f|x-\bar{x}|}{N}$, here $\bar{x}$ is median

M.D about mode = $\frac{\Sigma f|x-\bar{x}|}{N}$, here $\bar{x}$ is mode.

ex:- Calculate the mean deviation about arithmetic mean and co-efficient of mean deviation.

x	f	fx	$\lvert x-\bar{x}\rvert$	$f\lvert x-\bar{x}\rvert$
10	3	30	1.86	5.58
11	12	132	0.86	10.32
12	18	216	0.14	2.52
13	12	156	1.14	13.68
	$\Sigma=45$	$\Sigma fx=534$		$\Sigma=32.1$

So Mean = $\frac{\Sigma fx}{N} = \frac{534}{45} = 11.86$

Mean deviation $MD_x = \frac{\Sigma f|x-\bar{x}|}{N} = \frac{32.1}{45} = 0.71$

and co-efficient of $MD_x = \frac{MD}{\bar{x}} = \frac{0.71}{11.86} = 0.06$

For Median

x	f	CF
10	3	3
11	12	15
12	18	33
13	12	45
	N=45	

So Median = $\left(\frac{N+1}{2}\right)^{th}$ term

$= \left(\frac{45+1}{2}\right)^{th}$ term

$= 23$

So

x	f	CF	$\lvert x-\bar{x}\rvert$	$f\lvert x-\bar{x}\rvert$
10	3	3	13	39
11	12	15	12	144
12	18	33	11	198
13	12	45	10	120
	$\Sigma=45$			$\Sigma=501$

So Mean deviation $(MD_x) = \frac{\Sigma f|x-\bar{x}|}{N}$

$= \frac{501}{45} = 11.13$

Co-efficient of $MD_x = \frac{MD}{\bar{x}}$

$= \frac{11.13}{23} = 0.48$

For continuous Series

In case of continuous series, the formula of mean deviation are same as discrete series.

The mid point of class intervals have to found out and they are taken as 'M'

Mean Deviation from Mean $(MD_{\bar{x}}) = \frac{\Sigma f|M-\bar{x}|}{N} = \frac{\Sigma f|D|}{N}$

Mean Deviation from Median $(MD_{Me}) = \frac{\Sigma f|M-Me|}{N} = \frac{\Sigma f|D|}{N}$

where M = mid point, $\bar{x}$ = Arithmetic mean & Me = Median

* Calculate the mean deviation from mean for the following data.

class	f	M	fM	$\lvert M-\bar{x}\rvert$	$f\lvert M-\bar{x}\rvert$
2-4	3	3	9	2.2	6.6
4-6	4	5	20	0.2	0.8
6-8	2	7	14	1.8	3.6
8-10	1	9	9	3.8	3.8
	$\Sigma=10$		$\Sigma=52$		14.8

So Mean $\frac{\Sigma fM}{N} = \frac{52}{10} = 5.2$

Hence the mean deviation, $MD_{\bar{x}} = \frac{\Sigma f|M-\bar{x}|}{N} = \frac{14.8}{10} = 1.48$

Assignments

class	f	M	fM	CF	$\lvert M-\bar{M}e\rvert$	$f\lvert M-\bar{M}e\rvert$
50-100	7	75	525	7	125	875
100-150	18	125	2250	25	75	1350
150-200	25	175	4375	50	25	625
200-250	31	225		81	25	775
250-300	15	275		96	75	1125
300-350	4	325		100	125	500
	100					$\Sigma=5250$

$(M.D_{Me}) = \frac{\Sigma f|M-Me|}{N} = \frac{5250}{100} = 52.50$

Co-efficient of $(MD.Me) = \frac{MD}{Me} = \frac{52.50}{200} = 0.26\frac{25}{}$

Median class, $\frac{N}{2} = \frac{100}{2} = 50$, So it lies in class (150-200) having f & CF are 25, 50.

Median $= L_1 + \frac{L_2-L_1}{f_1}(m-c)$

$= 150 + \frac{200-150}{25}(50-25) = 150 + \frac{50}{25} \times 25 = 200$

Assignment

(a)

Q.1 Calculate the mean deviation from mean

X	f	M	fM	$\lvert M-\bar{X}\rvert$	$f\lvert M-\bar{X}\rvert$
0-10	5	5	25	22	110
10-20	8	15	120	12	96
20-30	15	25	375	2	30
30-40	16	35	560	8	128
40-50	6	45	270	18	108
	$\Sigma = 50$		$\Sigma = 1350$		$\Sigma = 472$

So Mean, $\bar{X} = \frac{\Sigma fM}{N} = \frac{1350}{50} = 27$

Hence Mean Deviation $(MD_X) = \frac{\Sigma f\lvert M-\bar{X}\rvert}{N}$

$= \frac{472}{50} = 9.44$

And, Co-efficient of $MD_X = \frac{MD}{Mean} = \frac{9.44}{27} = 0.35$

(b) calculate the mean deviation from the mean for the following data

X	f	M	fM	$\lvert M-\bar{X}\rvert$	$f\lvert M-\bar{X}\rvert$	
0-10	6	5	30	28.4	152.4	170.4
10-20	5	15	75	18.4	77	92
20-30	8	25	200	8.4	43.2	67.2
30-40	15	35	525	1.6	64	24
40-50	7	45	315	11.6	102.2	81.2
50-60	6	55	330	21.6	147.6	129.6
60-70	3	65	195	31.6	103.8	94.8
	$N = 50$		$\Sigma = 1670$		$\Sigma = 695.2$	$\Sigma = 659.2$

So Mean, $\bar{X} = \frac{\Sigma fM}{N} = \frac{1520}{50} = 30.4$ $\frac{1670}{50} = 33.4$

Thus Mean Deviation $(MD_X) = \frac{\Sigma f\lvert M-\bar{X}\rvert}{N} = \frac{695.2}{50}$ $\frac{659.2}{50} = 13.184$

Q.8(a) Mean deviation may be calculated from the arithmetic mean or the median or the mode. Which of these three measures is the minimum?

→ Mean deviation is minimum, when taken about median.

(b) Find out mean deviation and its co-efficient from median.

x	f	c.f	fx	$\lvert x-\bar{x}\rvert$	$f\lvert x-\bar{x}\rvert$
4	2	2	2	6	12
6	1	3	3	4	4
8	3	6	6	2	6
10	6	12	12	0	0
12	4	16	16	2	8
14	3	19	19	4	12
16	1		20	6	6
	$N=20$				$\Sigma = 42$

$= 48$

So Median $= \left(\frac{N+1}{2}\right)^{th}$ term

$= \left(\frac{19+1}{2}\right)^{th}$ term

$= 10^{th}$ term

$= 10$

$\frac{48}{20} = 2.4$

Hence Mean deviation from Median $= \frac{\Sigma f\lvert x-\bar{x}\rvert}{N} = \frac{42}{19} = 2.21$

and co-efficient of $(MD_{Me}) = \frac{MD}{\text{Median}} = \frac{2.21}{10} = 0.22$

Q.9

x	f	fx	$\lvert x-\bar{x}\rvert$	$f\lvert x-\bar{x}\rvert$
5	8	40	24	192
15	12	180	14	168
25	10	250	4	40
35	8	280	6	48
45	3	135	16	48
55	2	110	26	52
65	7	455	36	252
	50	1450		$\Sigma = 800$

So Mean $= \frac{\Sigma fx}{N} = \frac{1450}{50} = 29$

So Mean deviation $= \frac{\Sigma f\lvert x-\bar{x}\rvert}{N}$

$= \frac{800}{50} = 16$

For median

X	f	CF	|X- Me|	f|X- Me|
5	8	8	20	160
15	12	20	15	180
25	10	30	0	0
35	8	38	10	80
45	3	41	20	60
55	2	43	30	60
65	7	50	40	280
				Σ = 820

Hence MD of median = $\frac{\Sigma f|X-Me|}{N}$

$= \frac{820}{50}$

$= 16.4$

and co-efficient of $(MD_{Me}) = \frac{MD}{Me}$

$= \frac{16.4}{25}$

$= 0.65$

Median = $(\frac{N+1}{2})^{th}$ term

$= (\frac{50+1}{2})^{th}$ term

$= (\frac{51}{2})^{th}$ term

= 25.5 in CF i.e 25

10.

X	f	M	CF	|M- Me|	f|M- Me|
17-19.5	9	18.25	9	~~22.62~~ 20	~~203.58~~ 180
20-25.5	16	~~[illegible]~~ 22.75	25	~~18.12~~ 15.5	~~289.92~~ 248
26-35.5	12	30.75	37	~~10.12~~ 7.5	~~121.44~~ 90
36-40.5	26	38.25	63	~~2.62~~ 0	~~68.12~~ 0
41-50.5	14	45.75	77	~~4.88~~ 7.5	~~68.32~~ 105
51-55.5	12	53.25	89	~~12.38~~ 15	~~148.56~~ 180
56-60.5	6	58.25	95	~~17.38~~ 20	~~104.28~~ 120
61-70.5	5	65.75	100	~~24.88~~ 27.5	~~124.4~~ 137.5
					Σ = ~~1128.62~~ = 1060.5

Here Median class, $\frac{N}{2} = \frac{100}{2} = 50$, So it lies in class (51-55.5) having frequency ~~20~~ 12 and CF ~~89~~ 63

~~Median = l1 + (l2-l1)/f1 (m-c)~~

$= 51 + \frac{55.5 - 51}{12}(50 - 63)$

$= 51 + \frac{4.5}{12}(-27)$

$= 51 - \frac{121.5}{12}$

$= 51 - 10.125$

$= 40.875$

Thus Mean deviation from median, $(MD_{Me}) = \frac{\Sigma f|M-Me|}{N}$

$Me = 36 + \frac{50-36}{26}(50-37)$

$= 36 + \frac{4.5}{26} \times 13$

$= 36 + \frac{58.5}{26}$

$= 36 + 2.25$

$= 38.25$

$= \frac{1060.5}{100}$

$= 10.605$

11.

X	f	M	cf	$\lvert M-Me \rvert$	$f\lvert M-Me \rvert$
20-25	6	22.5	6	20	120
25-30	12	27.5	18	15	180
30-40	17	35	35	7.5	127.5
40-45	30	42.5	65	0	0
45-50	10	47.5	75	5	50
50-55	10	52.5	85	10	100
55-60	8	57.5	93	15	120
60-70	5	65	98	22.5	112.5
70-80	2	75	100	32.5	65
					$\Sigma = 875$

Median class $\frac{N}{2} = 50^{th}$, (40-45)

Median, $l + \frac{\frac{N}{2} - cf}{f_1}(m-c)$

$= 40 + \frac{5}{30}(50-35)$

$= 40 + 2.5$

$= 42.5$

$MD_{me} = \frac{\Sigma f|M-Me|}{N}$

$= \frac{875}{100}$

$= 8.75$

Coefficient of $MD_{me} = \frac{8.75}{42.5} = 0.205$

Co-relation

In a bivariant distribution if there is any relationship between the two variables, then we can study the co-relation between the variables.

For ex:- ① Height, weight

② Family income and expenditure of luxury items

③ Price & supply of a commodity

④ Price & demand " "

⑤ sales of woolen items & the season

There are two types of co-relation

Positive correlation

Negative correlation

The correlation coefficient is defined as 'r' and the range of lies in between $-1 \leq r \leq 1$

when $r = 1$, then it is positively correlated

when $r = -1$ then it is negatively correlated

(if both the variables going in same direction then it means +ve correlation)

where $r = 0$, then there is correlation.

Positive Correlation

when both the variable increased or decreased in same direction, then we can say that there is a positive correlation

Negative correlation

When both the variable increased or decreased in different direction, then there is negative correlation.

Formula for 'r'

$$r = \frac{\Sigma dx\,dy}{\sqrt{\Sigma dx^2 \cdot \Sigma dy^2}}$$

where $dx = x - \bar{x}$

$dy = y - \bar{y}$

This r is known as Karl Pearson's co-efficient of correlation.

Q. calculate the 'r' between expenditure on advertisement and sales from the following data (6)

$r = \frac{\Sigma dxdy}{\sqrt{\Sigma dx^2 \Sigma dy^2}}$ $dx = x - \bar{x}$ $dy = y - \bar{y}$

expense on adv.	sales	dx $x - \bar{x}$	dy $y - \bar{y}$	$dxdy$	dx^2	dy^2
39	47	-26	-19	494	676	361
65	53	0	-13	0	0	169
62	58	-3	-8	24	9	64
90	86	25	20	500	625	400
82	62	17	-4	-68	289	16
75	68	10	2	20	100	4
25	60	-40	-6	240	1600	36
98	91	33	25	825	1089	625
36	51	-29	-15	435	841	225
78	84	13	18	234	169	324
		$\Sigma = 0$	$\Sigma = 0$	2704	5398	2224

Here Mean of $\bar{x} = \frac{650}{10} = 65$

Mean of $\bar{y} = \frac{660}{10} = 66$

$$r = \frac{\Sigma dxdy}{\sqrt{\Sigma dx^2 . \Sigma dy^2}} = \frac{2704}{\sqrt{5398 \times 2224}} = \frac{2704}{\sqrt{12005152}}$$

$$= \frac{2704}{3464.84}$$

$$r = 0.78$$

So there is a fairly +ve correlation between expenditure on adv. and sales.

Therefore sales increased with an increase in the advertisement expenses

Q: Calculate the ~~coef~~ 'r' for ages of husbands and wives

Husband X	Wives	$x-\bar{x}$	$y-\bar{y}$	$dxdy$	dx^2	dy^2
23	18	-8.1	-7.7	62.37	65.61	59.29
27	22	-4.1	-3.7	15.17	16.81	13.69
28	23	-3.1	-2.7	8.37	9.61	7.29
29	24	-2.1	-1.7	3.57	4.41	2.89
30	25	-1.1	-0.7	0.77	1.21	0.49
31	26	-0.1	0.3	0.03	0.01	0.09
33	28	1.9	2.3	4.37	3.61	5.29
35	29	3.9	3.3	12.87	15.21	10.89
36	30	4.9	4.3	21.07	24.01	18.49
39	32	7.9	6.3	49.77	62.41	39.69
				178.59	202.89	168.1

$$\bar{X} = \frac{311}{10} = 31.1$$

$$\bar{Y} = \frac{257}{10} = 25.7$$

$$r = \frac{\Sigma dxdy}{\sqrt{\Sigma dx^2 \cdot \Sigma dy^2}}$$

$$= \frac{178.59}{\sqrt{32076.0}}$$

$$= \frac{178.59}{179.10}$$

$$= 0.99$$

Here $r = 0.9$, so there is a strongly positive correlation between the ages of husbands and wives.

Find the co-relation coefficient

x	y	dx $x-\bar{x}$	dy $y-\bar{y}$	$dxdy$	dx^2	dy^2
45	25	-20	-19	380	400	361
48	32	-17	-12	204	289	144
50	35	-15	-9	135	225	81
55	30	-10	-14	140	100	196
65	42	0	-2	0	0	4
70	50	5	6	30	25	36
75	45	10	1	10	100	1
72	55	7	11	77	49	121
80	60	15	16	240	125	256
85	65	20	21	420	400	441
				1636	1813	1641

$\bar{x} = \frac{645}{10} = 64.5 = 65$

$\bar{y} = \frac{439}{10} = 43.9 = 44$

$r = \frac{\Sigma dxdy}{\sqrt{\Sigma dx^2 \cdot \Sigma dy^2}}$

$= \frac{1636}{\sqrt{2646933}}$

$= \frac{1636}{1613.4}$

$= 1.01$

$\frac{1636}{\sqrt{2975133}}$

2975133

1724.8

$\frac{1636}{\sqrt{2646933}}$

$\frac{1636}{1626.9}$

$= 0.94$

$= \frac{1636}{1724.8}$

$= 0.93$

Here $r = 0.94$

Q. 13

x	y	dx	dy	dxdy	dx^2	dy^2
6	10	-12	-9	108	144	81
8	12	-10	-7	70	100	49
12	15	-6	-4	24	36	16
15	15	-3	-4	12	9	16
18	18	0	1	0	0	1
20	25	2	6	12	4	36
24	22	6	3	18	36	9
28	26	10	7	70	100	49
31	28	13	9	117	169	81
168	171			431	598	338

$\bar{X} = \frac{162}{9} = 18$

$\bar{y} = \frac{171}{9} = 19$

So $r = \frac{\Sigma dx.dy}{\sqrt{\Sigma dx^2 . \Sigma dy^2}}$

$= \frac{431}{\sqrt{202124}} = \frac{431}{449.5} = 0.95$

Q.14

x	y	dx	dy	dxdy	dx^2	dy^2
80	45	17	-13	-221	289	169
60	71	-3	13	-39	9	169
51	60	-12	2	-24	144	4
69	57	6	-1	-6	36	1
58	62	-5	4	-20	25	16
62	58	-1	0	0	1	0
64	48	1	-10	-10	1	100
72	50	9	-8	-72	81	64
56	62	-7	4	-28	49	16
58	69	-5	11	-55	25	121
630	582			-475	660	660

Mean, $\bar{X} = \frac{630}{10} = 63$

Mean, $\bar{y} = \frac{582}{10} = 58.2 \simeq 58$

So $r = \frac{\Sigma dxdy}{\sqrt{\Sigma dx^2 . \Sigma dy^2}} = \frac{-475}{\sqrt{435600}}$

$= \frac{-475}{660}$

$= -0.65$

Q.3

X_1	X_2	dx	dy	$dxdy$	dx^2	dy^2
10	9	0	-1	0	0	4
6	4	-4	-7	28	16	49
9	6	-1	-5	5	1	25
10	9	0	-2	0	0	4
12	11	2	0	0	4	0
13	13	3	2	6	9	4
11	8	1	-3	-3	1	9
9	4	-1	-7	7	1	49
				$\Sigma = 43$	$\Sigma = 32$	$\Sigma = 144$

Here $\bar{X}_1 = \frac{80}{8} = 10$

$\bar{X}_2 = \frac{64}{8} = 8$

Thus $r = \frac{\Sigma dx\,dy}{\sqrt{\Sigma dx^2 . \Sigma dy^2}}$

$= \frac{43}{\sqrt{4608}}$

$= \frac{43}{67.88}$

$=$

X	Y
10	
9	

Q_1 and Q_3 of an individual series is calculated as $\frac{N+1}{4}$ and $3\left(\frac{N+1}{4}\right)^{th}$ term of the series.

X
2
4
7
8
9
10
N=6

$Q_1 = \left(\frac{N+1}{4}\right)^{th}$

$= \left(\frac{6+1}{4}\right)$

$= \left(\frac{7}{4}\right)^{th}$ term

$= 1.7^{th}$ term

= 1st term + .75(2nd − 1st)

= 2 + .75(4−2)

= 2 + 1.5

= 3.5

$Q_3 = 3\left(\frac{N+1}{4}\right)$

$= 3 \times 1.75$

$= 5.25$

* Relation between standard deviation, MD, QD and range.

$\Rightarrow$ 4SD = 5MD = 6QD

and SD = ~~$\frac{1}{4}$ range~~ $\frac{1}{6}$ range

Q. Find Mean of data is 25.6 & coefficient of variance 18.75. Find the SD

$\rightarrow \bar{X} = 25.6$, CV = 18.75

$CV = \frac{\sigma}{X} \times 100$

$\Rightarrow 18.75 = \frac{\sigma}{25.6} \times 100$

$\Rightarrow 480 = 100\sigma$

$\Rightarrow \sigma = \frac{480}{100} = 4.8$

Q Quartile of distribution 47.3 & 52.7

MD and range

$Q_1 = 47.3$, $Q_3 = 52.7$

$QD = \frac{52.7 - 47.3}{2} = \frac{5.4}{2} = 2.7$

$6QD = 5MD$

$MD = \frac{6QD}{5} = \frac{16.2}{5} = 3.24$

Now

$4SD = 5MD$

$SD = \frac{5MD}{4} = \frac{16.2}{4} = 4.05$

So range $= 6SD$

$= 6 \times 4.05$

$= 24.3$

X	Y	dx	dy	dxdy	dx^2	dy^2
10	9	0	-1	0	0	1
6	4	-4	-4	16	16	16
9	6	-1	-2	2	1	4
10	9	0	1	0	0	1
12	11	2	3	6	4	9
13	13	3	5	15	9	25
11	8	1	0	0	1	0
9	4	-1	-4	4	1	16
80	64			43	32	72

$\rightarrow \bar{X} = \frac{80}{8} = 10$

$\bar{Y} = \frac{64}{8} = 8$

$r = \frac{43}{\sqrt{32 \times 72}} = \frac{43}{\sqrt{2304}} = \frac{43}{48}$

$= 0.89$

Q Find r' using Karl where $\bar{x}=6, \bar{y}=8$

x	y	dx	dy	dxdy	dx^2	dy^2
6	9	0	1	0	0	1
2	11	-4	3	-12	16	9
10	?5	4	a-8(-3)	4a-32	16	$(a-8)^2 9$
4	8	-2	0	0	4	0
8	7	2	-1	-2	4	1
35+x				4a-56	40	$a^2+64+16+1$
				-26		20

Here $\bar{x}=\frac{30}{5}=6$

$\bar{y}=8$

$dx=x-\bar{x}$

$dy=y-\bar{y}$ So $r=\frac{4a-56}{40\times(a^2+81)}$

$\frac{35+x}{5}=8$

$\Rightarrow 35+x=40$

$\Rightarrow x=5$

$= \frac{4a-56}{\sqrt{40a^2+3240}}$

$= \frac{20-56}{\sqrt{40\times25+3240}}$

$= \frac{-26}{\sqrt{1000+3240}} = \frac{-26}{\sqrt{800}}$

$= \frac{-26}{\sqrt{4240}} = \frac{-26}{\sqrt{28.28}}$

$= \frac{-26}{65.11} = -0.91$

Q.

Karl Pearson co-efficient also defined as $\frac{\text{co-variance of } xy}{\sigma x \cdot \sigma y}$

$$r = \frac{cov(x,y)}{\sigma x \cdot \sigma y}$$

$$= \frac{\Sigma dx.dy}{\sqrt{\Sigma dx^2 \cdot \Sigma dy^2}}$$

of x, y

Co-variance is defined as the relationship between two variable x and y and mathematically

$$cov(x,y) = \frac{\Sigma(x-\bar{x})\,\Sigma(y-\bar{y})}{n}$$

$$\sigma x = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n}}$$

$$r = \frac{cov(x,y)}{\sigma x \cdot \sigma y} = \frac{\Sigma(x-\bar{x})(y-\bar{y})}{n} \times \sqrt{\frac{n}{\Sigma(x-\bar{x})^2}} \sqrt{\frac{n}{\Sigma(y-\bar{y})^2}}$$

$$= \frac{\Sigma dx \cdot \Sigma dy}{n} \times \frac{n^2}{\sqrt{\Sigma(x-\bar{x})^2 \cdot \Sigma(y-\bar{y})^2}}$$

$$= \frac{\Sigma dx.dy}{\sqrt{\Sigma dx^2 \cdot \Sigma dy^2}}$$

Calculate r from the following series, in x, n = 15

(X)

	X	Y
no. of obs.	15	15
AM	25	18
S.D	3.01	3.03
$\Sigma(x-\bar{x})^2$ / $\Sigma(y-\bar{y})^2$	136	138
$\Sigma(x-\bar{x})(y-\bar{y})$	122	

$$r = \frac{\Sigma dx.dy}{\sqrt{\Sigma dx^2 . \Sigma dy^2}}$$

$$= \frac{122}{\sqrt{136 \times 138}}$$

$$= \frac{122}{\sqrt{18768}}$$

$$= \frac{122}{136.99}$$

$$r = 0.89$$

Here $\frac{\Sigma dx.dy}{\Sigma (x-\bar{x})^2 . \sigma x}$

$$Cov(x,y) = \frac{\Sigma (x-\bar{x})(y-\bar{y})}{n}$$

$$= \frac{122}{15} = 8.13$$

$$\frac{Cov(x,y)}{\sigma x . \sigma y} = \frac{122}{15}$$

$$= \frac{0.89}{3.01 \times \sigma y} = \frac{122}{15}$$

$$\Rightarrow (0.89) \times 15 = 122 (0.31 \times \sigma y)$$

$$\Rightarrow 13.35 = 367.22 \sigma y$$

$$\Rightarrow \sigma y = \frac{1335}{367.22}$$

$$\frac{Cov(x,y)}{\sigma x . \sigma y} = \frac{\Sigma (x-\bar{x})(y-\bar{y})}{\Sigma (x-\bar{x})^2 . \Sigma (y-\bar{y})^2}$$

$$r = \frac{Cov(x,y)}{\sigma x . \sigma y}$$

$$\& \; Cov(x,y) = \frac{\Sigma (x-\bar{x})(y-\bar{y})}{n} = \frac{122}{15} = 8.13$$

$$=$$

Now $r = \frac{8.13}{\sigma x . \sigma y}$

$$\Rightarrow 0.89 = \frac{8.13}{3.01 \sigma y}$$

$$\Rightarrow (0.89)(3.01)\sigma y = 8.13$$

~~Cov(x,y)~~ $\sigma^2(x,y)$ $\sigma x = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n}}$

$r = 0.48$ $\sigma^2 = c'$ σx^2

$Cov(x,y) = 36.$

$\sigma x^2 = 16 \Rightarrow \sigma x = 4.$

$\sigma x =$

$$\frac{Cov(x,y)}{\sigma x \cdot \sigma y} = \frac{36}{4 \times \sigma y} = 0.48$$

$$\Rightarrow 36 = 0.48 \times 4\sigma y$$

$$\Rightarrow 1.92\sigma y = 36$$

$$\Rightarrow \sigma y = \frac{36}{1.92} = 18.75$$

Q.

$r = 0.8$

$\Sigma xy = 16$

$\sigma y = 2.5$

$\Sigma x^2 = 90$

where x and y are the deviation from their respective mean, find n,

Q. ~~Cov beta~~ $Cov(x,y) = 96$

$\sigma^2 x : \sigma^2 y = 4 : 9$

Find σx & σy

Q. $Cov(x,y) = 144$

Q.1

$r = 0.8$

$\Sigma xy = \Sigma(X-\bar{X})(Y-\bar{Y}) = 16$

$\sigma_y = 2.5$

$\Sigma x^2 = \Sigma(X-\bar{X})^2 = 90$

Thus $r = \dfrac{\Sigma(X-\bar{X})(Y-\bar{Y})}{}$

As $\Sigma x^2 = 90$

$\Rightarrow \Sigma(X-\bar{X})^2$

Correlation coefficient also defined as $r = \frac{n\Sigma xy - \Sigma x \cdot \Sigma y}{\sqrt{\{n\Sigma x^2 - (\Sigma x)^2\}}}$

$$r = \frac{n\Sigma xy - \Sigma x \cdot \Sigma y}{\sqrt{n\cdot\Sigma x^2 - (\Sigma x)^2 (n\Sigma y^2 - (\Sigma y)^2)}}$$

The merit of this formula is if $\bar{x}$ & $\bar{y}$ both will come in the form of decimal, then we can use this formula.

	x	y	xy	x^2	y^2
	6	9	54	36	81
	2	11	22	4	121
	10	5	50	100	25
	4	8	32	16	64
	8	7	56	64	49
			214	220	340

$\bar{x} = \frac{30}{5} = 6$

$\bar{y} = \frac{40}{5} = 8$

$$r = \frac{n\cdot\Sigma xy - \Sigma x\cdot\Sigma y}{\sqrt{(5\times220 - 30^2)(5\times340 - 40^2)}}$$

$$= \frac{5\cdot214 - 30\times40}{\sqrt{(5\times220 - 30^2)(5\times340 - 40^2)}}$$

$$= \frac{1070 - 1200}{\sqrt{(1100 - 900)(1700 - 1600)}}$$

$$= \frac{-130}{\sqrt{200\times100}}$$

$$= \frac{-130}{\sqrt{20,000}} = \frac{-130}{141} = -0.92$$

Q

x	y	xy	x^2	y^2
1	1	1	1	1
2	4	8	4	16
3	9	27	9	81
4	16	64	16	256
10	30	100	30	354

$$r = \frac{4 \times 100 - 300}{\sqrt{(4 \times 30 - 100)(4 \times 354 - 900)}}$$

$$= \frac{400 - 300}{\sqrt{20 \times 1416 - 900}}$$

$$= \frac{100}{\sqrt{26520}} \quad \frac{100}{10320}$$

$$= \frac{100}{162.84} \quad \frac{100}{101.58}$$

$$= 0.98$$

$$r = \frac{n\Sigma uv - \Sigma u . \Sigma v}{\sqrt{n.\Sigma u^2 - (\Sigma u)^2 (n\Sigma v^2 - (\Sigma v)^2}}$$

$$u = \frac{x - A}{h}$$

$$v = \frac{y - A}{h}$$

Continuous Series

x	y		v	uv	u^2	v^2
17.5	75	-2	5	-10	4	25
22.5	60	-1	2	-2	1	4
27.5	50	0	0	0	0	0
32.5	50	1	0	0	1	0
37.5	45	2	-1	-2	4	1
42.5	40	3	-2	-6	9	4
		3		-20	19	34

$$r = \frac{6 \times (-20) - 3 \times 4}{\sqrt{(6 \times 19 - 9)(6 \times 34 - 16)}} = \frac{-120 - 12}{\sqrt{(114 - 9)(204 - 16)}} = \frac{-132}{\sqrt{105 \times 188}}$$

$$= \frac{-132}{\sqrt{19740}} = \frac{-132}{140.4} = -0.94$$

X	Y	XY	X^2	Y^2
2	18	36	4	324
4	12	48	16	144
5	10	50	25	100
6	8	48	36	64
8	7	56	64	49
11	5	55	121	25
36	60	293	266	706

$r = \frac{6 \times 293 - 36 \times 60}{\sqrt{(6 \times 266 - (36)^2)(6 \times 706 - 60^2)}}$

$= \frac{1758 - 2160}{\sqrt{(1596 - 1225) \times (4236 - 3600)}}$

$= \frac{-402}{\sqrt{371 \times 636}}$

$= \frac{-342}{\sqrt{235,956}}$

$= \frac{-342}{485.75}$

$\frac{-402}{\sqrt{190,800}} = \frac{-402}{436.8}$

$= -0.92$

2. calculate 'r' from the following data

$n = 10$, $\Sigma x = 140$, $\Sigma y = 150$, $\Sigma (x-10)^2 = 180$ & $\Sigma (y-15)^2 = 215$

$\Sigma (x-10)(y-15) = 60$

Here $r = \frac{cov(x,y)}{\sigma x \cdot \sigma y} = \frac{6}{4.24 \times 4.63} = 0.30$

$cov(xy) = \frac{\Sigma (x-10)(y-15)}{10} = \frac{60}{10} = 6$

$\sigma x = \sqrt{\frac{\Sigma (x-10)^2}{10}} = \sqrt{\frac{180}{10}} = \sqrt{18} = 4.24$

$\sigma y = \sqrt{\frac{\Sigma (y-15)^2}{n}} = \sqrt{\frac{215}{10}} = \sqrt{21.5} = 4.63$

Q. calculate r, use uv method.

X	Y	$U = X - A$	$V = Y - A$	U^2	V^2	UV
23	18	-7	-7	49	49	49
27	22	-3	-3	9	9	9
28	23	-2	-2	4	4	4
29	24	-1	-1	1	1	1
(30)	(25)	0	0	0	0	0
31		1	0	1		1
33	26	3	1	9	1	9
35	28	5	3	25	9	
36	29	6	4	36	16	20
39	30	9	5	81	25	30
311	32	11	7	215	163	63
	257		7			186

$$r = \frac{n \cdot \Sigma UV - \Sigma U \cdot \Sigma V}{\sqrt{(n\Sigma U^2 - (\Sigma U)^2)\,[n\Sigma V^2 - (\Sigma V)^2]}}$$

$$= \frac{10 \times 186 - 11 \times 7}{\sqrt{(10 \times 215 - 121)\,(10 \times 163 - 49)}}$$

$$= \frac{1860 - 77}{\sqrt{(2150 - 121)\,(1630 - 49)}}$$

$$= \frac{1783}{\sqrt{2029 \times 1581}}$$

$$= \frac{1783}{\sqrt{3207849}}$$

$$= \frac{1783}{1791.04}$$

$$\approx 0.99$$

What do you mean by correlation? What are the diff types of co-relation. (8)
write do two uses of correlation related to biz using suitable examples.

Q.2

X	Y	XY	U	V	UV	U^2	V^2
50	11	550	-15	-5	75	225	25
50	13	650	-15	-3	45	225	9
55	14	770	-10	-2	20	100	4
60	16	960	-5	0	0	25	0
(65)	(16)	1040	0	0	0	0	0
65	15		0	-1	0	0	1
65	15		0	-1	0	0	1
60	14		-5	-2	10	25	4
60	13		-5	-3	15	25	9
50	13		-15	-3	45	225	9
			-70	-20	210	850	62

$$r = \frac{n\Sigma uv - \Sigma u \cdot \Sigma v}{\sqrt{n\Sigma u^2 - (\Sigma u)^2 \times n\Sigma v^2 - (\Sigma v)^2}}$$

$$= \frac{10 \times 210 - (-70 \times -20)}{\sqrt{(10 \times 850 - 4900) \times (10 \times 62 - 400)}}$$

$$= \frac{2100 - 1400}{\sqrt{(8500 - 4900)(620 - 400)}}$$

$$= \frac{700}{\sqrt{3600 \times 220}}$$

$$= \frac{700}{\sqrt{792,000}}$$

$$= \frac{700}{889.94}$$

$$= 0.78$$

Q.7

x	y	d(x-y)	d^2
1	6	-5	25
6	8	-2	4
3	3	0	0
9	7	2	4
5	2	3	9
2	1	1	1
7	5	2	4
10	9	1	1
8	4	4	16
4	10	-6	36
			$\Sigma = 100$

Thus $\rho = 1 - \frac{6\Sigma d^2}{n(n^2-1)}$

$= 1 - \frac{6 \times 100}{10 \times 99}$ (10, 33)

$= 1 - \frac{20}{33}$

$= 1 - 0.6060$

$= 0.394$

Q.8

x	y	d	d^2
1	2	-1	1
2	10	-8	64
3	3	0	0
4	4	0	0
5	5	0	0
6	7	-1	1
7	2	5	25
8	6	2	4
9	8	1	1
10	11	-1	1
11	15	-4	16
12	9	3	9
13	14	-1	1
14	12	2	4
15	16	-1	1
16	13	3	9
			$\Sigma = 136$

Thus $\rho = 1 - \frac{6\Sigma d^2}{n(n^2-1)}$

$= 1 - \frac{6 \times 136}{16 \times 255}$

$= 1 - \frac{816}{4080}$

$= 1 - 0.2$

$= 0.8$

Q.9 (18)

x	y	d	d^2
1	12	-11	121
2	9	-7	49
3	6	-3	9
4	10	-6	36
5	3	2	4
6	5	1	1
7	4	-3	9
8	7	1	1
9	8	1	1
10	2	8	64
11	11	0	0
12	1	11	121
			$\Sigma = 416$

Thus $\rho = 1 - \frac{6\Sigma d^2}{n(n^2-1)}$

$= 1 - \frac{6 \times 416}{12 \times 143}$

$= 1 - \frac{2496}{1716}$

$= 1 - 1.454$

$= -0.454$

∴ As $\rho = -0.454$, then this shows a moderate negative correlation between two judges rankings. Means one judge's rankings increase, the other's decrease and ~~vice-v~~ Vice-Versa.

Q.10

R_1	R_2	R_3	d_{12}	d_{12}^2	d_{23}	d_{23}^2	d_{13}	d_{13}^2
1	4	6	-3	9	-2	4	-5	25
5	8	7	-3	9	1	1	-2	4
4	7	8	-3	9	-1	1	-4	16
8	6	1	2	4	5	25	7	49
9	5	5	4	16	0	0	4	16
6	9	10	-3	9	-1	1	-4	16
10	10	9	0	0	1	1	1	1
7	3	2	4	16	1	1	5	25
3	2	3	1	1	-1	1	0	0
2	1	4	1	1	-3	9	-2	4
				$\Sigma = 74$		$\Sigma = 44$		$\Sigma = 156$

Hence $\rho_{12} = 1 - \frac{6\Sigma d_{12}^2}{n(n^2-1)} = 1 - \frac{6 \times 74}{10 \times 99} = 1 - \frac{444}{990} = 1 - 0.448 = 0.5515$

$\rho_{23} = 1 - \frac{6\Sigma d_{23}^2}{n(n^2-1)} = 1 - \frac{6 \times 44}{10 \times 99} = 1 - \frac{4}{15} = 1 - 0.26 = 0.73$

$\rho_{13} = 1 - \frac{6\Sigma d_{13}^2}{n(n^2-1)} = 1 - \frac{6 \times 156}{10 \times 99} = 1 - \frac{936}{990} = 1 - 0.945 = 0.054$

∴ 1st judge and 2nd judge has the nearest approach to beauty.

Q.11

Judge 'A'	Judge 'B'	Judge 'C'	d_{AB}	d_{AB}^2	d_{BC}	d_{BC}^2	d_{AC}	d_{AC}^2
6	5	4	1	1	1	1	2	4
5	8	9	-3	9	-1	1	-4	16
3	4	8	-1	1	-4	16	-5	25
10	7	1	3	9	6	36	9	81
2	10	2	-8	64	8	64	0	0
4	2	3	2	4	-1	1	1	1
9	1	10	8	64	-9	81	-1	1
7	6	5	1	1	1	1	2	4
8	9	7	-1	1	2	4	1	1
1	3	6	-2	4	-3	9	-5	25
				$\Sigma = 158$		$\Sigma = 214$		$\Sigma = 158$

So $\rho_{AB} = 1 - \frac{6\Sigma d_{AB}^2}{n(n^2-1)} = 1 - \frac{6 \times 158}{10 \times 99} = 1 - \frac{948}{990} = 1 - 0.957 = 0.042$

$\rho_{BC} = 1 - \frac{6\Sigma d_{BC}^2}{n(n^2-1)} = 1 - \frac{6 \times 214}{10 \times 99} = 1 - \frac{1284}{990} = 1 - 1.296 = -0.296$

$\rho_{AC} = 1 - \frac{6\Sigma d_{AC}^2}{n(n^2-1)} = 1 - \frac{6 \times 158}{10 \times 99} = 1 - \frac{948}{990} = 1 - 0.957 = 0.042$

∴ The judges (A & B) and (A & C) have the nearest approach to common tastes in beauty.

Q.12

Equity shares X	Preference shares Y	Rank of X	Rank of Y	d	d^2
97.5	75.1	4	6	-2	4
99.4	75.9	1	5	-4	16
98.6	77.1	2	3	-1	1
96.2	78.2	6	2	4	16
95.1	79	7	1	6	36
98.4	74.8	3	7	-4	16
97.1	76.2	5	4	1	1
					$\Sigma = 90$

Thus $\rho = 1 - \frac{6\Sigma d^2}{n(n^2-1)}$

$= 1 - \frac{6 \times 90}{7 \times 48} = 1 - \frac{540}{336} = 1 - 1.607 = -0.607$

Q.16

X	Y	U	V	UV	U^2	V^2
46	12	-1	-12	12	1	144
33	13	-12	-9	108	144	81
41	(24)	-4	0	0	16	0
38	16	-7	-8	56	49	64
36	15	-9	-9	81	81	81
(45)	14	0	-10	0	0	100
34	21	-11	-3	33	121	9
37	17	-8	-7	56	64	49
50	19	5	-5	-25	25	25
40	19	-5	-5	25	25	25
		-52	-68	346	526	578

$$r = \frac{n\Sigma UV - \Sigma U \cdot \Sigma V}{\sqrt{n\Sigma U^2 - (\Sigma U)^2} \times \sqrt{n\Sigma V^2 - (\Sigma V)^2}}$$

$$= \frac{10 \times 346 - (-52)\times(-68)}{\sqrt{(10 \times 526 - 2704) \times (10 \times 578 - 4624)}}$$

$$= \frac{3460 - 3536}{\sqrt{(5260 - 2704)(5780 - 4624)}}$$

$$= \frac{-76}{\sqrt{2556 \times 1156}}$$

$$= \frac{-76}{\sqrt{2954736}}$$

$$= \frac{-76}{1718.93}$$

$$= -0.0442$$

Q.17 → Given $\bar{X} = 20, \bar{Y} = 70$

X	Y	U	V	UV	U^2	V^2
14	84	-6	14	-84	36	196
16	78	-4	8	-32	16	64
17	70	-3	0	0	9	0
18	75	-2	5	-10	4	25
19	66	-1	-4	4	1	16
20	67	0	-3	0	0	9
21	62	1	-8	-8	1	64
22	58	2	-12	-24	4	144
23	60	3	-10	-30	9	100
		-10	-10	-184	80	618

$$r = \frac{n\Sigma UV - \Sigma U \cdot \Sigma V}{\sqrt{n\Sigma U^2 - (\Sigma U)^2 \times n\Sigma V^2 - (\Sigma V)^2}}$$

$$= \frac{9 \times (-184) - (-10)(-10)}{\sqrt{(9 \times 80 - 100)(9 \times 618 - 100)}}$$

$$= \frac{-1656 - 100}{\sqrt{(720 - 100)(5562 - 100)}}$$

$$= \frac{-1756}{\sqrt{620 \times 5462}}$$

$$= \frac{-1756}{\sqrt{3386440}}$$

$$= \frac{-1756}{1840.2}$$

$$= -0.954$$

Q.18

X	Y	U	V	UV	U^2	V^2
75	62	-1	2	-2	1	4
81	68	5	8	40	25	64
70	65	-6	5	-30	36	25
(76)	(60)	0	0	0	0	0
77	69	1	9	9	1	81
81	72	5	12	60	25	144
84	76	8	16	128	64	256
75	72	-1	12	-12	1	144
		11	64	193	153	718

$$r = \frac{n\Sigma UV - \Sigma U \cdot \Sigma V}{\sqrt{n\Sigma U^2 - (\Sigma U)^2} \times \sqrt{n\Sigma V^2 - (\Sigma V)^2}}$$

$$= \frac{8 \times 193 - 11 \times 64}{\sqrt{(8 \times 153 - 121) \times (8 \times 718 - 4096)}}$$

$$= \frac{1544 - 704}{\sqrt{(1224 - 121)(5744 - 4096)}}$$

$$= \frac{840}{\sqrt{1103 \times 1648}}$$

$$= \frac{840}{\sqrt{1817744}}$$

$$= \frac{840}{1348.23}$$

$$= 0.623$$

Q.19

X	Y	U	V	UV	U^2	V^2
100	98	1	6	6	1	36
101	99	2	7	14	4	49
103	99	4	7	28	16	49
102	97	3	5	15	9	25
100	95	1	3	3	1	9
(99)	(92)	0	0	0	0	0
97	95	-2	3	-6	4	9
98	94	-1	2	-2	1	4
96	90	-3	-2	6	9	4
95	91	-4	-1	4	16	1
		1	30	68	51	186
					61	

$$\Rightarrow r = \frac{n\Sigma UV - \Sigma U \cdot \Sigma V}{\sqrt{n\Sigma U^2 - (\Sigma U)^2} \times \sqrt{n\Sigma V^2 - (\Sigma V)^2}}$$

$$= \frac{10 \times 68 - 30}{\sqrt{(10 \times 51 - 1)(10 \times 186 - 900)}}$$

$$= \frac{680 - 30}{\sqrt{509 \times 960}}$$

$$= \frac{650}{\sqrt{488640}}$$

$$= \frac{650}{221.05} =$$

$$\frac{10 \times 68 - 30}{\sqrt{(609 \times 10 \times 186 - 900)}} = \frac{650}{}$$

$$= \frac{650}{\sqrt{609 \times 960}} = \frac{650}{\sqrt{584640}}$$

$$= \frac{650}{764.61}$$

$$= 0.850$$

Now, $N = 10$ & $\Sigma X = 991$, $\Sigma Y = 950$

So $\bar{X} = \frac{991}{10} = 99.1$

$\bar{y} = \frac{950}{10} = 95$

Thus,

dx	dy	dxdy	dx^2	dy^2
0.9	3	2.7	0.81	9
1.9	4	7.6	3.61	16
3.9	4	5.6	15.21	16
2.9	2	5.8	8.41	4
0.9	0	0	0.81	0
-0.1	-3	0.3	0.01	9
-2.1	0	0	4.41	0
-1.1	-1	1.1	1.21	1
-3.1	-5	15.5	9.61	25
-4.1	-4	16.6	16.81	16
		$\Sigma = 65.2$	$\Sigma = 60.9$	$\Sigma = 96$

Thus $r = \frac{\Sigma dxdy}{\sqrt{\Sigma dx^2 \times \Sigma dy^2}}$

$$= \frac{65.2}{\sqrt{60.9 \times 96}}$$

$$= \frac{65.2}{\sqrt{5846.4}}$$

$$= \frac{65.2}{76.46} = 0.852$$

Q.20

X	Y	$U = \frac{X-120}{10}$	$V = Y-19$	UV	U^2	V^2
80	15	-4	-4	16	16	16
90	15	-3	-4	12	9	16
100	16	-2	-3	6	4	9
110	19	-1	0	0	1	0
120	17	0	-2	0	0	4
130	18	1	-1	1	1	1
140	16	2	-3	-6	4	9
150	18	3	-1	-3	9	1
160	19	4	0	0	16	0
		0	-18	26	60	56

$$r = \frac{n\Sigma UV - \Sigma U \cdot \Sigma V}{\sqrt{n\Sigma U^2 - (\Sigma U)^2 \times n\Sigma V^2 - (\Sigma V)^2}}$$

$$= \frac{9 \times 26 - 0}{\sqrt{(9 \times 60 - 0)(9 \times 56 - 324)}}$$

$$= \frac{234}{\sqrt{540 \times 180}}$$

$$= \frac{234}{\sqrt{97200}} = \frac{234}{311.77} = 0.750$$

Q.21

x	y	dx	dy	dxdy	dx^2	dy^2
13	54	4	-8	-32	16	64
12	53	3	-9	-27	9	81
9	57	0	-5	0	0	25
9	61	0	-1	0	0	1
8	67	-1	5	-5	1	25
6	72	-3	10	-30	9	100
6	70	-3	8	-24	9	64
		0	0	-118	44	360

Here $N = 7$, $\Sigma X = 63$, $\Sigma Y = 434$

So $\bar{x} = 9$, $\bar{y} = \frac{434}{7} = 62$

$$\gamma = \frac{\Sigma dx\,dy}{\sqrt{\Sigma dx^2 \Sigma dy^2}}$$

Q.21

X	Y	U	V	UV	U^2	V^2
13	54	4	-3	-12	16	9
12	53	3	-4	-12	9	16
(9)	(57)	0	0	0	0	0
9	61	0	4	0	0	16
8	67	-1	10	-10	1	100
6	72	-3	15	-45	9	225
6	70	-3	13	-39	9	169
		0	35	-118	44	535

$$r = \frac{n\Sigma UV - \Sigma U \cdot \Sigma V}{\sqrt{n\Sigma U^2 - (\Sigma U)^2 \times n\Sigma V^2 - (\Sigma V)^2}}$$

$$= \frac{7 \times (-118) - 0}{\sqrt{(7 \times 44 - 0)(7 \times 535 - 1225)}} = \frac{-826}{\sqrt{308 \times 2520}} = \frac{-826}{880.9} = -0.937$$

Q.22

X	Y	U	V	UV	U^2	V^2
23	18	-5	-9	45	25	81
27	10	-1	-17	17	1	289
28	22	0	-5	0	0	25
(28)	(27)	0	0	0	0	0
28	21	0	-6	0	0	36
30	29	2	2	4	4	4
30	27	2	0	0	4	0
33	29	5	2	10	25	4
35	28	7	1	7	49	1
38	29	10	2	20	100	4
		20	-30	103	208	444

$$r = \frac{n\Sigma UV - \Sigma U \cdot \Sigma V}{\sqrt{n\Sigma U^2 - (\Sigma U)^2 \times n\Sigma V^2 - (\Sigma V)^2}}$$

$$= \frac{10 \times 103 + 600}{\sqrt{(10 \times 208 - 400)(10 \times 444 - 900)}}$$

$$= \frac{1030 + 600}{\sqrt{1680 \times 3540}}$$

$$= \frac{1630}{\sqrt{5947200}}$$

$$= \frac{1630}{2438.6} = 0.66$$

Q.22

X	Y	$x-\bar{x}$	$y-\bar{y}$	$dx\,dy$	$(x-\bar{x})^2$	$(y-\bar{y})^2$
23	18	-7.22	-7.44	53.71	52.12	55.35
27	20	-3.22	-5.44	17.51	10.36	29.59
28	22	-2.22	-3.44	7.63	4.92	11.83
29	27	-1.22	1.56	-1.90	1.48	2.43
30	29	-0.22	3.56	-0.78	0.04	12.67
31	27	0.78	1.56	1.21	0.60	2.43
33	29	2.78	3.56	9.89	7.72	12.67
35	28	4.78	2.56	12.23	22.84	6.55
36	29	5.78	3.56	20.57	33.40	12.67
272	229			120.07	133.48	$\Sigma = 146.19$

Here $N = 9$, $\Sigma X = 272$, $\Sigma Y = 229$

Then $\bar{X} = \frac{272}{9} = 30.22$

$\bar{Y} = \frac{229}{9} = 25.44$

Now $r = \frac{\Sigma dx.dy}{\sqrt{\Sigma dx^2 . \Sigma dy^2}} = \frac{120.17}{\sqrt{133.48 \times 146.19}} = \frac{120.17}{\sqrt{19513.44}} = \frac{120.17}{139.69} = 0.860$

Q.23 → Given $\bar{x} = 44$, $\bar{y} = 26$

X	Y	$U = X-44$
43	29	
44	31	
46	19	
40	18	
44	19	
42	27	
45	27	
42	29	
38	41	
40	30	
42	26	
57	10	

Q. Coefficient of range correlation bet^n 2 variables is 0.8 and if the sum of the Squares of difference is 33. The find n

$r = 0.8$

$d^2 = 33$

$n = ?$

$$r = 1 - \frac{6\Sigma d^2}{n(n^2-1)}$$

$$\Rightarrow 0.8 = 1 - \frac{6 \times 33}{n^3 - n} \qquad 1 - \frac{6 \times 33}{n(n^2-1)}$$

$$\Rightarrow (0.8)(n(n^2-1)) = -1$$

$$\Rightarrow 0.8 = 1 - \frac{198}{n(n^2-1)}$$

$$\Rightarrow 0.8 \times n(n^2-1) = n(n^2-1) - 198$$

$$\Rightarrow 0.8 \times n(n^2-1) - n(n^2-1) = 0 - 198$$

$$\Rightarrow n(n^2-1)[0.8 - 1) = -198$$

$$\Rightarrow n(n^2-1) = \frac{-198}{-0.2}$$

$$\Rightarrow n(n^2-1) = 990$$

$$\Rightarrow n(n+1)(n-1) = 990$$

$$\Rightarrow (n+1)(n^2 - n + 1) = 990$$

$$\Rightarrow n^3 - n - 990 = 0$$

$$\Rightarrow n^3 - 100n + 99n - 990 = 0$$

$$\Rightarrow n(n^2 - 100) + 99(n - 10) = 0$$

$$\Rightarrow n(n-10)(n+10) + 99(n-10) = 0$$

$$\Rightarrow (n-10)[n(n+10) + 99] = 0$$

$$\Rightarrow n - 10 = 0 \text{ or } n(n+10) + 99 = 0$$

$$\Rightarrow n = 10 \text{ or } n^2 + 10n + 99 = 0$$

$$n = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-10 \pm \sqrt{100 - 396}}{2}$$

$$= \frac{-10 \pm \sqrt{-296}}{2}$$

Q. $r = 0.143$, $\sum d^2 = 48$

$$r = 1 - \frac{6\sum d^2}{n(n^2-1)}$$

$$\Rightarrow 0.143 = 1 - \frac{6 \times 48}{n(n^2-1)}$$

$$\Rightarrow 0.143 = 1 - \frac{288}{n^3-n}$$

$$\Rightarrow 0.143(n^3-n) = (n^3-n) - 288$$

$$\Rightarrow 0.143(n^3-n) - (n^3-n) + 288 = 0$$

$$\Rightarrow n^3 - n(0.143 - 1) = -288$$

$$\Rightarrow n^3 - n = \frac{-288}{-0.857} = 336.05$$

$$\Rightarrow n(n^2-1) = 336.$$

$$\Rightarrow n^3 - n - 336 = 0$$

$$\Rightarrow n^3 - 49n + 48n - 336 = 0$$

$$\Rightarrow n(n^2-49) + 48(n-7) = 0$$

$$\Rightarrow n(n+7)(n-7) + 48(n-7) = 0$$

$$\Rightarrow (n-7)\left[n(n+7) + 48\right] = 0$$

$$\Rightarrow (n-7) = 0 \quad \text{or} \quad n(n+7) + 48 = 0$$

$$\Rightarrow n = 7$$

$$\Rightarrow n^2 + 7n + 48 = 0$$

$$\Rightarrow n = \frac{-7 \pm \sqrt{49 - 192}}{2}$$

$$= \frac{-7 \pm 11.95}{2}$$

$$= 7$$

As $n = 5, n^3 = 125$

$n = 6, n^3 = 216$

$n = 7, n^3 = 343$

Probable Error

Since we are finding the correlation between two variable based on sample of a population / data. So this shows that we are not taking the whole population / data. so there must be some error in our calculation and by the help of we can define a limit where the correlation coefficient lies, whatever sample will dreaw from the population, and that error is known as probable error.

$$P.E = \frac{2}{3}\left(\frac{1-r^2}{\sqrt{n}}\right) = 0.6745\left(\frac{1-r^2}{\sqrt{n}}\right)$$

and limit is defined as $r \pm P.E$

If $r = 0.36$ and $n = 10$, find out P.E & the range of the Correlation Coeffice

$$P.E = \frac{2}{3}\frac{1-r^2}{\sqrt{n}}$$

$$= \frac{2}{3}\frac{1-(0.36)^2}{\sqrt{10}}$$

$$= \frac{2}{3}\frac{1-0.1296}{3.16}$$

$$= \frac{2}{3} \times \frac{0.87}{3.16}$$

$$= 0.66 \times \frac{0.87}{3.16}$$

$$= \frac{0.5742}{3.16} = 0.18$$

limit $= r \pm PE$

$= 0.36 + 0.18$ and $0.36 - 0.18$

$= 0.54$ and 0.18

This is called range of the population correlation coefficient.

Standard Error

$$\frac{1-r^2}{\sqrt{n}}$$

The value of r is significant or reliable if $r > 6 \times P.E$ and insignificant $r < P.E$

r is significant means the relationship bet^n both the variables are reliable or realistic or significant.

And the value of r is insignificant means, the relnship bet^n both the variable are not reliable.

Assignment

Q.31 (a)

Given $\Sigma X = 125$, $\Sigma Y = 100$

$\Sigma X^2 = 650$, $\Sigma Y^2 = 436$

$\Sigma XY = 520$, $n = 25$

Thus $r(X,Y) = \dfrac{n\Sigma XY - \Sigma X \cdot \Sigma Y}{\sqrt{n\Sigma X^2 - (\Sigma X)^2 (n\Sigma Y^2 - (\Sigma Y)^2}}$

$= \dfrac{25 \times 520 - 125 \times 100}{\sqrt{25 \times 650 - (125)^2 \times (25 \times 436 - (100)^2)}}$

$= \dfrac{13000 - 12500}{\sqrt{(16250 - 15625)(10900 - 10000)}}$

$= \dfrac{500}{\sqrt{625 \times 900}} = \dfrac{500}{25 \times 30} = \dfrac{2}{3} = 0.67$

(b) Given $n = 9$, $\Sigma X = 45$, $\Sigma X^2 = 285$, $\Sigma Y = 108$, $\Sigma Y^2 = 1356$, $\Sigma XY = 597$

Hence $r(X,Y) = \dfrac{n\Sigma XY - \Sigma X \cdot \Sigma Y}{\sqrt{[n\Sigma X^2 - (\Sigma X)^2][n\Sigma Y^2 - (\Sigma Y)^2]}}$

$= \dfrac{9 \times 597 - 45 \times 108}{\sqrt{(9 \times 285 - 2025)(9 \times 1356 - 11664)}}$

$= \dfrac{5373 - 4860}{\sqrt{(2565 - 2025)(12204 - 11664)}}$

$= \dfrac{513}{\sqrt{540 \times 540}}$

$= \dfrac{513}{540}$

$= 0.95$

Q.32

Given $\bar{x} = 5.5$ $\bar{y} = 4$

$\Sigma x^2 = 385$, $\Sigma y^2 = 192$

$\Sigma (x+y)^2 = 947$, $n = 10$

→ As $\Sigma (x+y)^2 = 947$

$\Rightarrow \Sigma x^2 + \Sigma y^2 + 2\Sigma xy = 947$

$\Rightarrow 385 + 192 + 2\Sigma xy = 947$

$\Rightarrow 577 + 2\Sigma xy = 947$

$\Rightarrow 2\Sigma xy = 947 - 577$

$\Rightarrow \Sigma xy = \frac{370}{2} = 185$

~~Now $r(x,y) =$~~

Again, $n = 10$, $\bar{x} = 5.5$

as we know $\bar{x} = \frac{\Sigma x}{n}$

$\Rightarrow \Sigma x = n\bar{x}$

$= 10 \times 5.5$

$= 55$

Similarly, $\Sigma y = n\bar{y}$

$= 10 \times 4$

$= 40$

Hence $r(x,y) = \frac{n\Sigma xy - \Sigma x \Sigma y}{\sqrt{n\Sigma x^2 - (\Sigma x)^2}\ \sqrt{n\Sigma y^2 - (\Sigma y)^2}}$

$= \frac{10 \times 185 - 40 \times 55}{\sqrt{(10 \times 385 - 3025)(10 \times 192 - 1600)}}$

$= \frac{1850 - 2200}{\sqrt{(3850 - 3025)(1920 - 1600)}}$

$= \frac{-350}{\sqrt{825 \times 320}}$

$= \frac{-350}{\sqrt{264000}}$

$= \frac{-350}{513.8}$

$= -0.681$

Q.33

→ Given $N=30$, $\Sigma X=120$, $\Sigma X^2=600$, $\Sigma Y=90$, $\Sigma Y^2=250$, $\Sigma XY=356$

Two pairs of observation, which is (8,10) & (12,7) wrongly taken, while the ~~right~~ correct pair is (8,12) & (10,8).

→ As we know Correct sum = Wrong Sum − Wrongly taken Obs. + Correct Obs.

Thus we need to find out correct sum for all the data given.

Now

Correct $\Sigma X = 120 - 8 - 12 + 8 + 10 = 118$

Correct $\Sigma Y = 90 - 10 - 7 + 12 + 8 = 93$

Correct $\Sigma X^2 = 600 - 64 - 144 + 64 + 100 = 556$

Correct $\Sigma Y^2 = 250 - 100 - 49 + 144 + 64 = 309$

Correct $\Sigma XY = 356 - (8\times10) - (12\times7) + (8\times12) + (10\times8)$

$= 356 - 80 - 84 + 96 + 80$

$= 368$

Thus correct $r(X,Y) = \dfrac{n\Sigma XY - \Sigma X \Sigma Y}{\sqrt{n\Sigma X^2 - (\Sigma X)^2} \times n\Sigma Y^2 - (\Sigma Y)^2}$

$= \dfrac{30\times 368 - 118\times 93}{\left[(30\times 556 - (118)^2)(30\times 309 - (93)^2)\right]^{1/2}}$

$= \dfrac{11040 - 10974}{\left[(16680 - 13924)(9270 - 8649)\right]^{1/2}}$

$= \dfrac{66}{(2756 \times 621)^{1/2}}$

$= \dfrac{66}{(1711476)^{1/2}}$

$= \dfrac{66}{1308.2}$

$= 0.0504$

Q.34

Given $n = 20$, $r_{xy} = 0.3$

$\bar{X} = 15$, $\bar{Y} = 20$

$\sigma_x = 4$, $\sigma_y = 5$

At the time of calculations one pair $(x=27, y=30)$ was wrongly taken as $(x=17, y=35)$

Now $\Sigma X = n \cdot \bar{X} = 20 \times 15 = 300$

$\Sigma Y = n \cdot \bar{Y} = 20 \times 20 = 400$

Again, we know ~~$\sigma_x = \frac{\Sigma X - \bar{X}}{n}$~~

$\sigma_x^2 = \frac{\Sigma X^2}{n} - (\bar{X})^2$

$\Rightarrow 16 = \frac{\Sigma X^2}{20} - (15)^2$

$\Rightarrow \frac{\Sigma X^2}{20} = 16 + 225 = 241$

$\Rightarrow \Sigma X^2 = 4820$

~~$\sigma_x^2 = \frac{\Sigma X^2}{n} - (\bar{X})^2$~~

~~$\Rightarrow n\sigma_x^2 = \Sigma X^2 - (\bar{X})^2$~~

~~$\Rightarrow \Sigma X^2 = n\sigma_x^2 - (\bar{X})^2$~~

~~$\Rightarrow \Sigma X^2 = 20 \times 16 - 225$~~

~~$= 320 - 225$~~

~~$= 95$~~

Similarly $\Sigma Y^2 = n[(\bar{Y})^2 + \sigma_y^2]$

$= 20[25 + 400]$

$= 20 \times 425$

$= 8500$

~~Similarly $\Sigma Y^2 = n\sigma_y^2 - (\bar{Y})^2$~~

~~$= 20 \times 25 - 400$~~

~~$= 500 - 400$~~

~~$= 100$~~

~~$\Sigma(X-\bar{X})(Y-\bar{Y})$~~

~~n~~ $\Sigma XY =$

~~$\frac{\Sigma XY}{n} =$~~

Now we need to find ~~all~~ correct sum for all the data

Correct $\Sigma X = 300 - 17 + 27 =$ ~~290~~ 310

Correct $\Sigma Y = 400 - 35 + 30 =$ ~~405~~ 395

Correct $\Sigma X^2 =$ ~~485~~ $4820 - 17^2 + 27^2$

$= 4820 - 289 + 729 = 5260$

Correct $\Sigma Y^2 = 8500 - 35^2 + 30^2 = 8500 - 1225 + 900 = 8175$

Correct $\Sigma XY = 6120 - 17 \times 35 + 27 \times 30 = 120 - 595 + 810 = 6335$

Again $r = \frac{cov(x,y)}{\sigma_x \cdot \sigma_y}$

$\Rightarrow 0.3 = \frac{cov(x,y)}{4 \times 5} = \frac{cov\,xy}{20}$

$\Rightarrow cov(x,y) = 6$

$\Rightarrow \frac{\Sigma XY}{n} = 6120$

~~$\Rightarrow \Sigma XY = 6 \times 20 = 120$~~

~~$\Sigma(XY - \bar{X}Y - \bar{Y}X + \bar{X}\bar{Y})$~~

~~$\bar{X}\bar{Y}$~~

Hence $r(x,y) = \frac{n\Sigma XY - \Sigma X \Sigma Y}{\left[n\Sigma X^2 - (\Sigma X)^2 \times n\Sigma Y^2 - (\Sigma Y)^2\right]^{1/2}}$

$= \frac{20 \times 6335 - 310 \times 395}{\left\{[20 \times 5260 - (310)^2][20 \times 8175 - (395)^2]\right\}^{1/2}}$

$= \frac{126700 - 122450}{\left[(105200 - 96100)(163500 - 156025)\right]^{1/2}}$

$= \frac{4250}{(9100 \times 7475)^{1/2}}$

$= \frac{4250}{(68022500)^{1/2}}$

$= \frac{4250}{8247.57}$

$= 0.515$

Q.34

Given $n = 20$, $r(x,y) = 0.3$, $\bar{x} = 15$, $\bar{y} = 20$, $\sigma_x = 4$ & $\sigma_y = 5$

At the time of calculations one pair $(x=27, y=30)$ was wrongly taken as $(x=17, y=35)$

Now $\Sigma X = n\bar{x} = 20 \times 15 = 300$

$\Sigma Y = n \cdot \bar{y} = 20 \times 20 = 400$

Again, we know $\sigma_x^2 = \frac{\Sigma x^2}{n} - (\bar{x})^2$

$\Rightarrow 16 = \frac{\Sigma x^2}{20} - (15)^2$

$\Rightarrow 16 = \frac{\Sigma x^2}{20} - 225$

$\Rightarrow \Sigma x^2 = 20(16+225)$

$= 4820$

and $\sigma_y^2 = \frac{\Sigma y^2}{n} - (\bar{y})^2$

$\Rightarrow 25 = \frac{\Sigma y^2}{20} - 400$

$\Rightarrow \Sigma y^2 = 20(25+400)$

$= 8500$

And $r = 0.3$ (given)

$\Rightarrow \frac{cov(x,y)}{\sigma_x \sigma_y} = 0.3$

$\Rightarrow \frac{cov(x,y)}{4 \times 5} = 0.3$

$\Rightarrow cov(x,y) = 0.3 \times 20 = 6$

Again $cov(x,y) = 6$

$\Rightarrow \frac{\Sigma(x-\bar{x})(y-\bar{y})}{n} = 6$

$\Rightarrow \Sigma(xy - x\bar{y} - \bar{x}y + \bar{x}\bar{y}) = 120$

$\Rightarrow \Sigma xy - \bar{y}\Sigma x - \bar{x}\Sigma y + n\bar{x}\bar{y} = 120$

$\Rightarrow \Sigma xy = 20 \times 300 - 15 \times 400 + 20 \times 300 = 120$

$\Rightarrow \Sigma xy - 6000 - 6000 + 6000 = 120$

$\Rightarrow \Sigma xy = 6120$

Therefore the correct $r(x,y)$

$$= \frac{n\Sigma xy - \Sigma x \Sigma y}{\left[n\Sigma x^2 - (\Sigma x)^2 \times n\Sigma y^2 - (\Sigma y)^2\right]^{1/2}}$$

$$= \frac{20 \times 6335 - 310 \times 395}{\left\{\left[20 \times 5260 - (310)^2\right]\left[20 \times 8175 - (395)^2\right]\right\}^{1/2}}$$

$$= \frac{126700 - 122450}{\left[(105200 - 96100)(163500 - 156025)\right]^{1/2}}$$

$$= \frac{4250}{(9100 \times 7475)^{1/2}}$$

$$= \frac{4250}{(68022500)^{1/2}}$$

$$= \frac{4250}{8247.57} = 0.515$$

Now we need to find all the correct sum of the data.

Correct $\Sigma X = 300 - 17 + 27 = 310$

Correct $\Sigma Y = 400 - 35 + 30 = 395$

Correct $\Sigma x^2 = 4820 - 17^2 + 27^2 = 4820 - 289 + 729 = 5260$

Correct $\Sigma y^2 = 8500 - 35^2 + 30^2 = 8500 - 1225 + 900 = 8175$

Correct $\Sigma xy = 6120 - 17 \times 35 + 27 \times 30 = 6120 - 595 + 810 = 6335$

Q.35

Given $n = 12$, $\Sigma XY = 334$

$\Sigma X = 30$, $\Sigma Y = 5$

$\Sigma X^2 = 670$, $\Sigma Y^2 = 285$

A pair $(X=11, Y=4)$ was copied wrongly, correct pair is $(X=10, Y=14)$

→ We need to find out correct sum of all the data first.

Correct $\Sigma X = 30 - 11 + 10 = 29$

Correct $\Sigma Y = 5 - 4 + 14 = 15$

Correct $\Sigma X^2 = 670 - 121 + 100 = 649$

Correct $\Sigma Y^2 = 285 - 16 + 196 = 465$

Correct $\Sigma XY = 334 - 44 + 140 = 430$

Thus Correct $r(X,Y) = \dfrac{n\Sigma XY - \Sigma X \Sigma Y}{\left[n\Sigma X^2 - (\Sigma X)^2 \times n\Sigma Y^2 - (\Sigma Y)^2\right]^{1/2}}$

$= \dfrac{12 \times 430 - 15 \times 29}{\left[(12 \times 649 - 29^2) \times (12 \times 465 - 15^2)\right]^{1/2}}$

$= \dfrac{5160 - 435}{\left[(7788 - 841)(5580 - 225)\right]^{1/2}}$

$= \dfrac{4715}{(6947 \times 5355)^{1/2}}$

$= \dfrac{4715}{(37201185)^{1/2}}$

$= \dfrac{4715}{6099.27}$

$= 0.77$

Q.37

X	Y	U	V	UV	U^2	V^2
7	18	2	6	12	4	36
6	16	1	4	4	1	16
(5)	14	0	2	0	0	4
4	(12)	-1	0	0	1	0
3	10	-2	-2	4	4	4
2	6	-3	-6	18	9	36
1	8	-4	-4	16	16	16
		-7	0	54	35	112

Now $r(X,Y) = \dfrac{n\Sigma UV - \Sigma U.\Sigma V}{\left\{[n\Sigma U^2-(\Sigma U)^2][n\Sigma V^2-(\Sigma V)^2]\right\}^{1/2}}$

$= \dfrac{7\times 54 - 0}{[(7\times(-7)-49)(7\times 112-0)]^{1/2}}$

$= \dfrac{378}{(-98\times 784)^{1/2}}$

$=$

Q.37

X	Y	$x-\bar{x}$	$y-\bar{y}$	dxdy	$(x-\bar{x})^2$	$(y-\bar{y})^2$
7	18	3	6	18	9	36
6	16	2	4	8	4	16
5	14	1	2	2	1	4
4	12	0	0	0	0	0
3	10	-1	-2	2	1	4
2	6	-2	-6	12	4	36
1	8	-3	-4	12	9	16
				54	28	112

$\bar{X} = \dfrac{\Sigma X}{n} = \dfrac{28}{7} = 4$

$\bar{Y} = \dfrac{\Sigma Y}{n} = \dfrac{84}{7} = 12$

Thus $r = \dfrac{\Sigma dx.dy}{(\Sigma dx^2.\Sigma dy^2)^{1/2}}$

$= \dfrac{54}{(28\times 112)^{1/2}}$

$= \dfrac{54}{(3136)^{1/2}}$

$= \dfrac{54}{56}$

$= 0.964$

And P.E $= 0.6745\left(\dfrac{1-r^2}{\sqrt{n}}\right)$

$= 0.6745\times\dfrac{1-.93}{\sqrt{7}}$

$= \dfrac{0.6745\times 0.07}{2.645}$

$= \dfrac{0.047}{2.645} = 0.0178$

Q.38

X	Y	$x-\bar{x}$	$y-\bar{y}$	$dxdy$	$(x-\bar{x})^2$	$(y-\bar{y})^2$	R_x	R_y	$D=R_x-R_y$	D^2
15	$\frac{200}{250}\times100=80$	-2	40	-80	4	1600	6	1	5	25
16	$\frac{150}{200}\times100=75$	-1	35	-35	1	1225	5	2	3	9
(17)	$\frac{90}{150}\times100=60$	0	20	0	0	400	4	3	1	1
18	$\frac{48}{120}\times100=$ (40)	1	0	0	1	0	3	4	-1	1
19	$\frac{30}{100}\times100=30$	2	-10	-20	4	100	2	5	-3	9
20	$\frac{12}{80}\times100=15$	3	-25	-75	9	625	1	6	-5	25
		3	-60	-210	19	3950				$\Sigma D^2=70$

$$r=\frac{\Sigma dxdy}{\sqrt{\Sigma dx^2.\Sigma dy^2}}=\frac{n\Sigma UV-\Sigma U.\Sigma V}{\left[n\Sigma U^2-(\Sigma U)^2 \times n\Sigma V^2-(\Sigma V)^2\right]^{1/2}}$$

$$=\frac{-210}{\sqrt{19\times3950}}=\frac{6\times(-210)-180}{\left[(6\times19-9)(6\times3950-3600)\right]^{1/2}}$$

$$=\frac{-210}{\sqrt{75050}}=\frac{-1260-180}{(105-20100)^{1/2}}=\frac{-1440}{}$$

$$=\frac{-210}{273.95}$$

$$=$$

$$R_{xy}=1-\frac{6\Sigma D^2}{6(6^2-1)}$$

$$=1-\frac{420}{210}$$

$$=1-2=(-1)$$

$$P.E=0.6745\left(\frac{1+1}{\sqrt{n}}\right)$$

$$=0.6745\times\frac{2}{}$$

Q.39

X	Y	U	V	UV	U^2	V^2
50	45	3	5	15	9	25
60	52	13	12	156	169	144
75	50	28	10	280	784	100
84	65	37	25	925	1369	625
(47)	(40)	0	0	0	0	0
52	65	5	25	125	25	625
59	50	12	10	120	144	100
44	60	-3	20	-60	9	400
33	32	-14	-8	112	196	64
46	51	-1	11	-11	1	121
		70	110	1662	2706	2204

$$r=\frac{n\Sigma UV-\Sigma U.\Sigma V}{\left[(n\Sigma U^2-(\Sigma U)^2)(n\Sigma V^2-(\Sigma V)^2)\right]^{1/2}}$$

$$=\frac{10\times1662-70\times110}{\left[(10\times2706-4900)(10\times2204-12100)\right]^{1/2}}$$

$$=\frac{16620-7700}{\left[(27060-4900)(22040-12100)\right]^{1/2}}$$

$$=\frac{8920}{(22160\times9940)^{1/2}}$$

Repeated Range correlation Coefficient

$$r = 1 - \frac{6\left(\Sigma d^2 + \Sigma \frac{m_i(m_i^2-1)}{12}\right)}{n(n^2-1)}$$, where m_i is the repeation of variable

X	Y	Rx	Ry	Rx-Ry	D^2
24	37	6	1	5	25
29	35	3	2	1	1
19	16	8.5	9.5	-1	1
14	26	10	4	6	36
30	23	1.5	5	-3.5	12.25
19	27	8.5	3	5.5	30.25
27	19	5	8	-3	9
30	20	1.5	7	5.5	30.25
20	16	7	9.5	-2.5	6.25
28	11	4	11	-7	49
11	21	11	6	5	25
					$\Sigma = 225$

$m_1 = 2$

$m_2 = 2$

$m_3 = 2$

$$r = 1 - \frac{6\left(225 + \frac{2(2^2-1)}{12} + \frac{2(2^2-1)}{12} + \frac{2(2^2-1)}{12}\right)}{11(11^2-1)}$$

$$= 1 - \frac{6(225 + 0.5 + 0.5 + 0.5)}{11(120)}$$

$$= 1 - \frac{6(225 + 1.5)}{1320}$$

$$= 1 - \frac{6 \times 226.5}{1320}$$

$$= 1 - \frac{1359}{1320}$$

$$= 1 - 1.029$$

$$= -0.029$$

Q.

X	Y	Rx	Ry	Rx-y	d^2
48	13	3	5.5	-2.5	6.25
33	13	5	5.5	-0.5	0.25
40	24	4	1	3	9
9	6	10	8.5	1.5	2.25
16	15	8	4	4	16
16	4	8	10	-2	4
65	20	1	2	-1	1
24	9	6	7	-1	1
16	6	8	8.5	-0.5	0.25
57	19	2	3	-1	1
					41

$m_1 = 3, m_2 = 2, m_3 = 2$

$$r = 1 - \frac{6\left(41 + \frac{3(3^2-1)}{12} + \frac{2(2^2-1)}{12} + \frac{2(2^2-1)}{12}\right)}{10(100-1)}$$

$$= 1 - \frac{6(41 + 2 + 0.5 + 0.5)}{990}$$

$$= 1 - \frac{6(44)}{990}$$

$$= 1 - \frac{264}{990}$$

$$= 1 - 0.266$$

$$= 0.734$$

P.E < 1

P.E > 6P.E

P.E < 1

$r = -1 \quad 0$

$P.E = .6745$

$r \pm P.I$

$-1 + 0$

Q.8 (i)

	R1	R2	R3	
	A	B	C	Supply
C1	1	2 [5]	7	4	5 → 0
C2	2	3 [2]	3 [8]	1	8 → 6 → 0
C3	3	5	4 [3]	7 [4]	7 → 4 → 0
C4	4	1	6	2 [14]	14
Demand	7 → 2 → 0	9 → 3	18 → 14		34

⇒

	S1	S2	S3	Supply
D1	2 [5]	7	4	5
D2	3 [2]	3 [6]	1	8
D3	5	4 [3]	7 [4]	7
D4	1	6	2 [14]	14
Demand	7	9	18	34

→ It is a balanced T.P as row sum = column sum = 34.

The given T.P has 12 cells

According to NWCM, 1st allocation made at the cell (1,1), with the allocation min (7,5) = 5. The 2nd allocation made at the cell (2,1), with the allocation min (2,8) = 2. We continue this process again and again until all transportation cost has been used. Finally the number of occupied cell is $m+n-1 = 4+3-1 = 6$.

The transportation cost according to above route

$= 2\times5 + 3\times2 + 3\times6 + 4\times3 + 7\times4 + 2\times14$

$= 102.$

(ii)

	S1	S2	S3	Supply
D1	2 [5]	7	4	5 → 0
D2	3	3	1 [8]	8 → 0
D3	5	4	7	7
D4	1	6	2 [10]	14 → 4
Demand	7 → 2	9	18 → 10 → 0	34

(ii)

	S1	S2	S3	Supply
D1	2	7 [2]	4 [3]	5 → 2
D2	3	3	1 [8]	8 → 0
D3	5	4 [7]	7	7 → 0
D4	1 [7]	6	2 [7]	14 → 7 → 0
Demand	7 → 0	9 → 2	18 → 10 → 3	34

⇒

	S1	S2	S3	Supply
D1	2	7 [2]	4 [3]	5
D2	3	3	1 [8]	8
D3	5	4 [7]	7	7
D4	1 [7]	6	2 [7]	14
Demand	7	9	18	34

→ According to LCM, we search for an element having the least cost. Here we observe the cell (2,3) having the least cost, so we allocate their min (18,8) = 8 and cross the 2nd column. Then the next allocation is at the cell (4,1) with allocation min (7,14) = 7 and cross the 1st row. We continue this process until all transportation has been used. Finally the no. of occupied cell is m+n-1 = 6

The T.P cost according to this route = $7\times2 + 4\times3 + 1\times8 + 4\times7 + 1\times7 + 2\times7$

$= 83.$

(iii)

→ The largest penalty associated with 1st and 2nd row. So we choose 1st row and allocate min(7,5) = 5 to the cell(1,1) having the minimum cost and cross-up the 1st row.

The new revise T.P. as follows.

		S_1	S_2	S_3	Supply
R_1	D_1	2 [5]	7	4	~~7~~ 2 (2) ←
R_2	D_2	3	3	1	8 (2)
R_3	D_3	5	4	7	7 (1)
R_4	D_4	1	6	2	14 (1)
Demand		~~7~~ 2	9	18	
		(1)	(1)	(1)	

The largest penalty associated with 1st ~~and~~ row and 1st column, So we choose 1st row and allocate min(~~8~~,18) = ~~8~~ to the cell (1,3) having the minimum cost and cross up the 1st row.

		S_1	S_2	S_3	Supply
R_1	D_2	3	3	1 [18]	~~8~~ 0 (2) ←
R_2	D_3	5	4	7	7 (1)
R_3	D_4	1	6	2	14 (1)
Demand		2	9	~~18~~ 10	
		(2)	(1)	(1)	

The new revise T.P. as follows.

The largest penalty (5) associated with 3rd column. So we allocate min(10,4) = 4 to the cell (2,3) having the minimum cost and cross up the 2nd row.

		S_1	S_2	S_3	Supply
R_1	D_3	5	4	7 [10]	7 (1)
R_2	D_4	1	6	2	~~14~~ 10 (1)
Demand		2	9	~~10~~ 6	
		(4)	(2)	(5) ↑	

The new revise T.P as follows.

The largest penalty (7) associated with 3rd column. So we allocate min (6,7) = 6 to the cell and cross up the 3rd column.

		S_1	S_2	S_3	Supply
R_1	D_1	5	4	~~7~~ 1 [6]	7 (1)
Demand		~~2~~ 5	9	~~6~~ 0	
				7 ↑	

5	4		7 (1)
1 [2]	6		~~14~~ 12 (5) ←
2 0	9		
(4)	(2)		

The new revise T.P. as follows

The largest penalty (5) associated with 1st column. So we allocate min (2,7) = 2 to the cell and cross up the 1st column

		S_1	S_2	Supply
R_1	D_1	5	4	7 (1)
Demand		~~2~~ 5 ↑	9	

Find out the penalty of each row and each column i.e. the difference between smallest cost and next to smallest cost

identify the highest penalty among the penalties and start allocation in the corresponding smallest cost of respective rows or column and continue this process up to demand fullfilled and supply satisfaction

Q.8 Assignments

	D_1	D_2	D_3	Supply
O_1	2 [5]	7	4	~~5~~ 0
O_2	3	3 [2]	1 [6]	~~8~~ 2
O_3	5 [7]	4	7	7
O_4	1 [2]	6	2 [12]	~~14~~ ~~12~~ 0
Demand	~~7~~ ~~2~~ 0	9	~~18~~ ~~6~~ 0	

	P_1	P_2	P_3	P_4
~~5~~ 0	(2)	–	–	–
~~8~~ 2	(2)	(2)	(2)	(2)
7	(1)	(1)	(3)	(3)
~~14~~ ~~12~~ 0	(1)	(1)	(4) ←	–
P_1	(1)	(1)	(1)	
P_2	(2)↑	(1)	(1)	
P_3	–	(1)	(1)	
P_4	–	(1)	(6)↑	

The T.P cost = $2\times5 + 3\times2 + 1\times6 + 4\times7 + 1\times2 + 2\times12$

= 76

Q.9

	D_1	D_2	D_3	D_4	Sup
O_1	5 [10]	4	2 [10]	6	~~20~~ ~~10~~ 0
O_2	8	8 [30]	5	7	~~30~~ 0
O_3	5	9 [10]	4 [10]	6 [30]	~~50~~ 40
Demand	~~10~~ 0	~~40~~ ~~10~~ 0	~~20~~ ~~10~~ 0	30	

	P_1	P_2	P_3	P_4
~~20~~ ~~10~~ 0	(2)	(2)	–	–
~~30~~ 0	(2)	(2)	(2)	–
~~50~~ 40	(1)	(2)	(2)	(2)
P_1	(3)↑	(1)	(2)	(1)
P_2	–	(1)	(2)↑	(1)
P_3	–	(6)↑	(1)	(1)
P_4	–	(9)↑	(4)	(6)

∴ The T.P. cost = $5\times10 + 2\times10 + 3\times30 + 9\times10 + 4\times10 + 6\times30$

= 470

Q.10

	D_1	D_2	D_3	P_1	P_2
O_1	2 [10]	2	3	10	(1) (0)
O_2	4	1 [15]	2	~~15~~ 0	(1) (3) ←
O_3	1 [10]	3	1 [30]	~~40~~ 10	(2) (2) ←
	20	~~15~~ 0	~~30~~ 0		
P_1	(1)	(1)	(1)		
P_2	(1)	(1)	—		

The T.P cost = $2\times10 + 1\times15 + 1\times10 + 1\times30$

$= 20 + 15 + 10 + 30$

$= 75$

	D_1	D_2	D_3	D_4	P_1	P_2	P_3
O_1	5 [10]	4	2 [10]	6	~~10~~ ~~20~~ 0	(2)	(2) —
O_2	8	3 [30]	5	7	~~30~~ 0	(2)	(2) (2)
O_3	5	9 [10]	4 [10]	6 [30]	50	(2)	(2) (2)
	~~10~~ 0	~~40~~ 10	~~20~~ 10	30			
P_1	(3)↑	(1)	(2)	(1)			
P_2	—	(1)	(2)	(1)			
P_3	—	(6)↑	(1)	(1)			

T.P. Cost = $5\times10 + 2\times10 + 3\times30 + 9\times10 + 4\times10 + 6\times30$

$= 50 + 20 + 90 + 90 + 40 + 180$

$=$

Q.10

	D_1	D_2	D_3	P_1	P_2	P_3
O_1	2	2 [10]	3	~~10~~ 0	(1)	(1) (1) ←
O_2	4	1 [5]	2 [10]	15	(1)	(1) (1)
O_3	1 [20]	3	1 [20]	~~40~~ ~~20~~ 0	(2) ←	(2) ←
	~~20~~ 0	~~15~~ 5	30 10			
P_1	(1)	(1)	(1)			
P_2	—	(1)	(1)			
P_3	—	(1)	(1)			

The T.P cost = $2\times10 + 1\times5 + 2\times10 + 1\times20 + 1\times20$

$= 20 + 5 + 20 + 20 + 20$

$= 85$

	D_1	D_2	D_3	D_4	P_1	P_2	P_3	P_4
O_1	5	4 [10]	2 [10]	6	~~20~~ ~~10~~ 0	(2)	(2)	(2) (4) ←
O_2	8	3 [30]	5	7	~~30~~ 0	(2)	(2) ←	— —
O_3	5 [10]	9	4 [10]	6 [30]	~~40~~ ~~50~~	(1)	(2)	(2) (2)
	~~10~~ 0	~~40~~ ~~10~~ 0	~~20~~ 10	30				
P_1	(3)↑	(1)	(2)	(1)				
P_2	—	(1)	(2)	(1)				
P_3	—	(5)↑	(2)	(0)				
P_4	—	—	(2)	(0)				

	D_1	D_2	D_3	D_4	P_1	P_2	P_3
O_1	5	4	2 [20]	6	~~20~~ 0	(2)	(2) —
O_2	8	3 [30]	5	7	~~30~~ 0	(2)	(2) (4) ←
O_3	5 [10]	9 [10]	4	6 [30]	~~50~~ ~~40~~	(2)	(2) (3)
	~~10~~ 0	~~40~~ 10	~~20~~ 0	30			
P_1	(3)↑	(1)	(2)	(1)			
P_2	—	(1)	(2)↑	(1)			
P_3	—	(6)↑	—	(1)			

T.P cost = $2\times20 + 3\times30 + 5\times10 + 9\times10 + 6\times30$

$= 450$

∴ T.P cost = $4\times10 + 2\times10 + 3\times30 + 5\times10 + 4\times10 + 6\times30$

$=$

Unbalanced T.P

A T.P. is said to be unbalanced, when sum of demand is not equal to supply. In this situation we have to make it balance by adding a dummy row or column as per the requirement, with proper supply and demand.

The entries of dummy row or dummy column are 0.

	D_1	D_2	D_3	Supply
O_1	2	3	4	70
O_2	5	6	7	50
demand	0	0	0	10
	20	30	80	

sum of supply = 120
" of demand = 130

	D_1	D_2	D_3	
O_1	2	3	4	70
O_2	5	6	7	50
O_3	0	0	0	10
	20	30	80	

→

	D_1	D_2	D_3	
O_1	2	1	0	30
O_2	3	2	0	40
O_3	4	3	0	70
	50	60	80	

	D_1	D_2	D_3
O_1	2	2	3
O_2	4	1	2
O_3	1	3	1

Amit
Amit

Initial Basic Feasible Solution

The Sol^n of a ~~trans~~ TP using NWCR, LCM or VAM is knows as IBFS.

optimal Sol^n

Optimal sol of a TP is is the best Sol^n

Step-1 Optimal Sol^n is solved by MODI method (Modified Distributed Method)

Step-2 First find out the Sol^n of the T.P using anyone of the IBFS method.

Step-3 check whether the Sol^n is degenerate or non degenerate.

(i) A IBFS is said to be degenerate or degeneracy if the no. of allocation is less than $m+n-1$)

where m is no. of row

and n is no. of column.

(ii) An IBFS is said to be non-degenerate if it is equal to $m+n-1$)

Step-4 The MODI method is applicable when a TP is non-degenerate.

	D_1	D_2	D_3	D_4	Supply
O_1	21	16	25	13 (11)	11
O_2	17 (6)	18	14 (3)	23 (4)	13
O_3	32	17 (10)	18 (9)	41	19
Dem	6	10	12	15	

$\Delta ij = c_{ij} - (u_i + v_j)$

Allocated cell

$c_{14} = u_1 + v_4 = 13$

$c_{21} = u_2 + v_1 = 17$

$c_{23} = u_2 + v_3 = 14$

$c_{24} = u_2 + v_4 = 23$

$c_{32} = u_3 + v_2 = 17$

$c_{33} = u_3 + v_3 = 18$

Non allocated cell

c_{11} $\Delta_{11} = c_{11} - (u_1 + v_1)$
$= 21 - (-10 + 17) = 14$

c_{12} $\Delta_{12} = c_{12} - (u_1 + v_2)$ > 0
$= 16 - (-10 + 13) = 13$

c_{13} $\Delta_{13} = c_{13} - (u_1 + v_3)$ > 0
$= 25 - (-10 + 14) = 21$ > 0

c_{22} $\Delta_{22} = 18 - (0 + 13) = 5$

c_{31} $\Delta_{31} = 32 - (4 + 17) = 11$ > 0

c_{34} $\Delta_{34} = 41 - (4 + 23)$
$= 14 > 0$

Define a eq^n $c_{ij} = u_i + v_j$, where u_i represent row
v_j " column.

Let $u_2 = 0$, so $v_4 = 13$, $v_1 = 17$, $v_3 = 14$, $v_4 = 23$, $v_2 = 17$, $v_3 = 18$

$u_1 = -10$, $u_2 = 0$, $u_3 = 4$

$v_1 = 17$, $v_2 = 13$, $v_3 = 14$, $v_4 = 23$

To solve u_i & v_j of the allocated cell take anyone of the variable as 0, where no. of allocation is more.

when all Δij are +ve over Sol^n is optimal

T.P cost = $11 \times 13 + 17 \times 6 + 14 \times 3 + 23 \times 4 + 17 \times 10 + 18 \times 9$

Solve the T.P using MODI method.

8.

	D_1	D_2	D_3	D_4	Supply
O_1	2	2	2	1	3
O_2	10	8	5	4	7
O_3	7	6	6	8	5
De	4	3	4	4	

→

	D_1	D_2	D_3	D_4	Supply	P_1	P_2	P_3
O_1	2 [3]	2	2	1	3	(1)	–	–
O_2	10	8	5 [3]	4 [4]	7	(1)	(1)	(3) ←
O_3	7 [1]	6 [3]	6 [1]	8	5	(1)	(1)	(1)
Demand	4	3	4	4				

P_1 (5)↑ (4) (4) (3)

P_2 (3) (2) (1) (4)↑

P_3 (3) (2) (1) –

The T.P. Cost = $2\times3 + 5\times3 + 4\times4 + 7\times1 + 6\times3 + 6\times1$

= 68

Here $m = 3$ and $n = 4$, so $m+n-1 = 6$

and also no. of occupied cell is 6

Thus this sol^n is non-degenerate.

As the 3rd contain more no. of occupied cell, thus let $U_3 = 0$

Allocated cell

$c_{11} = U_1 + V_1 = 2$, $U_1 = -5$

$c_{23} = U_2 + V_3 = 5$, $U_2 = -1$

$c_{24} = U_2 + V_4 = 4$, $V_4 = 5$

$c_{31} = U_3 + V_1 = 7$, $V_1 = 7$

$c_{32} = U_3 + V_2 = 6$, $V_2 = 6$

$c_{33} = U_3 + V_3 = 6$, $V_3 = 6$

Non Allocated cell

~~$c_{21} = \Delta_{21} = c_{21} - (U_2+V_1) = 2-(-1+7) =$~~

$c_{12} = \Delta_{12} = c_{12} - (U_1+V_2) = 2-(-5+6) = 1 > 0$

$c_{13} = \Delta_{13} = c_{13} - (U_1+V_3) = 2-(-5+6) = 1 > 0$

$c_{14} = \Delta_{14} = c_{14} - (U_1+V_4) = 1-(-5+5) = 1 > 0$

$c_{21} = \Delta_{21} = c_{21} - (U_2+V_1) = 10-(-1+7) = 4 > 0$

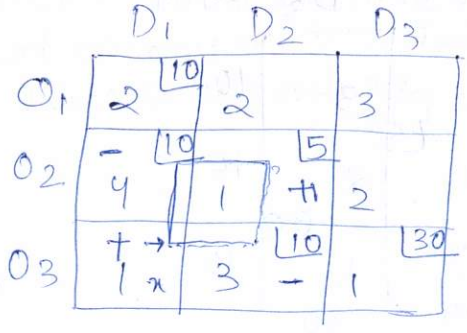
$c_{22} = \Delta_{22} = c_{22} - (U_2+V_2) = 8-(-1+6) = 3 > 0$

$c_{34} = \Delta_{34} = c_{34} - (U_3+V_4) = 8-(0+5) = 3 > 0$

∴ Since all $\Delta_{ij} > 0$, the ~~solution is optimum~~ T.P. has been reached the optimum solution is $x_{11}=3$, $x_{23}=3$, $x_{24}=4$, $x_{31}=1$, $x_{32}=3$, $x_{33}=1$ and the T.P. cost = 68 respectively

Q.

	D_1	D_2	D_3	Supply
O_1	2 (10)	2	3	~~10~~ 0
O_2	4 (10)	1 (5)	2	~~15~~ 5 0
O_3	1	3 (10)	1 (30)	40
Demand	~~20~~ ~~10~~ 0	~~15~~ 10	30	



T.P cost = 20 + 40 + 5 + 30 + 30

= 125

~~Allocated cell~~ This soln is non-degenerency

As u_2 & v_3 contains maximum occupied cell, we choose u_2 and let $u_2 = 0$

Allocated cell

$c_{11} = u_1 + v_1 = 2, u_1 = -2$

$c_{21} = u_2 + v_1 = 4, v_1 = 4$

$c_{22} = u_2 + v_2 = 1, v_2 = 1$

$c_{32} = u_3 + v_2 = 3, u_3 = 2$

$c_{33} = u_3 + v_3 = 1, v_3 = -1$

Non Allocated cell

$c_{12}, A_{12} = c_{12} - (u_1 + v_2) = 2 - (-2 + 1) = 3 > 0$

$c_{13}, A_{13} = c_{13} - (u_1 + v_3) = 3 - (-2 - 1) = 6 > 0$

$c_{23}, A_{23} = c_{23} - (u_2 + v_3) = 2 - (0 + -1) = 3 > 0$

$c_{31}, A_{31} = c_{31} - (u_3 + v_1) = 1 - (2 + 4) = -5 < 0$

As we know that when $A_{ij} \geq 0$, our soln is optimal, but in this case $A_{31} < 0$, so the soln is not optimal.

Loop

Find out the highest (-ve) from Aij and start a loop, the loop moves horizontally and vertically where the corner of the loop must be an allocated cell and again it will comeback to its initial cell for when it has started

Assign (+ve) and (-ve) ~~sign~~ sign in the corner of the loop, start with +ve sign

Define a parameter θ = ~~max (negative~~ min{Allocated -ve cell} =

θ = min{10, 10} = 10

Add θ to the +ve cell and subtract it from the -ve cell.

	D_1	D_2	D_3	Supply
O_1	[10] 2	2	3	10
O_2	4	[15] 1	2	15
O_3	[10] 1	3	[30] 1	10
Demand	20	15	30	

Now the no. of occupied cell (4) is not equal to m+n-1.

So this T.P is degeneracy.

Now to make this T.P non-degeneracy, we have to make a cell occupied, So for that we choose an empty cell i.e. C(1,2) which is the smallest among the unoccupied cell and also the cell is in closed path.

And we allocate the small +ve value ε to the cell.

	D_1	D_2	D_3
O_1	[10] 2	[ε] 2	3
O_2	4	[15] 1	2
O_3	[10] 1	3	[30] 1

Now the T.P. is non-degeneracy and to find the optimal sol^n for this T.P, we use UV method.

As 1st row contains maximum no. of occupied cell, so we choose $U_1 = 0$

Allocated cell

$C_{11} = U_1 + V_1 = 2$, $V_1 = 2$

$C_{12} = U_1 + V_2 = 2$, $V_2 = 2$

$C_{22} = U_2 + V_2 = 1$, $U_2 = -1$

$C_{31} = U_3 + V_1 = 1$, $U_3 = -1$

$C_{33} = U_3 + V_3 = 1$, $V_3 = 2$

Non Allocated cell (Net evalution)

$\Delta_{13} = C_{13} - (U_1 + V_3) = 3 - (0+2) = 1 > 0$

$\Delta_{21} = C_{21} - (U_2 + V_1) = 4 - (-1+2) = 3 > 0$

$\Delta_{23} = C_{23} - (U_2 + V_3) = 2 - (-1+2) = 1 > 0$

$\Delta_{32} = C_{32} - (U_3 + V_2) = 3 - (-1+2) = 2 > 0$

$\therefore$ Since all $\Delta_{ij} > 0$, the T.P has been reached the optimal solution.

$x_{11} = 10$, $x_{12} = \varepsilon$, $x_{22} = 15$, $x_{31} = 10$, $x_{33} = 30$

and the T.P. Cost $= 2\times10 + 2\times\varepsilon + 1\times15 + 1\times10 + 1\times30$

$= 75 + 2\varepsilon$

$= 75$ ($\because \varepsilon$ is very small +ve no.)

when there is degeneracy occur in a T.P, to convert into non-degenerate we have to assign a very small value ε to the smallest non-allocated cell present in the matrix and that cell must be in independent position means from that particular cell there is no possibility of forming a loop.

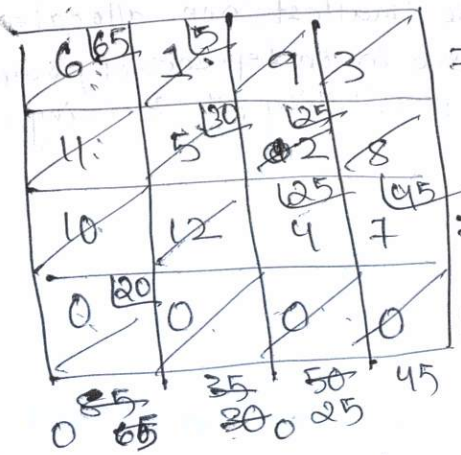
Q.

	D_1	D_2	D_3	D_4	Supply
O_1	6	1	9	3	70
O_2	11	5	2	8	55
O_3	10	12	4	7	70
Demand	85	35	50	45	195 / 215

$O_1 \rightarrow O_2$

	D_1	D_2	D_3	D_4		P_1	P_2	P_3
O_1	6 (65)	1 (5)	9	3	~~70~~ 5	(2)	(2)	(2)
O_2	11	5	2	8	55	(3)	(3)	(3)
O_3	10	12	4	7	70	(3)	(3)	(3)
O_4	0 (20)	0	0	0	~~20~~ 0	(0)	—	—
	~~85~~ 65	35	50	45				

P_1	(4) ↑ (4)	(2)	(4)	
P_2	(4) ↑ (4)	(2)	(4)	
P_3	—	(4) ↑ (2)	(4)	



	P_1	P_2	P_3	P_4	P_5	
70	(2)	(2)	(2)	—	—	0
55	(3)	(3)	(3)	(3)	(6)	250
70	(3)	(3)	(3)	(3)	(3)	
20	(0)	—	—	—	—	0

P_1 (6) (1) (4) (3)

P_2 (4) (4) (2) (4)

P_3 — (4) (2) (4)

P_4 — (7) (2) (1)

P_5 — — (2) (1)

T.P cost $= 6\times65 + 1\times5 + 5\times30 + 2\times25 + 0 + 4\times25 \times 7\times45$

$= 1010$

This T.P is non-degeneracy.

Now let $U_1 = 0$

Allocated cell

$C_{11} = (U_1+V_1) = 6$, $V_1 = 6$

$C_{12} = (U_1+V_2) = 1$, $V_2 = 1$

$C_{22} = (U_2+V_2) = 5$, $U_2 = 4$

$C_{23} = (U_2+V_3) = 2$, $V_3 = -2$

$C_{33} = (U_3+V_3) = 4$, $U_3 = 6$

$C_{34} = (U_3+V_4) = 7$, $V_4 = 1$

$C_{41} = (U_4+V_1) = 0$, $U_4 = -6$

Non Allocated cell (Net ev

$\Delta_{13} = C_{13} - (U_1+V_3) = 9+2 = 11 > 0$

$\Delta_{14} = C_{14} - (U_1+V_4) = 3-(0+1) = 2 > 0$

$\Delta_{21} = C_{21} - (U_2+V_1) = 11-(4+6) = 1 > 0$

$\Delta_{24} = C_{24} - (U_2+V_4) = 8-(4+1) = 3 > 0$

$\Delta_{31} = C_{31} - (U_3+V_1) = 10-(6+6) = -2 < 0$

$\Delta_{32} = C_{32} - (U_3+V_2) = 12-(6+1) = 5 > 0$

$\Delta_{42} = C_{42} - (U_4+V_2) = 0-(-6+1) = 5 > 0$

$\Delta_{43} = C_{43} - (U_4+V_3) = 0-(-6-2) = 8 > 0$

$\Delta_{44} = C_{44} - (U_4+V_4) = 0-(-6+1) = 5 > 0$

65	6	1	9	3
30	11	5	2	8
25	10	12	4	7
45	0	0	0	0
20				

	6	1	9	3
	11	5	2	8
	10	12	4	7
	0	0	0	0

$\min(65, 30, 25) = 25$

40	6	1	9	3
30	11	5	2	8
25	10	12	4	7
45	0	0	0	0
20				

Let $v_1 = 0$

T.P is non-degeneracy

$c_{11} = (v_1 + v_1) = 6$, $v_1 = 6$

$c_{12} = (v_1 + v_2) = 1$, $v_2 = 1$

$c_{22} = (v_2 + v_2) = 5$, $v_2 = 4$

$c_{23} = (v_2 + v_3) = 2$, $v_3 = -2$

$c_{31} = (v_3 + v_1) = 10$, $v_3 = 4$

$c_{34} = (v_3 + v_4) = 7$, $v_4 = 3$

$c_{41} = (v_4 + v_1) = 0$, $v_4 = -6$

Non Allocated (net evalution)

$A_{13} = c_{13} - (v_1 + v_3) = 9 - (0 - 2) = 11 > 0$

$A_{14} = c_{14} - (v_1 + v_4) = 3 - (0 + 3) = 0$

$A_{21} = c_{21} - (v_2 + v_1) = 11 - (4 + 6) = 1 > 0$

$A_{24} = c_{24} - (v_2 + v_4) = 8 - (4 + 3) = 1 > 0$

$A_{32} = c_{32} - (v_3 + v_2) = 12 - (4 + 1) = 7 > 0$

$A_{33} = c_{33} - (v_3 + v_3) = 4 - (4 - 2) = 2 > 0$

$A_{42} = c_{42} - (v_4 + v_2) = 0 - (-6 + 1) = 5 > 0$

$A_{43} = c_{43} - (v_4 + v_3) = 0 - (-6 - 2) = 8 > 0$

$A_{44} = c_{44} - (v_4 + v_4) = 0 - (-6 + 3) = 3 > 0$

Since all $A_{ij} > 0$, the T.P has reached its optimal solution.

and the T.P cost $= 6 \times 40 + 1 \times 30 + 5 \times 5 + 2 \times 50 + 10 \times 25 + 7 \times 45 + 0$

$= 240 + 30 + 25 + 100 + 250 + 315$

$= 960$

Q.

	D_1	D_2	D_3	D_4	Supply
O_1	3	1	7	4	250
O_2	2	6	5	9	350
O_3	8	3	3	2	400
Demand	200	300	350	150	

As this T.P is a balanced, so ~~use~~ we use VAM method direct.

	D_1	D_2	D_3	D_4		P_1	P_2	P_3	P_4
O_1	3	1 (250)	7	4	~~250~~ 0	(2)	(3) ←	—	—
O_2	2 (200)	6	5 (150)	9	~~350~~ 150	(3) ←	(1)	(1)	(1)
O_3	8	3 (150)	3 (200)	2 (150)	~~400~~ ~~250~~ 200	(1)	(1)	(1)	(0)
	~~200~~ 0	~~300~~ 150 0	350	150 0					

P_1	(1)	(2)	(2)	(2)
P_2	—	(2)	(2)	(2)
P_3	—	(3)	(2)	(7) ↑
P_4	—	(3) ↑	(2)	—

The T.P. Cost = $1\times250 + 2\times200 + 5\times150 + 3\times150 + 3\times200 + 2\times150$

= 2450

Now using MODI method to find out optimal solution, (This sol? is non-degeneracy)

Here, 3rd contains maximum no. of allocation, so we choose $U_3 = 0$ to find out values of ~~U_i and~~ U_i, V_j and A_{ij}

Allocated cell

$C_{12} = U_1 + V_2 = 1$, $U_1 = -2$

$C_{21} = U_2 + V_1 = 2$, $V_2 = 0$

$C_{23} = U_2 + V_3 = 5$, $U_2 = 2$

$C_{32} = U_3 + V_2 = 3$, $V_2 = 3$

$C_{33} = U_3 + V_3 = 3$, $V_3 = 3$

$C_{34} = U_3 + V_4 = 2$, $V_4 = 2$

Non Allocated cell (Net evalution)

$A_{11} = C_{11} - (U_1 + V_1) = 3 - (-2 + 0) = 5 > 0$

$A_{13} = C_{13} - (U_1 + V_3) = 7 - (-2 + 3) = 6 > 0$

$A_{14} = C_{14} - (U_1 + V_4) = 4 - (-2 + 2) = 4 > 0$

$A_{22} = C_{22} - (U_2 + V_2) = 6 - (2 + 3) = 1 > 0$

$A_{24} = C_{24} - (U_2 + V_4) = 9 - (2 + 2) = 5 > 0$

$A_{31} = C_{31} - (U_3 + V_1) = 8 - (0 + 0) = 8 > 0$

Since $A_{ij} > 0$, So the T.P. has been reached the optimal solution and

$x_{12} = 1$, $x_{21} = 2$, $x_{23} = 5$, $x_{32} = 3$, $x_{33} = 3$, $x_{34} = 2$

and the T.P cost = $1\times250 + 2\times200 + 5\times150 + 3\times150 + 3\times200 + 2\times150$

= 2450

Q.2

	I	II	III	IV	V	Supply
A	8	6 (15)	6 (85)	9	7	~~100~~ 15
B	7	8 (20)	9	6 (105)	10	~~125~~ ~~20~~ 0
C	5 (60)	6 (45)	8	5	6 (70)	~~175~~ ~~115~~ 45
	~~60~~ 0	~~80~~ 60 0	~~85~~ 0	~~105~~ 0	~~70~~ 0	

	P_1	P_2	P_3	P_4	P_5
	(2)	(1)	(1)	(1)	(1)
	(1)	(2) ←	(1)	(2) ←	-
	(1)	(1)	(2)	(0)	(0)

P_1	(2) ↑	(2)	(2)	(1)	(1)
P_2	-	(2)	(2)	(1)	(1)
P_3	-	(2)	(2) ↑	-	(1)
P_4	-	(2)	-	-	(1)
P_5	-	(0)	-	-	(1) ↑

	I	II	III	IV	V	
A	8	6 (15)	6 (85)	9	7	100
B	7	8 (20)	9	6 (105)	10	125
C	5 (60)	6 (45)	8	5	6 (70)	175
	60	80	85	105	70	

→ The T.P. cost = $6\times15 + 6\times85 + 8\times20 + 5\times60 + 6\times45 + 6\times105 + 6\times70$

= 2380

Here the no. of occupied cell = $m+n-1 = 7$, so this solution is non-degenerecy. Now to find optimal sol^n for this T.P, we use MODI method ~~and~~.

Let $U_3 = 0$, because 3rd row contains maximum no. of occupied cell.

Allocated cell

$c_{12} = U_1 + V_2 = 6, U_1 = 0$

$c_{13} = U_1 + V_3 = 6, V_3 = 6$

$c_{22} = U_2 + V_2 = 8, U_2 = 2$

$c_{24} = U_2 + V_4 = 6, V_4 = 4$

$c_{31} = U_3 + V_1 = 5, V_1 = 5$

$c_{32} = U_3 + V_2 = 6, V_2 = 6$

$c_{35} = U_3 + V_5 = 6, V_5 = 6$

Non Allocated cell (Net evalution)

$\Delta_{11} = c_{11} - (U_1 + V_1) = 8 - (0+5) = 3 > 0$

$\Delta_{14} = c_{14} - (U_1 + V_4) = 9 - (0+4) = 5 > 0$

$\Delta_{15} = c_{15} - (U_1 + V_5) = 7 - (0+6) = 1 > 0$

$\Delta_{21} = c_{21} - (U_2 + V_1) = 7 - (2+5) = 0$

$\Delta_{23} = c_{23} - (U_2 + V_3) = 9 - (2+6) = 1 > 0$

$\Delta_{25} = c_{25} - (U_2 + V_5) = 10 - (2+6) = 2 > 0$

$\Delta_{33} = c_{33} - (U_3 + V_3) = 8 - (0+6) = 2 > 0$

$\Delta_{34} = c_{34} - (U_3 + V_4) = 5 - (0+4) = 1 > 0$

Since all $\Delta_{ij} \geq 0$, the T.P. has been reached the optimal solution and

$x_{12} = 6, x_{13} = 6, x_{22} = 8, x_{24} = 6, x_{31} = 5, x_{32} = 6, x_{35} = 6$

and the T.P. cost = $6\times15 + 6\times85 + 8\times20 + 5\times60 + 6\times45 + 6\times105 + 6\times70$

= 2380

Q → New T.P after adding variable cost to the Transportation cost, ~~is~~ given below.

	I	II	III	IV	V	
A	22	20	20	23	21	100
B	23	24	25	22	26	125
C	20	21	23	20	21	175
	60	80	85	105	70	

As this T.P is balanced, so we use MODI method directly,

	I	II	III	IV	V	supply	P_1	P_2	P_3	P_4
A	22	20	85 20	23	15 21	~~100~~ 15	(0)	(1)	(1)	(1)
B	23	80 24	25	22	45 26	~~125~~ 45	(1)	(1)	(2)	(2) ←
C	60 20	21	23	105 20	10 21	~~175~~ ~~115~~ 10	(0)	(0)	(1)	(0)
Demand	~~60~~ 0	~~80~~ 0	~~85~~ 0	~~105~~ 0	70					

	I	II	III	IV	V
P_1	(2)	(1)	(3) ↑	(2)	(0)
P_2	(2) ↑	(1)	–	(2)	(0)
P_3	–	(1)	–	(2) ↑	(0)
P_4	–	(1)	–	–	(0)

The T.P. Cost according to this route = $20\times85 + 21\times15 + 24\times80 + 26\times45 + 20\times60 + 20\times105 + 21\times10$

$= 8615$

Also this ~~P.P.~~ solution is non-degenerecy (No. of allocated cell = 7 = m+n-1)

Now to find optimal solution, we use MODI method / UV method.

And the 3rd row contains maximum no. of allocated cell, so we take $U_3 = 0$.

Allocated cell

$C_{13} = U_1 + V_3 = 20$, $V_3 = 20$

$C_{15} = U_1 + V_5 = 21$, $V_1 = 0$

$C_{22} = U_2 + V_2 = 24$, $V_2 = 19$

$C_{25} = U_2 + V_5 = 26$, $U_2 = 5$

$C_{31} = U_3 + V_1 = 20$, $V_1 = 20$

$C_{34} = U_3 + V_4 = 20$, $V_4 = 20$

$C_{35} = U_3 + V_5 = 21$, $V_5 = 21$

Non Allocated cell (Net evalution)

$\Delta_{11} = C_{11} - (U_1 + V_1) = 22 - (0+20) = 2 > 0$

$\Delta_{12} = C_{12} - (U_1 + V_2) = 20 - (0+19) = 1 > 0$

$\Delta_{14} = C_{14} - (U_1 + V_4) = 23 - (0+20) = 3 > 0$

$\Delta_{21} = C_{21} - (U_2 + V_1) = 23 - (5+20) = -2 < 0$

$\Delta_{23} = C_{23} - (U_2 + V_3) = 25 - (5+20) = 0$

$\Delta_{24} = C_{24} - (U_2 + V_4) = 22 - (5+20) = -3 < 0$

$\Delta_{32} = C_{32} - (U_3 + V_2) = 21 - (0+19) = 2 > 0$

Hence Δ_{21} and Δ_{24} are less than zero, so the sol^n not reached optimal.

Hence we need to ~~create~~ start a loop from C_{24} cell as its value is highest -ve.

	I	II	III	IV	V
A	22	20	20 [85]	23	21 [15]
B	23	24 [80]	25	22 [+ve]	26 [45] [-ve]
C	20 [60]	21	23	20 [105] [-ve]	21 [10] [+ve]

⇒

	I	II	III	IV	V
A	22	20	20 [85]	23	21 [15]
B	23	24 [80]	25	22 [45]	26
C	20 [60]	21	23	20 [60]	21 [55]

θ = min {Allocated -ve cell}
= min {105, 45} = 45

And the new T.P is non-degeneracy as No. of occupied cell = 7 = (m+n-1).

Again to find optimal soln for the new. TP we use UV method.
and $v_3 = 0$ (3rd row contains max. allocated cell)

Allocated cell

$c_{13} = u_1 + v_3 = 20, v_3 = 20$

$c_{15} = u_1 + v_5 = 21, u_1 = 0$

$c_{22} = u_2 + v_2 = 24, v_2 = 22$

$c_{24} = u_2 + v_4 = 22, u_2 = 2$

$c_{31} = u_3 + v_1 = 20, v_1 = 20$

$c_{34} = u_3 + v_4 = 20, v_4 = 20$

$c_{35} = u_3 + v_5 = 21, v_5 = 21$

Non Allocated cell (Net evalution)

$\Delta_{11} = c_{11} - (u_1 + v_1) = 22 - (0+20) = 2 > 0$

$\Delta_{12} = c_{12} - (u_1 + v_2) = 20 - (0+22) = -2 < 0$

$\Delta_{14} = c_{14} - (u_1 + v_4) = 23 - (0+20) = 3 > 0$

$\Delta_{21} = c_{21} - (u_2 + v_1) = 23 - (2+20) = 1 > 0$

$\Delta_{23} = c_{23} - (u_2 + v_3) = 25 - (2+20) = 3 > 0$

$\Delta_{25} = c_{25} - (u_2 + v_5) = 26 - (2+21) = 3 > 0$

$\Delta_{32} = c_{32} - (u_3 + v_2) = 21 - (0+22) = -1 < 0$

$\Delta_{33} = c_{33} - (u_3 + v_3) = 23 - (0+20) = 3 > 0$

Since Δ_{12} and Δ_{32} are negative, the soln not reached optimal.
We need to start loop from cell (1,2) as it is the highest -ve value.

	I	II	III	IV	V
A	22	20 [85] (+)	20	23	21 [15] (-)
B	23	24 [80] (-)	25	22 [45] (+)	26
C	20 [60]	21	23	20 [60] (-)	21 [55] (+)

⇒

	I	II	III	IV	V
A	22	20 [15]	20 [100]	23	21
B	23	24 [65]	25	22 [60]	26
C	20 [60]	21	23	20 [45]	21 [70]

$\theta = \{80, 60, 15\} = 15$

Allocated cell

$c_{12} = u_1 + v_2 = 20$, $u_1 = -2$

$c_{13} = u_1 + v_3 = 20$, $v_3 = 22$

$c_{22} = u_2 + v_2 = 24$, $v_2 = 22$

$c_{24} = u_2 + v_4 = 22$, $u_2 = 2$

$c_{31} = u_3 + v_1 = 20$, $v_1 = 20$

$c_{34} = u_3 + v_4 = 20$, $v_4 = 21$

$c_{35} = u_3 + v_5 = 21$, $v_5 = 21$

22	20	20	23	21
23	24	25	22	26
20	21	23	20	21

Non Allocated cell (Net evalution)

$\Delta_{11} = c_{11} - (u_1 + v_1) = 22 - (-2 + 20) = 4 > 0$

$\Delta_{14} = c_{14} - (u_1 + v_4) = 23 - (-2 + 20) = 5 > 0$

$\Delta_{15} = c_{15} - (u_1 + v_5) = 21 - (-2 + 21) = 2 > 0$

$\Delta_{21} = c_{21} - (u_2 + v_1) = 23 - (2 + 20) = 1 > 0$

$\Delta_{23} = c_{23} - (u_2 + v_3) = 25 - (2 + 22) = 1 > 0$

$\Delta_{25} = c_{25} - (u_2 + v_5) = 26 - (2 + 21) = 3 > 0$

$\Delta_{32} = c_{32} - (u_3 + v_2) = 21 - (0 + 22) = -1 < 0$

$\Delta_{33} = c_{33} - (u_3 + v_3) = 23 - (0 + 22) = 1 > 0$

Q

	D_1	D_2	D_3	D_4	D_5
O_1					
O_2					
O_3					

Maximization

Generally T.P is a minimization problem. If the given problem is in maximization form of a T.P then first convert the max. to minimization by subtracting all the element of the T.P from the highest element of T.P table, then follow as usual method to find the sol;

Q. Find the T.P cost where the matrix is a maximizat

	D_1	D_2	D_3	D_4	supply
O_1	2	2	2	4	1000
O_2	3	5	3	2	700 →
O_3	4	3	2	1	900
Dem	900	800	500	400	

P_1	D_1	D_2	D_3	D_4	P_1	P_2	P_3
O_1	3	3 100	3 500	1 400	~~1000~~ 600 ←	(2)	(0)	(0)
O_2	2	0 700	2	3	~~700~~ 0	(2)	(2)	–
O_3	1 900	2	3	4	~~900~~ 0	(1)	(1)	(1)
	~~900~~ 0	~~800~~ 100	500	~~400~~ 0				
P_1	(1)	(2)	(1)	(2)				
P_2	(1)	(2) ↑	(1)	–				
P_3	(2) ↑	(1)	(0)	–				

T.P Cost = $3\times100 + 3\times500 + 1\times400 + 0\times700 + 1\times900$

$= 300 + 1500 + 400 + 900$

$= 3100$

Q.

2 (100)	2	2 (500)	4 (400)
3	5 (700)	3	2
4 (900)	3	2	1

T.P cost = $2\times100 + 2\times500 + 4\times400 + 5\times700 + 4\times900$

= 9900

Q. Solve the

	D_1	D_2	D_3	D_4	Suppl
O_1	15	1	42	33	23
O_2	80	42	26	81	44 →
O_3	90	40	66	60	33
	23	31	16	30	

	(23)			23	P1 (9) P2 (18) P3 (18) P4 (32)
75	89	48	57		
(6)	(8)		(30)	44	(1) (1) (1) (39)
10	48	64	9	38 8	
(17)		(16)			
0	50	24	30	33 17 0	(24) (30) ←
23 0	31	16 0	38		

P1	(10)	(2)	(24)↑	(21)
P2	(10)	(2)	–	(21)
P3	(65)↑	(41)	–	(48)
P4	–	(41)	–	(48)↑

	(23)			23
15	1	42	33	
(6)	(8)		(30)	44 = 5855
80	42	26	81	
(17)		(16)		33
90	40	66	60	

Q.

	A	B	C	D	
1	40	25	22	33	100
2	44	35	30	30	30 →
3	38	38	28	30	70
	40	20	60	30	

	A	B	C	D	E	
1	40	25	22	33	0	100
2	44	35	30	30	0	30
3	38	38	28	30	0	70
	40	20	60	30	50	

	A	B	C	D	E	P1
			20	30	50	
1	4	19	22	11	44	~~100~~ 70
	30					
2	0	9	14	14	44	~~30~~ 0 ←
	10	20	40			
3	6	6	16	14	44	~~70~~ 50 40 0
	40	20	60	30	50	
P1	~~(4)~~	~~(3)~~	~~(2)~~	~~(3)~~	~~(0)~~	
	~~40~~ 0	~~10~~ 0	~~20~~ 0	~~60~~ 20	~~30~~ 0	50

	P1	P2	P3	P4	P5
	(7)	(7)	(7)	(11) ←	(22)
	(9)	–	–	–	–
	(0)	(0)	(8) ←	(2)	(28) ←

P1 (4) (3) (2) (3) (0)

P2 (2) (13) (6) (3) (0)

P3 (2) – (6) (3) (0)

P4 – – (6) (3) (0)

P5 – – (6) – (0)

→

		20	30	50
40	25	22	33	0
30				
44	35	30	30	0
10	20	40		
38	38	28	30	0

= 22×20 + 33×30 + 0×50 + 44×30 + 38×10 + 38×20 + 28×40

= 440 + 990 + 0 + 1320 + 380 + 760 + ~~920~~ 1120

= 5010

4	19	22 (20)	11 (30)	44 (50)
0 (30)	9	14	14	44
6 (10)	6 (20)	16 (40)	14	44
40	20	60	30	50

This is a non-degeneracy T.P.
as $m+n-1 = 7$ = occupied cell

Let $U_1 = 0$

11 lakh
470,000
12.6 lakh
7.54.000

Allocated cell

$C_{13} = U_1 + V_3 = 22, V_3 = 22$

$C_{14} = U_1 + V_4 = 11, V_4 = 11$

$C_{15} = U_1 + V_5 = 44, V_5 = 44$

$C_{21} = U_2 + V_1 = 0, U_2 = -12$

$C_{31} = U_3 + V_1 = 6, V_1 = 12$

$C_{32} = U_3 + V_2 = 6, V_2 = 12$

$C_{33} = U_3 + V_3 = 16, U_3 = -6$

Non Allocated cell (Net evalution)

$\Delta_{11} = C_{11} - (U_1 + V_1) = 4 - (0+12) = -8 < 0$

$\Delta_{12} = C_{12} - (U_1 + V_2) = 19 - (0+12) = 7 > 0$

~~$\Delta_{32} = C_{32} - (U_3 + V_2) = 9 - (-6+12) = 3 > 0$~~

~~$\Delta_{33} = C_{33} - (U_3 + V_3) = 14 - (-6+22) = -2 < 0$~~

~~$\Delta_{34} = C_{34} - (U_3 + V_4) = 14 - (-6+11) = 9 > 0$~~

~~$\Delta_{35} = C_{35} - (U_3 + V_5) = 44 - (-6+44) = 6 > 0$~~

$\Delta_{22} = C_{22} - (U_2 + V_2) = 9 - (-12+12) = 9 > 0$

$\Delta_{23} = C_{23} - (U_2 + V_3) = 14 - (-12+22) = 4 > 0$

$\Delta_{24} = C_{24} - (U_2 + V_4) = 14 - (-12+11) = 13 > 0$

$\Delta_{25} = C_{25} - (U_2 + V_5) = 44 - (-12+44) = 12 > 0$

$\Delta_{34} = C_{34} - (U_3 + V_4) = 14 - (-6+11) = 9 > 0$

$\Delta_{35} = C_{35} - (U_3 + V_5) = 44 - (-6+44) = 6 > 0$

As $\Delta_{11} < 0$ so the T.P has not reached optimal solution.

Hence we need to start loop from C_{11} cell

+ →		(-) 22 (20)	11 (30)	44 (50)
4 ↓	19			
30		↑		
0 ↓	9	14	14	44
10	←	(+)		
6 (-)	6 →	16 (20)	14 (40)	44

→

10		10	30	50
4	19	22	11	44
30				
0	9	14	14	44
	20	50		
6	6	16	14	44

$(U_1 = 0)$

20			30	50
4	19	22	11	44
10				
0	9	14	14	44
30	ε	60		
6	6	16	14	44

$(U_1 = 0)$

Here $\theta = \min\{\cancel{30, 20, 20}\ 10, 20\} = \cancel{20}\ 10$

Allocated

$C_{11} = U_1 + V_1 = 4, V_1 = 4$

$C_{14} = U_1 + V_4 = 11, V_4 = 11$

$C_{15} = U_1 + V_5 = 44, V_5 = 44$

$C_{21} = U_2 + V_1 = 0, U_2 = -4$

~~$C_{31} = U_3 + V_1 = 6, U_3 = 2$~~

$C_{32} = U_3 + V_2 = 6, V_2 = \cancel{12}\ 12$

$C_{33} = U_3 + V_3 = 16, V_3 = \cancel{-6}\ -6$

$C_{13} = U_1 + V_3 = 22, V_3 = 22$

Non Allocated

~~$\Delta_{12} = C_{12} - (U_1 + V_2) = 19 - (0+4) = 15 > 0$~~

~~$\Delta_{13} = C_{13} - (U_1 + V_3) = 22 - (0+14) = 8 > 0$~~

~~$\Delta_{21} = C_{21} - (U_2 + V_2) = 9 - (-4+4) = 9 > 0$~~

$\Delta_{22} = C_{22} - (U_2 + V_3) = 14 - (-4+14) = 4 > 0$

$\Delta_{24} = C_{24} - (U_2 + V_4) = 14 - (-4+11) = 7 > 0$

$\Delta_{25} = C_{25} - (U_2 + V_5) = 44 - (-4+44) = 4 > 0$

$\Delta_{34} = C_{34} - (U_3 + V_4) = 14 - (2+11) = 1 > 0$

$\Delta_{35} = C_{35} - (U_3 + V_5) = 44 - (-2+44) = 2 > 0$

$19 - (0+12) = 7 > 0$

$\Delta_{22} = 9 - (-4+12) = 9 - 8 = 1 > 0$

$\Delta_{23} = 14 - (-4+22) = -4 < 0$

$\Delta_{24} = 14 - (-4+11) = 7 > 0$

$\Delta_{25} = 44 - (-4+44) = 4 > 0$

$\Delta_{31} = 6 - (-6+4) = 8 > 0$

$\Delta_{34} = 14 - (-6+11) = 9 > 0$

$\Delta_{35} = 44 - (-6+44) = 6 > 0$

20			30	50
40	25	22	33	0
10				
44	35	30	30	0
30	ε	60		
38	38	28	30	0

T.P. cost = $40\times20 + 33\times30 + 0\times50 + 44\times10 + 38\times30 + 38\times\varepsilon + 28\times60$

$= 800 + 990 + 0 + 440 + 1140 + 38\varepsilon + 1680$

$= 5050 + 38\varepsilon$

$= 5050$

Q.

40	25	22	33	100
44	35	30	30	30 →
38	38	28	30	70
40	20	60	30	

4	19	22	11	100
0	9	14	14	30 →
6	6	16	14	70
40	20	60	30	

4	19	22	11	0	
0	9	14	14	0	
6	6	16	14	0	
40	20	60	30	50	

Ans)

10		60	30	
4	19	22	11	0
30				
0	9	14	14	0
	20			50
6	6	16	14	0
~~40~~ 10	~~20~~ 0	60	30	~~50~~ 0

	P_1	P_2	P_3
100	(4)	(7)	(7)
~~30~~ 0	(0)	(9) ←	
~~70~~ 20 ①	(6) ←	(0)	(0)

P_1	(4)	(3)	(2)	(3)	(0)
P_2	(4)	(3)	(2)	(3)	(-)
P_3	(2)	(13)↑	(6)	(3)	(-)

10		60	30	
4	19	22	11	0
30				
0	9	14	14	0
	20			50
6	6	16	14	0

~~The T.P cost~~

Allocated cell

~~$c_{11} = u_1 + v_1 = 4, v_1 = 4$~~

~~$c_{13} = u_1 + v_3 = 22, v_3 = 22$~~

~~$c_{14} = u_1 + v_4 = 11, v_4 = 11$~~

~~$c_{21} = u_2 + v_1 = 0, u_2 = -4$~~

~~$c_{31} = u_3 + v_1 = 6, u_3 = -4$~~

~~$c_{32} = u_3 + v_2 = 6, v_2 = 10$~~

~~$c_{35} = u_3 + v_5 = 0, v_5 = 4$~~

10		60	30	
4	19	22	11	0
30				
0	9	14	14	0
E	20			50
6	6	16	14	0

($u_1 = 0$)

Allocated cell

$c_{11} = u_1 + v_1 = 4, v_1 = 4$

$c_{13} = u_1 + v_3 = 22, v_3 = 22$

$c_{14} = u_1 + v_4 = 11, v_4 = 11$

$c_{21} = u_2 + v_1 = 0, u_2 = -4$

$c_{31} = u_3 + v_1 = 6, u_3 = 2$

$c_{32} = u_3 + v_2 = 6, v_2 = 4$

$c_{35} = u_3 + v_5 = 0, v_5 = -2$

Non Allocated Net evolution

$\Delta_{12} = c_{12} - (u_1 + v_2) = 19 - (0+4) = 15 > 0$

$\Delta_{15} = c_{15} - (u_1 + v_5) = 0 - (0-2) = 2 > 0$

$\Delta_{22} = c_{22} - (u_2 + v_2) = 9 - (-4+4) = 9 > 0$

$\Delta_{23} = c_{23} - (u_2 + v_3) = 14 - (-4+22) = -4 < 0$

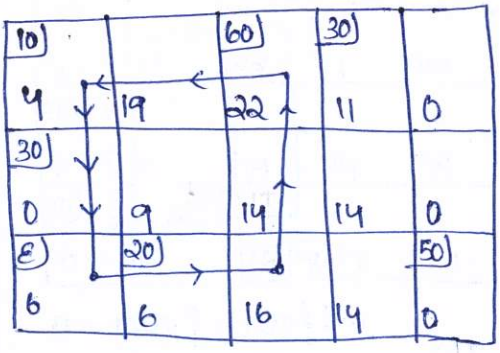
$\Delta_{24} = c_{24} - (u_2 + v_4) = 14 - (-4+11) = 7 > 0$

$\Delta_{25} = c_{25} - (u_2 + v_5) = 0 - (-4-2) = 6 > 0$

$\Delta_{33} = c_{33} - (u_3 + v_3) = 16 - (2+22) = -8 < 0$ ✓

$\Delta_{34} = c_{34} - (u_3 + v_4) = 14 - (2+11) = 1 > 0$

$\Delta_{35} = c_{35} - (u_3 + v_5) = 0 - (2-2) = 0$



		[20]	[30]	[50]	100
4	19	22	11	44	
[30]					30
0	9	14	14	44	
[10]	[20]	[40]			70
6	6	16	14	44	
40	20	60	30	50	

This is a non-degeneracy T.P as $m+n-1 = 7$ = occupied cell

Let $U_1 = 0$

Allocated cell ($U_1 = 0$)

$C_{13} = U_1 + V_3 = 22, V_3 = 22$

$C_{14} = U_1 + V_4 = 11, V_4 = 11$

$C_{15} = U_1 + V_5 = 44, V_5 = 44$

$C_{21} = U_2 + V_1 = 0, U_2 = -12$

$C_{31} = U_3 + V_1 = 6, V_1 = 12$

$C_{32} = U_3 + V_2 = 6, V_2 = 12$

$C_{33} = U_3 + V_3 = 16, U_3 = -6$

Non Allocated cell

$\Delta_{11} = C_{11} - (U_1 + V_1) = 4 - (0+12) = -8 < 0$

$\Delta_{12} = 19 - (0+12) = 7 > 0$

$\Delta_{22} = 9 - (-12+2) = 9 > 0$

$\Delta_{23} = 14 - (-12+22) = 4 > 0$

$\Delta_{24} = 14 - (-12+11) = 15 > 0$

$\Delta_{25} = 44 - (-12+44) = 12 > 0$

$\Delta_{34} = 14 - (-6+11) = 9 > 0$

$\Delta_{35} = 44 - (-6+44) = 6 > 0$

As $\Delta_{11} < 0$, so the T.P has not reached optimal, hence we need to start a loop from C_{11}

(+)		(-) [20]	[30]	[50]
4	19	22	11	44
[30]				
0	9	14	14	44
[10]	[20]	(+)		
6 (-)	6	16	[40]	14	44

$\theta = \min\{10, 20\} = 10$

→

[10]		[10]	[30]	[50]
4	19	22	11	44
[30]				
0	9	14	14	44
	[20]	[50]		
6	6	16	14	44

This New T.P is Non-degeneracy

Allocated cell ($U_1 = 0$)

$C_{11} = U_1 + V_1 = 4, V_1 = 4$

$C_{13} = U_1 + V_3 = 22, V_3 = 22$

$C_{14} = U_1 + V_4 = 11, V_4 = 11$

$C_{15} = U_1 + V_5 = 44, V_5 = 44$

$C_{21} = U_2 + V_1 = 0, U_2 = -4$

$C_{32} = U_3 + V_2 = 6, V_2 = 12$

$C_{33} = U_3 + V_3 = 16, U_3 = -6$

Non Allocated

$\Delta_{12} = C_{12} - (U_1 + V_2) = 19 - (0+12) = 7 > 0$

$\Delta_{22} = 9 - (-4+12) = 1 > 0$

$\Delta_{23} = 14 - (-4+22) = -4 < 0$

$\Delta_{24} = 14 - (-4+11) = 7 > 0$

$\Delta_{25} = 44 - (-4+44) = 4 > 0$

$\Delta_{31} = 6 - (-6+4) = 8 > 0$

$\Delta_{34} = 14 - (-6+11) = 9 > 0$

$\Delta_{35} = 44 - (-6+44) = 6 > 0$

As $\Delta_{23} < 0$ so the T.P has not reached optimal and we have to start a loop from C_{23}

(+) 10		(-) 10	30	50
4	19	22	11	44
30				
0 (-)	9	14 (+)	14	44
	20	50		
6	6	16	14	44

$\theta = \min\{30, 10\} = 10$

→

20			30	50
4	19	22	11	44
20		10		
0	9	14	14	44
	20	50		
6	6	6	14	44

The New T.P is Non-degenerecy,

Allocated cell ($U_1 = 0$)

$c_{11} = U_1 + V_1 = 4, V_1 = 4$

$c_{14} = U_1 + V_4 = 11, V_4 = 11$

$c_{15} = U_1 + V_5 = 44, V_5 = 44$

$c_{21} = U_2 + V_1 = 0, U_2 = -4$

$c_{23} = U_2 + V_3 = 14, V_3 = 18$

$c_{32} = U_3 + V_2 = 6, V_2 = 8$

$c_{33} = U_3 + V_3 = 16, U_3 = -2$

Non Allocated cell (Net evalution)

$\Delta_{12} = c_{12} - (U_1 + V_2) = 19 - (0+8) = 11 > 0$

$\Delta_{13} = 22 - (0+18) = 4 > 0$

$\Delta_{22} = 9 - (-4+8) = 5 > 0$

$\Delta_{24} = 14 - (-4+11) = 7 > 0$

$\Delta_{25} = 44 - (-4+44) = 4 > 0$

$\Delta_{31} = 6 - (-2+4) = 4 > 0$

$\Delta_{34} = 14 - (-2+11) = 5 > 0$

$\Delta_{35} = 44 - (-2+44) = 2 > 0$

Since all $\Delta ij \geq 0$, the ~~solution~~ T.P has reached to optimal solution and the Profit cost = $40\times20 + 33\times30 + 0\times50 + 44\times20 + 30\times10 + 38\times20 + 28\times50$

$= 5130$

The Assignment Problem is a minimization problem, where we have to assign 'n' job to 'n' person in such a way that the total time will be minimum or total cost will be minimum.

To solve an Assignment Method, we have to follow Hungarian Method

Step-1 Row Reduction

Step-2 Column Reduction

Step-3 Assignment

Row Reduction

Find out the smallest element in each row and subtract it from every element of that row.

Column Reduction

After row reduction, find out the smallest element in each column and subtract from every element of that column.

Assignment Process

(Row Assignment) → Find out single zero in a row make an rectangle there and cross all other zeros of that corresponding column, and continue this process from first row to last row.

(Column Assignment) – After row assignment, find out a single zero in a column, make a rectangle there and cross all other zeros of that corresponding row and continue this process from first column to last column

Then if the no. of assignment are equal to the order of the matrix, then our soln is optimal

N.B: Check the no. of assignment must be 1 in each row and each column, otherwise you are wrong.

Q.

	J_1	J_2	J_3	J_4	J_5
P_1	10	3	3	2	8
P_2	9	7	8	2	7
P_3	7	5	6	2	4
P_4	3	5	8	2	4
P_5	9	10	9	6	10

row reduction →

	8	1	1	0	6
	7	5	6	0	5
	5	3	4	0	2
	1	3	6	0	2
	3	4	3	0	4

Column reduction

7	[0]	✗	✗	4	
6	4	5	[0]	3	
4	2	3	✗	[0]	
[0]	2	5	✗	✗	
2	1	2	✗	2	

↓

9	[0]	✗	2	6	
6	2	3	[0]	3	
4	✗	1	✗	[0]	
[0]	✗	3	✗	✗	
2	1	[0]	✗	2	

→

	Assi]	Cost
P_1	J_2	3
P_2	J_4	2
P_3	J_5	4
P_4	J_1	3
P_5	J_3	9
		21

In this problem, after assignment process is over the no. of assignments are 4 which is not equal to the order of matrix, so over sol^n is not optimal

Step-4 If the sol^n is not optimal, draw minimum no. of lines (horizontal & vertical) to cover all the zeros of the matrix Then find out the smallest uncovered element by the lines subtract it from uncovered ; and add it to the intersection elements of the line, Then we have a new Table and start the as usual assignment process to find the sol^n.

P_1
P_2
P_3
P_4
P_5

Here the no. of assignments are 5, and we have assigned

To solve an assignment problem, whether it is balanced or unbalanced. An assignment problem is said to be balanced if no. of rows or no. of column are equal. If we have an unbalance assignment problem then make it balance by adding a dummy row or dummy column as per the requirement, where the entries of dummy rows or column are Zero

Row Reduction

Column Reduction

$$\begin{array}{c} \\ M_1 \\ M_2 \\ M_3 \\ M_4 \end{array}\begin{array}{c} \begin{array}{cccc} J_1 & J_2 & J_3 & J_4 \end{array} \\ \begin{bmatrix} 5 & 8 & 4 & 10 \\ 7 & 5 & 7 & 4 \\ 11 & 9 & 10 & 8 \\ 6 & 6 & 7 & 3 \end{bmatrix} \end{array} \rightarrow \begin{bmatrix} 1 & 7 & 0 & 6 \\ 3 & 1 & 3 & 0 \\ 3 & 1 & 2 & 0 \\ 3 & 3 & 4 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 0 & 6 & 0 & 6 \\ 2 & 0 & 3 & 0 \\ 2 & 0 & 2 & 0 \\ 2 & 2 & 4 & 0 \end{bmatrix}$$

Assignment Process

Drawing lines

$$\rightarrow \begin{bmatrix} [0] & 6 & \cancel{0} & 6 \\ 2 & [0] & 3 & \cancel{0} \\ 2 & \cancel{0} & 2 & \cancel{0} \\ 2 & 2 & 4 & [0] \end{bmatrix} \quad \begin{bmatrix} \cancel{[0]} & \cancel{6} & \cancel{0} & \cancel{6} \\ 2 & [0] & 3 & \cancel{0} \\ 2 & \cancel{0} & 2 & \cancel{0} \\ 2 & 2 & 4 & [0] \end{bmatrix} \rightarrow \begin{bmatrix} 0 & 8 & 0 & 8 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 2 & 2 & 0 \end{bmatrix}$$

Row Reduction

Column Reduction

	M_1	M_2	M_3	M_4
J_1	5	7	11	6
J_2	8	5	9	6
J_3	4	7	10	7
J_4	10	4	8	3

$\Rightarrow$

0	2	6	1
3	0	4	1
0	3	6	3
7	1	5	0

$\rightarrow$

0	2	2	1
3	0	0	1
0	3	2	3
7	1	1	0

Assignment Process

[0]	2	2	1
3	[0]	✗	1
✗	3	2	3
7	1	1	[0]

(Not optimal)

(Drawing lines)

[0]	2	2	1
~~3~~	~~[0]~~	~~✗~~	~~1~~
✗	3	2	3
~~7~~	~~1~~	~~1~~	~~[0]~~

(New Table)

0	1	1	0
4	0	0	1
0	2	1	2
8	1	1	0

Row Reduction of New Table

0	1	1	0
4	0	0	1
0	2	1	2
8	1	1	0

Column Reduction

0	1	1	0
4	0	0	1
0	2	1	2
8	1	1	0

Assignment Process

✗	1	1	✗
~~4~~	~~[0]~~	~~✗~~	~~1~~
[0]	2	1	2
8	1	1	[0]

0	1	1	0
5	0	0	2
0	1	0	2
8	0	0	0

Q.

row reduction → column reduction → Assignment Process

$$\begin{bmatrix} 5 & 7 & 11 & 6 \\ 8 & 5 & 9 & 6 \\ 4 & 7 & 10 & 7 \\ 10 & 4 & 8 & 3 \end{bmatrix} \rightarrow \begin{bmatrix} 0 & 2 & 6 & 1 \\ 3 & 0 & 4 & 1 \\ 0 & 3 & 6 & 3 \\ 7 & 1 & 5 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 0 & 2 & 2 & 1 \\ 3 & 0 & 0 & 1 \\ 0 & 3 & 2 & 3 \\ 7 & 1 & 1 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} [0] & 2 & 2 & 1 \\ 3 & [0] & \cancel{0} & 1 \\ \cancel{0} & 3 & 2 & 3 \\ 7 & 1 & 1 & [0] \end{bmatrix}$$

Drawing lines → New Table → Assignment Process → Drawing lines

$$\begin{bmatrix} [0] & 2 & 2 & \cancel{1} \\ \cancel{3} & [0] & \cancel{0} & \cancel{1} \\ \cancel{0} & 3 & 2 & 3 \\ \cancel{7} & 1 & 1 & [0] \end{bmatrix} \rightarrow \begin{bmatrix} 0 & 1 & 1 & 1 \\ 4 & 0 & 0 & 2 \\ 0 & 2 & 1 & 3 \\ 7 & 0 & 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} [0] & 1 & 1 & 1 \\ 4 & [0] & \cancel{0} & 2 \\ \cancel{0} & 2 & 1 & 3 \\ 7 & \cancel{0} & \cancel{0} & [0] \end{bmatrix} \rightarrow \begin{bmatrix} [0] & 1 & 1 & 1 \\ \cancel{4} & [0] & \cancel{0} & \cancel{2} \\ \cancel{0} & 2 & 1 & 3 \\ \cancel{7} & \cancel{0} & \cancel{0} & [0] \end{bmatrix}$$

$$\begin{bmatrix} [0] & \cancel{0} & \cancel{0} & \cancel{0} \\ 5 & [0] & \cancel{0} & 2 \\ \cancel{0} & 1 & [0] & 2 \\ 8 & \cancel{0} & \cancel{0} & [0] \end{bmatrix} \rightarrow \begin{matrix} J_1 \rightarrow M_1 \\ J_2 \rightarrow M_2 \\ J_3 \rightarrow M_3 \\ J_4 \rightarrow M_4 \end{matrix}$$

and Cost = 5 + 5 + 10 + 3
= 23

When there is no single zero in row or column in a assignment table, make an assignment arbitrarily in such a way that there must be single assignment in each row and each column.

Maximization case in Assignment Problem

Like T.P., assignment problem is also a minimization problem, but sometime the given matrix will be a profit matrix, in that case we have to convert the maximization matrix to minimization matrix by subtracting all the elements of that matrix from the largest element of matrix, and then follow the hungarian procedure to find the solution, after getting the optimal assignment choose exact value of the initial matrix to get the solution.

	D_1	D_2	D_3	D_4	D_5
S_1	32	38	40	28	40
S_2	40	24	28	21	36
S_3	41	27	33	30	37
S_4	28	38	41	36	36
S_5	29	33	40	35	39

→ Minimization

$$\begin{bmatrix} 9 & 3 & 1 & 13 & 1 \\ 1 & 17 & 13 & 20 & 5 \\ 0 & 14 & 8 & 11 & 4 \\ 13 & 3 & 0 & 5 & 5 \\ 12 & 8 & 1 & 6 & 2 \end{bmatrix}$$

→ row reduction

$$\begin{bmatrix} 8 & 2 & 0 & 12 & 0 \\ 0 & 16 & 12 & 19 & 4 \\ 0 & 14 & 8 & 11 & 4 \\ 10 & 3 & 0 & 5 & 5 \\ 11 & 7 & 0 & 5 & 1 \end{bmatrix}$$

→ Column reduction

$$\begin{bmatrix} 8 & 0 & 0 & 7 & 0 \\ 0 & 14 & 12 & 14 & 4 \\ 0 & 12 & 8 & 6 & 4 \\ 13 & 1 & 0 & 0 & 5 \\ 11 & 5 & 0 & 0 & 1 \end{bmatrix}$$

→ Assignment Process

$$\begin{bmatrix} 8 & [0] & 7 & 7 \\ [0] & 14 & 12 & 14 & 4 \\ 12 & 8 & 6 & 4 \\ 13 & 1 & [0] & 5 \\ 11 & 5 & [0] & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & [0] & 1 & 8 \\ [0] & 13 & 12 & 14 & 3 \\ 11 & 8 & 6 & 3 \\ 13 & [0] & 4 \\ 11 & 4 & [0] \end{bmatrix}$$

$$\begin{bmatrix} 12 & [0] & 1 & 8 \\ 10 & 9 & 11 & [0] \\ [0] & 8 & 5 & 3 \\ 16 & [0] & 4 \\ 14 & 4 & [0] \end{bmatrix} \rightarrow$$

$S_1 \rightarrow D_2 \rightarrow 38 + 36 + 41 + 36 + 40$

$S_2 \rightarrow D_5 \quad = 191$

$S_3 \rightarrow D_1$

$S_4 \rightarrow D_4$

$S_5 - D_3$

Q: Sec-A (Central Tendency)

Q1 Find the sum of 'n' natural no.
Q2 Find the S.D of 'n' natural no. } (2 marks)
Q3 Correct mean

Sec-B Q1 missing frequency (mean, median, mode)
Q2 combined mean

S.Q check the consistency bet' two series,
Diff' between absolute measure & relative measure
Rel' bet' MD, SD, QD and range
Combined S.D
Correct S.D
Range of correlation of coefficient Types (Population correlation coefficient (π ± PE)

L.Q. Table
calculation of Q_1 and Q_3 of continuous series
L.Q from correlation

Let x_1 and x_2 ...

$\frac{n(n+1)}{2}$

Assignment

Q.1

	A	B	C	D
W	41	72	39	52
X	22	29	49	65
Y	27	39	60	51
Z	45	50	48	52

Row Reduction

2	33	0	13
0	7	27	43
0	12	33	24
0	5	3	7

Column Reduction

2	28	0	6
0	2	27	56
0	7	33	17
0	0	3	0

Assignment Process

~~2~~	~~28~~	[0]	6
[0]	2	27	56
~~0~~	7	33	17
~~0~~	[0]	~~3~~	~~0~~

New Table

4	28	[0]	6
~~0~~	[0]	25	54
[0]	5	31	15
2	~~0~~	3	[0]

Cost = 39 + 29 + 27 + 52
= 147

Q.1

	A	B	C	D
W	~~41~~	~~72~~	39	52
X	22	29	49	65
Y	27	39	60	51
Z	45	50	48	52

~~31~~	~~0~~	~~33~~	~~20~~
~~50~~	~~43~~	~~23~~	7
~~45~~	~~33~~	~~12~~	~~21~~
~~27~~	~~22~~	~~24~~	~~20~~

~~31~~	~~0~~	~~33~~	~~20~~
~~43~~	~~36~~	~~16~~	~~0~~
~~33~~	~~24~~	~~0~~	~~9~~
~~7~~	~~2~~	~~4~~	~~0~~

Q.2

1000	1200	400	7000
600	500	300	800
200	300	400	500
600	700	300	1000

Row Reduction

600	800	0	300
300	200	0	500
0	100	200	300
300	400	0	700

Column Reduction

600	700	0	0
300	100	0	200
0	0	200	0
300	300	0	400

Assignment Process

600	700	~~0~~	[0]
300	100	[0]	200
[0]	~~0~~	~~200~~	~~0~~
300	300	~~0~~	400

500	600	~~0~~	[0]
200	[0]	~~0~~	200
[0]	~~0~~	300	100
200	200	[0]	400

Cost = 700 + 500 + 200 + 300
= 1700

Q3

$$\begin{bmatrix} 7 & 9 & 3 & 3 & 2 \\ 6 & 1 & 6 & 6 & 5 \\ 1 & 5 & 2 & 2 & 4 \\ 6 & 6 & 9 & 4 & 2 \end{bmatrix} \rightarrow \begin{bmatrix} 5 & 7 & 1 & 1 & 0 \\ 5 & 0 & 5 & 5 & 4 \\ 0 & 4 & 6 & 7 & 4 \\ 4 & 4 & 7 & 2 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 5 & 7 & [0] & \cancel{4} & \cancel{4} \\ 5 & [0] & 4 & 4 & 4 \\ [0] & 1 & 5 & \cancel{6} & 4 \\ 4 & 4 & 6 & \cancel{1} & [0] \end{bmatrix}$$

Q3

$$\begin{bmatrix} 7 & 9 & 3 & 3 & 2 \\ 6 & 1 & 6 & 6 & 5 \\ 1 & 5 & 2 & 2 & 4 \\ 6 & 6 & 9 & 4 & 2 \end{bmatrix} \rightarrow \begin{bmatrix} 5 & 7 & 1 & 1 & 0 \\ 5 & 0 & 5 & 5 & 4 \\ 0 & 4 & 1 & 1 & 3 \\ 4 & 4 & 7 & 2 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 5 & 7 & [0] & \cancel{3} & \cancel{3} \\ 5 & [0] & 4 & 4 & 4 \\ [0] & 4 & \cancel{1} & [0] & 3 \\ 4 & 4 & 6 & 1 & [0] \end{bmatrix}$$

Cost = 3 + 1 + 1 +

Q3

$$\begin{bmatrix} 7 & 9 & 3 & 3 & 2 \\ 6 & 1 & 6 & 6 & 5 \\ 3 & 4 & 9 & 10 & 7 \\ 1 & 5 & 2 & 2 & 4 \\ 6 & 6 & 9 & 4 & 2 \end{bmatrix} \rightarrow \begin{bmatrix} 5 & 7 & 1 & 1 & 0 \\ 5 & 0 & 5 & 5 & 4 \\ 0 & 1 & 6 & 7 & 4 \\ 0 & 4 & 1 & 1 & 3 \\ 4 & 4 & 7 & 2 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 5 & 7 & [0] & \cancel{3} & \cancel{3} \\ 5 & [0] & 4 & 4 & 4 \\ [0] & 1 & 5 & 6 & 4 \\ \cancel{3} & 4 & \cancel{1} & [0] & 3 \\ 4 & 4 & 6 & 1 & [0] \end{bmatrix}$$

Cost = 3 + 1 + 3 + 2 + 2
= 11

Q4

$$\begin{bmatrix} 48 & 48 & 50 & 44 \\ 56 & 60 & 60 & 68 \\ 96 & 94 & 90 & 85 \\ 42 & 44 & 54 & 46 \end{bmatrix} \rightarrow \begin{bmatrix} 4 & 4 & 6 & 0 \\ 0 & 4 & 4 & 12 \\ 11 & 9 & 5 & 0 \\ 0 & 2 & 12 & 4 \end{bmatrix} \rightarrow \begin{bmatrix} 4 & 2 & 2 & [0] \\ \cancel{2} & 2 & [0] & 12 \\ 11 & 7 & 1 & \cancel{0} \\ \cancel{0} & [0] & 8 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 1 & 1 & [0] \\ [0] & 2 & \cancel{0} & 3 \\ 10 & 6 & [0] & \cancel{0} \\ \cancel{0} & [0] & 8 & 5 \end{bmatrix}$$

Cost = 44 + 56 + 90 + 44
= 234

Regression Analysis

In statistical study, regression analysis is a set of statistical process for estimating the relationships between a dependent variable and one or more independent variables.

This technique was developed by the British Biometrician Sir Francis Galton in 1877. He studied the average relationship between the two variables graphically, and named the line describing the relationship, the "Line of Regression"

Definition

Taro Yamane - "One of the most frequently used techniques in economics and business research to find a relation between two or more variables that are related is known as Regression Analysis."

Characteristics

- It consists of mathematical terms that are used to measure the average relationship between two or more closely related variables.
- It is used for estimating the unknown values of some dependent variable with reference to the known values of its related independent variables.
- It provides a mechanism for prediction or forecast of the values of one variables in terms of the other variables.
- It consists of two lines of equations.
  i.e. (i) Eqⁿ of X on Y
  (ii) Eqⁿ of Y on X

Regression Lines

(1) Y on X :

Here, x = given , y = to be determined
x = independent y = dependent

The eqⁿ of regression line Y on X is

$$\boxed{(y-\bar{y}) = b_{yx}(x-\bar{x})}$$

where, b_{yx} = regression coefficient Y on X

$$\Rightarrow b_{yx} = r \cdot \frac{\sigma_y}{\sigma_x}$$

$\bar{y}$ = mean of series Y
$\bar{x}$ = mean of series X
σ_y = S.D. of series Y
σ_x = S.D. of series X

(2) X on Y :

Here, y = given , x = to be determined
y = independent x = dependent

The eq^n of regression line X on Y is

$$(X - \bar{X}) = b_{xy}(Y - \bar{Y})$$

where, b_{xy} = regression coefficient X on Y

$$\Rightarrow b_{xy} = r \cdot \frac{\sigma_x}{\sigma_y}$$

Note :- Multiplying the regression coefficient of line Y on X and the regression coefficient of line X on Y, we get -

$$b_{yx} \cdot b_{xy} = r \cdot \frac{\sigma_y}{\sigma_x} \times r \cdot \frac{\sigma_x}{\sigma_y}$$

$$= r^2$$

$$\Rightarrow r^2 = b_{yx} \cdot b_{xy}$$

$$\Rightarrow r = \pm\sqrt{b_{yx} \cdot b_{xy}}$$

- The sign of both b_{yx} and b_{xy} should be either positive or negative.
- r^2 is positive or negative that depends upon the sign of regression coefficients.

Direct Method :-

$$b_{yx} = \frac{n\Sigma xy - \Sigma x \cdot \Sigma y}{n\Sigma x^2 - (\Sigma x)^2}, \quad b_{xy} = \frac{n\Sigma xy - \Sigma x \cdot \Sigma y}{n\Sigma y^2 - (\Sigma y)^2}$$

then find the regression line.

Shortcut Method

This method is useful for obtaining the equations of the lines of regression if the mean values $\bar{x}$ and $\bar{y}$ come out to be in fractions or if the values of x and y are large.

$$b_{yx} = \frac{cov(x,y)}{\sigma_x^2}$$

$\because b_{yx} = r \cdot \frac{\sigma_x}{\sigma_y}$

$$\Rightarrow b_{yx} = \frac{cov(x,y)}{\sigma_x \cdot \sigma_y} \cdot \frac{\sigma_y}{\sigma_x}$$

$$\Rightarrow b_{yx} = \frac{cov(x,y)}{\sigma_x^2}$$

$$\Rightarrow b_{yx} = \frac{\frac{\Sigma (X-\bar{X})(Y-\bar{Y})}{N}}{\frac{\Sigma (X-\bar{X})^2}{N}}$$

$$\Rightarrow \boxed{b_{yx} = \frac{\Sigma (X-\bar{X})(Y-\bar{Y})}{\Sigma (X-\bar{X})^2}}$$

We can also write it as,

$$\Rightarrow \boxed{b_{yx} = \frac{\Sigma dx \cdot dy}{\Sigma dx^2}}$$

Similarly, $b_{xy} = \frac{cov(x,y)}{\sigma_y^2}$

$$\Rightarrow b_{xy} = \frac{\frac{\Sigma (x-\bar{x})(y-\bar{y})}{N}}{\frac{\Sigma (y-\bar{y})^2}{N}}$$

$$\Rightarrow \boxed{b_{xy} = \frac{\Sigma (x-\bar{x})(y-\bar{y})}{\Sigma (y-\bar{y})^2}}$$

we can also convert it as

$$\boxed{b_{xy} = \frac{\Sigma dx \cdot dy}{\Sigma dy^2}}$$

$\because b_{xy} = r \cdot \frac{\sigma_x}{\sigma_y}$

$$\Rightarrow b_{xy} = \frac{cov(x,y)}{\sigma_x \cdot \sigma_y} \times \frac{\sigma_x}{\sigma_y}$$

$$\Rightarrow b_{xy} = \frac{cov(x,y)}{\sigma_y^2}$$

Q. Given the following values of X and Y

X: 3 5 6 8 9 11

Y: 2 3 4 6 5 8

Find the eq^n of regression line (i) Y on X (ii) X on Y by direct method.

X	Y	X^2	Y^2	XY
3	2	9	4	6
5	3	25	9	15
6	4	36	16	24
8	6	64	36	48
9	5	81	25	45
11	8	121	64	88
42	28	336	154	226

Here $\bar{X} = \frac{42}{6} = 7$

$\bar{Y} = \frac{28}{6} = 4.66$

According to Direct method,

$$b_{yx} = \frac{n\Sigma XY - \Sigma X \cdot \Sigma Y}{n\Sigma X^2 - (\Sigma X)^2}$$

$$= \frac{(6 \times 226) - (42 \times 28)}{(6 \times 336) - (42)^2} = \frac{1356 - 1176}{2016 - 1764}$$

$$= \frac{180}{252} = 0.71$$

And $b_{xy} = \dfrac{n\Sigma xy - \Sigma x \cdot \Sigma y}{n\Sigma y^2 - (\Sigma y)^2}$

$= \dfrac{(6 \times 226) - (42 \times 28)}{(6 \times 154) - (28)^2}$

$= \dfrac{180}{940 - 784}$

$= \dfrac{180}{156} = 1.15$

Now, Eq^n of regression line y on x is

$(y - \bar{y}) = b_{yx}(x - \bar{x})$

$\Rightarrow (y - 4.66) = 0.71 (x - 7)$

$\Rightarrow y - 4.66 = 0.71x - 4.97$

$\Rightarrow 0.71x - y = 0.31$

$\Rightarrow 0.71x - y - 0.31 = 0$

And Eq^n of regression line x on y is

$(x - \bar{x}) = b_{xy}(y - \bar{y})$

$\Rightarrow (x - 7) = 1.15 (y - 4.66)$

$\Rightarrow x - 7 = 1.15y - 5.35$

$\Rightarrow x - 1.15y = 1.65$

$\Rightarrow x - 1.15y - 1.65 = 0$

Q: Find the regression equation of y on x and x on y. where x and y are the marces obtained by 10 students as given below. (By shortcut method)

→

x	y	dx	dy	dx^2	dy^2	dxdy
20	20	-23	-26.5	529	702.25	609.5
45	60	2	13.5	4	182.25	27
65	55	22	8.5	484	72.25	187
40	45	-3	-1.5	9	2.25	4.5
55	75	12	28.5	144	812.25	342
35	35	-8	-11.5	64	132.25	92
15	25	-28	-21.5	784	462.25	602
80	90	37	43.5	7369	1892.25	1609.5
25	10	-18	-36.5	324	1332.25	657
50	50	7	3.5	49	12.25	24.5
430	465			3760	5528.5	4155

$\bar{x} = \dfrac{430}{10} = 43$

$\bar{y} = \dfrac{465}{10} = 46.5$

According to shortcut method,

$$b_{yx} = \frac{\Sigma dx.dy}{\Sigma dx^2} = \frac{4155}{3760} = 1.10$$

and $$b_{xy} = \frac{\Sigma dx.dy}{\Sigma dy^2} = \frac{4155}{5528.5} = 0.75$$

Now, Eq^n of regression line y on x is

$$(y-\bar{y}) = b_{yx}(x-\bar{x})$$

$$\Rightarrow (y-46.5) = 1.10(x-43)$$

$$\Rightarrow y - 46.5 = 1.10x - 47.3$$

$$\Rightarrow 1.1x - y = -0.8$$

$$\Rightarrow 1.1x - y + 0.8 = 0$$

Eq^n of regression line x on y is

$$(x-\bar{x}) = b_{xy}(y-\bar{y})$$

$$\Rightarrow (x-43) = 0.75(y-46.5)$$

$$\Rightarrow x - 43 = 0.75y - 34.87$$

$$\Rightarrow x - 0.75y = 8.13$$

$$\Rightarrow x - 0.75y - 8.13 = 0$$

Q. Out of the following two lines of regression, which one is the regression line of x only and which one is the regression line of y on x ?

$4x - 5y + 30 = 0$ and $20x - 9y - 107 = 0$.

→ Let us consider that,

$$4x - 5y + 30 = 0 \quad \text{——(1)}$$

$$20x - 9y - 107 = 0 \quad \text{——(2)}$$

Assume that,

Eq^n(1) is the regression line of x on y

Eq^n(2) is the regression line of y on x.

Then according to the form of the regression line of x on y, eq^n(1) becomes-

$$4x = 5y - 30$$

$$\Rightarrow x = \frac{5}{4}y - \frac{15}{2} \quad \text{——(3)}$$

Similarly, according to the form of the regression line of y on x, eq^n(2) becomes-

$$9y = 20x - 107$$

$$\Rightarrow y = \frac{20}{9}x - \frac{107}{9} \quad \text{——(4)}$$

From eq^n(3), $b_{xy} = \frac{5}{4}$

From eq^n(4), $b_{yx} = \frac{20}{9}$

Now, we know that,

$$r^2 = b_{xy} \cdot b_{yx}$$

Putting the values in above, we get -

$$r^2 = \frac{5}{4} \times \frac{20}{9} = \frac{25}{9}$$

$$\Rightarrow r^2 = 2.77$$

$$\Rightarrow r = 1.66$$

Here $r^2 > 1$, so that $r \notin [-1, 1]$

But, from Correlation Analysis, we know that,

'r' always lies between $[-1, 1]$ $\left[\because -1 \leq r \leq r\right]$

Hence, it is clear that,

Our assumption is wrong

So again we have to rewrite the given equations by assuming as below:

Eq^n(1) is the regression line of y on x

Eq^n(2) " " " " of x on y

Then, according to the forms of the regression line of y on x & x on y, we can write,

eq^n(1) becomes, $5y = 4x + 30$

$$\Rightarrow y = \frac{4}{5}x + 6 \quad \text{——— (5)}$$

eq^n(2), becomes, $20x = 9y + 107$

$$\Rightarrow x = \frac{9}{20}y + \frac{107}{20} \quad \text{——— (6)}$$

Now, From eq^n(5), $b_{yx} = \frac{4}{5}$

From eq^n(6), $b_{xy} = \frac{9}{20}$

we have, $r^2 = b_{yx} \cdot b_{xy}$

$$\Rightarrow r^2 = \frac{4}{5} \times \frac{9}{20} = \frac{9}{25}$$

$\Rightarrow r^2 = 0.36$

$\Rightarrow r = \pm 0.6$

Here $r^2 < 1$ so that, $r \in [-1, 1]$

whether, r is positive or negative that depends upon the sign of regression coefficients.

Here we have to take the value of r in positive

[$\because$ b_{xy} & b_{yx} both are positive]

So, our second assumption is right.

i.e $4x - 5y + 30 = 0$ is the regression line of y on x and $20x - 9y - 107 = 0$ is the regression line of x on y.

Q. What is Game Theory.

→ Game is defined as the activity between two or more person which involves with a certain set of rules. At the end of the game each person recieves some benefits or sufferes loss. The set of rules and procedures defines the game. A competitive situation is called game. The term game represents a conflict between two or more parties.

Game Theory

Module-II — Linear Programming Problem — (Quantitative Technique)

Linear Programming Problem is an optimization problem, where there is an objective function and subject to constraints and non-negative restriction.

There are 2 parts in LPP —

(I) Formulation of LPP

(II) Solution of LPP → (i) Graphical solution
(ii) Simplex Method

* Formulation of LPP

A manufacturing unit for a production company produce 2 types of products M_1 & M_2. Product M_1 requires, 4 hours of grinding and 2 hours of polishing, where product M_2 requires 2 hours of grinding and 5 hours of polishing. Then the company has 2 grinders and 3 polishers, each grinders works 40 hrs a week and each polishers works 60 hrs a week. After production the product M_1 is Rs. 3 profit and M_2 is Rs. 4 profit per unit. Formulate a mathematical modle to maximize the profit.

→ Step-1 Define the decision variables

Step-2 Construct the objective function

Step-3 Formulate the subject to constraint

Step-4 Declare the non-negative constraint-

→ (Step-1) Let X_1 and X_2 are two decision variables.

X_1 stands for product M_1

X_2 " " " M_2

Let X_1 & X_2 units of products produced in that company for M_1 and M_2

(Step-2) Since profit on 1 unit for product M_1 is 3 rupees

So total product profit on X_1 is $3X_1$

Similarly total product M_2 profit is $4X_2$

The objective function is here $Z = 3X_1 + 4X_2$

$$\text{Max } Z = 3X_1 + 4X_2$$

Step-3

	x_1	x_2	
Grinderer	4	2	$\leq$ 80 hrs
Polisherr	2	5	$\leq$ 180 hrs

Step-4

$x_1 \geq 0,\ x_2 \geq 0$

→ Max $z = 3x_1 + 4x_2$

$4x_1 + 2x_2 \leq 80$

$2x_1 + 5x_2 \leq 180$

$x_1, x_2 \geq 0$

Graphical Solution

If the LPP having two variables only, then it can be solve by graphical method, if the LPP having more than 2 variables or 2 variables, then it can be solve by simplex method.

There 4 types of Graphical solution

(1) Unique solution
(2) No solution
(3) Infinetly many solution
(4) Unbounded solution

→ Step-1 Express all the inequality constraints into equation

$4x_1 + 2x_2 = 80$

x_1	0	20
x_2	40	0

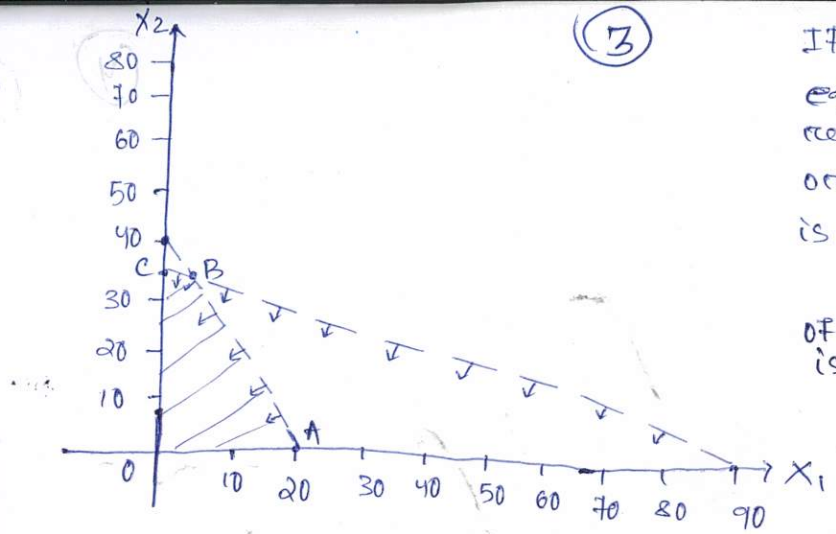
$(x_1, x_2) = (0, 40), (20, 0)$

$2x_1 + 5x_2 = 180$

x_1	0	90
x_2	36	0

$(x_1, x_2) = (0, 36)$ & $(90, 0)$

The solution of LPP in graphical technique will be found at the corner points of the feasible region.



If the constraint having less than equal to sign, then consider the region below the line towards the origin. Here the common region is OABC

The common region, which satisfy of the constraints of the given LPP is known common feasible region.

Points	Max $Z = 3X_1 + 4X_2$
O (0,0)	$Z = 0$
A (20,0)	$Z = 60$
B (2.5, 35)	$Z = 7.5 + 140 = 147.5$
C (0, 36)	$Z = 144$

Z is maximum at point B(2.5, 35)
So the optimal solution is $X_1 = 2.5$, $X_2 = 35$
Max $Z = 147.5$

$4X_1 + 2X_2 = 80$ — (1)
$2X_1 + 5X_2 = 180$ — (2)

multiplying (2) on eq^n (2) & subtract it with eq^n (1)

$4X_1 + 2X_2 = 80$
$4X_1 + 10X_2 = 360$
$-8X_2 = -280$
$X_2 = 35$

Putting X_2 on eq^n (1), we get -

$4X_1 + 2 \times 35 = 80$
$\Rightarrow 4X_1 = 80 - 70 = 10$
$\Rightarrow X_1 = \frac{10}{4} = 2.5$
$(X_1, X_2) = (2.5, 35)$

Q. Min $Z = 20X_1 + 10X_2$

$X_1 + 2X_2 \leq 40$
$3X_1 + X_2 \geq 30$
$4X_1 + 3X_2 \geq 60$
$X_1, X_2 \geq 0$

$X_1 + 2X_2 = 40$

X_1	0	40
X_2	20	0

(X_1, X_2) (0,20), (40,0)

$3X_1 + X_2 = 30$

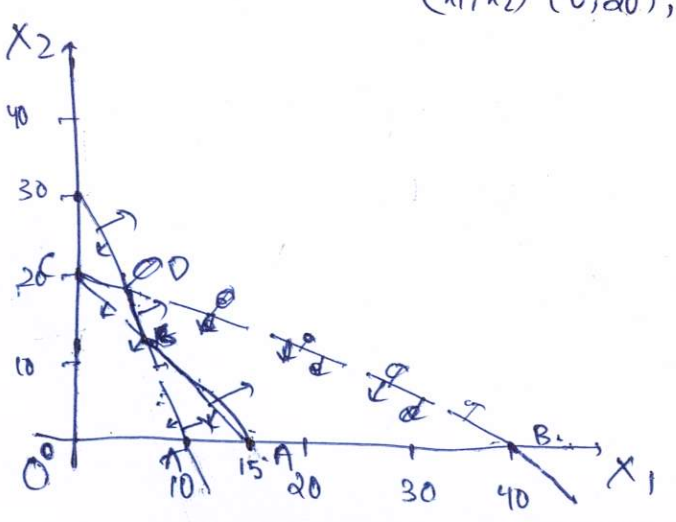
X_1	0	10
X_2	30	0

(0,30) (10,0)

$4X_1 + 3X_2 = 60$

X_1	0	15
X_2	20	0

(0,20), (15,0)



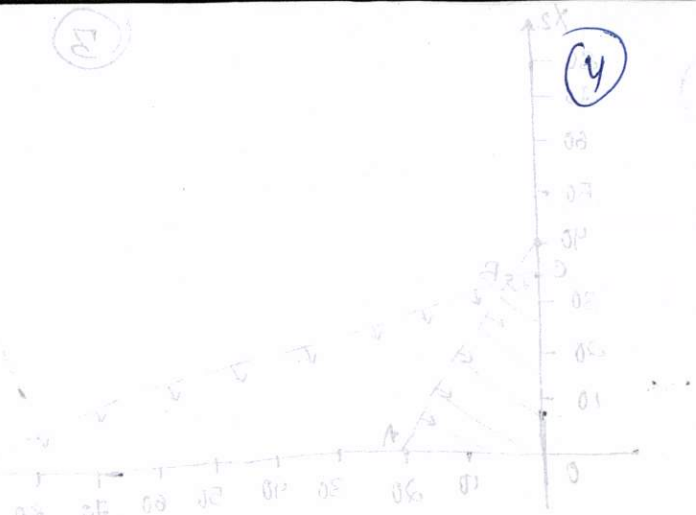
Points	Max Z
O (0,0)	0
A (10,0)	200
B (6,6)	180
C (0,20)	200

$3X_1 + 6X_2 = 120$
$3X_1 + X_2 = 30$
$5X_2 = 90$
$X_2 = 18$
$X_1 =$

$3X_1 + 6X_2 = 90$
$4X_1 + 3X_2 = 60$
$5X_1 = 30$
$X_1 = 6$
$24 + 3X_2 = 60$
$\Rightarrow 3X_2 = 36$
$\Rightarrow X_2 = 12$

(4)

Points	Max Z
A(15,0)	300
B(40,0)	800
C(4,18)	260
D(6,12)	240



Max Z = 240

$x_1 = 6, x_2 = 12$

Assignments

Q.1 Max $Z = X_1 - 3X_2$

$X_1 + X_2 \leq 300$

$X_1 - 2X_2 \leq 200$

$2X_1 + X_2 \leq 100$

$X_2 \leq 200$

$X_1, X_2 \geqslant 0$

→ $X_1 + X_2 = 300$

X_1	0	300
X_2	300	0

(0,300) & (300,0)

$X_1 - 2X_2 = 200$

X_1	0	200
X_2	−100	0

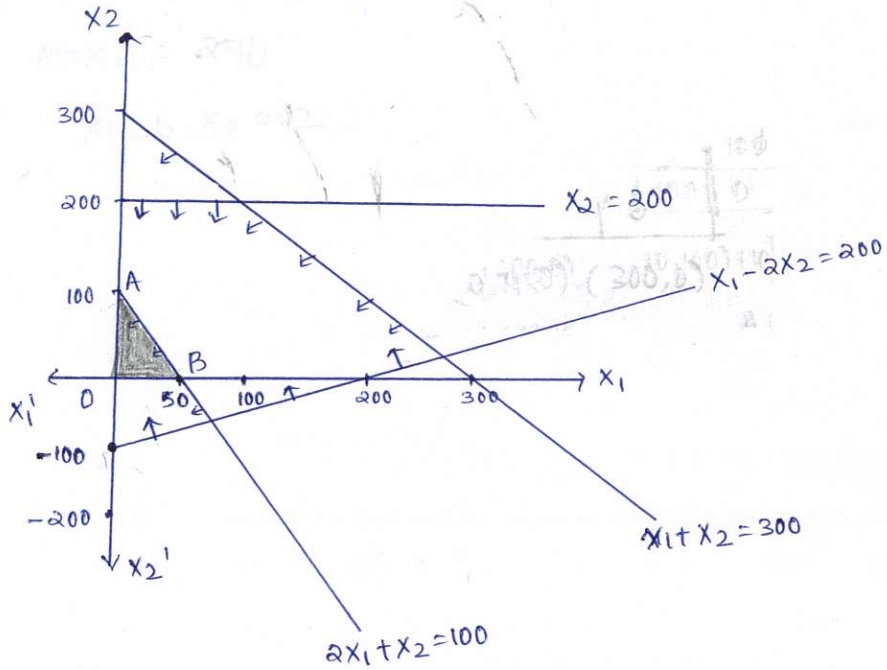
(0,−100) & (200,0)

$2X_1 + X_2 = 100$

X_1	0	50
X_2	100	0

(0,100) & (50,0)

and $X_2 = 200$ is ~~the~~ a line itself.



Here OAB is the common feasible region.

Points	Max $Z = X_1 - 3X_2$
O (0,0)	$Z = 0$
A (0,100)	$Z = 0 - 3\times100 = -300$
B (50,0)	$Z = 50 - 3\times0 = 50$ (Maximum)

Z is maximum at point B(50,0)

optimal solution is $X_1 = 50, X_2 = 0$

Max $Z = 50$

Q.2

$\text{Max } Z = 5x + 8y$

$3x + 2y \leq 36$

$x + 2y \leq 20$

$3x + 4y \leq 42$

$x, y \geq 0$

$3x+2y=36$

x	0	12
y	18	0

(0,18) & (12,0)

$x+2y=20$

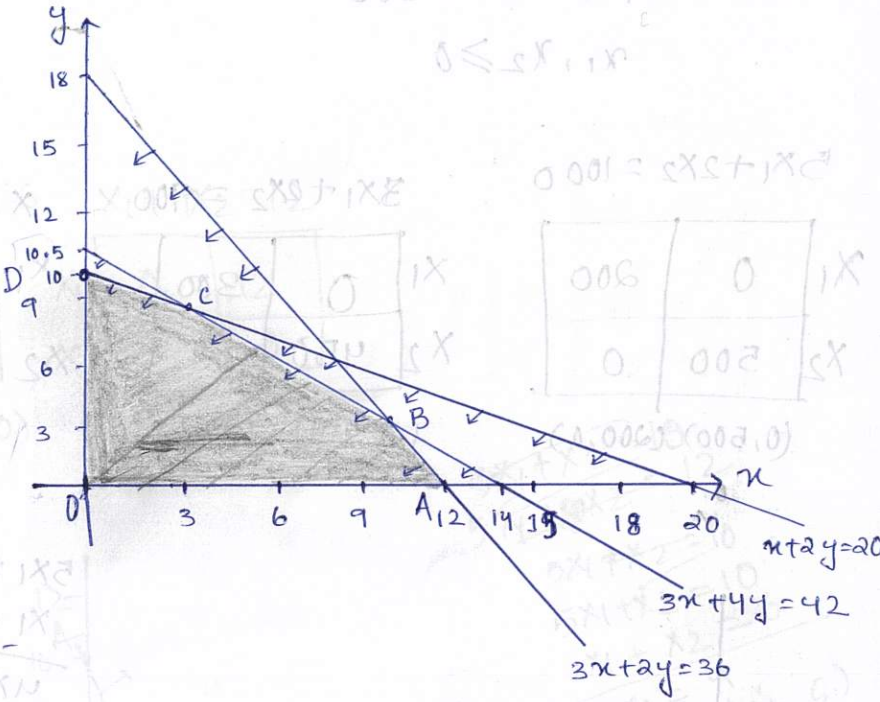
x	0	20
y	10	0

(0,10) & (20,0)

$3x+4y=12$

x	0	14
y	10.5	0

(0,10.5) & (14,0)



- The common region is OABCD

Now point B is intersection point of $(3x+2y=36)$ & $(3x+4y=42)$

Thus substracting both the eq^n, we get -

$$3x + 2y = 36$$
$$\underline{3x + 4y = 42}$$
$$-2y = -6$$
$$y = 3$$

Putting 'y' value on eq^n (1), we get -

$3x + 2 \times 3 = 36$

$\Rightarrow 3x = 30$

$\Rightarrow x = 10$

$\therefore B(10, 3)$

Similarly 'C' is intersection point of $(x+2y=20)$ and $(3x+4y=42)$

By solving them, we get

$3 \times$ eq^n (1) $\rightarrow$ $3x + 6y = 60$

$$\underline{3x + 4y = 42}$$
$$2y = 18$$
$$y = 9$$

Putting 'y' on eq^n (2), we get -

$3x + 36 = 42$

$\Rightarrow 3x = 6$

$\Rightarrow x = 2$

$\therefore C(2, 9)$

Points	Max $Z = 5x + 8y$
O (0,0)	$Z = 0$
A (12,0)	$Z = 60$
B (10,3)	$Z = 50 + 24 = 74$
C (2,9)	$Z = 10 + 72 = 82$ (Maximum).
D (0,10)	$Z = 80$

Z is maximum at point C(2,9),

optimal solution is $x = 2$, $y = 9$ and Max $Z = 82$

& the LPP has a unique solution.

Q. Max $z = 100x_1 + 40x_2$

s.t $5x_1 + 2x_2 \leq 1000$

$3x_1 + 2x_2 \leq 900$

$x_1 + 2x_2 \leq 500$

$x_1, x_2 \geq 0$

$5x_1 + 2x_2 = 1000$

x_1	0	200
x_2	500	0

(0,500) (200,0)

$3x_1 + 2x_2 = 900$

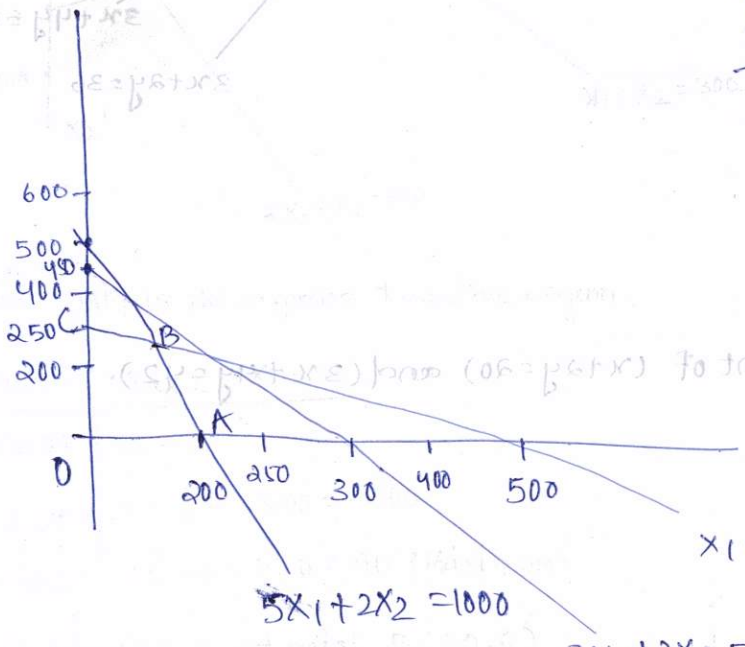
x_1	0	300
x_2	450	0

(0,450) (300,0)

$x_1 + 2x_2 = 500$

x_1	0	500
x_2	250	0

(0,250) (500,0)



$5x_1 + 2x_2 = 1000$

$x_1 + 2x_2 = 500$

$4x_1 = 500$

$x_1 = \frac{500}{4} = 125$

$125 + 2x_2 = 500$

$2x_2 = 375$

$x_2 = 187.5$

point	Max $z = 100x_1 + 40x_2$
0 (0,0)	$z = 0$
A (200,0)	$z = 20{,}000$
B (125, 187.5)	$z = 100 \times 125 + 40 \times 187.5 = 12500 + 7500 = 20{,}000$
C (0,250)	$z = 18{,}000$

This problem has infinetly many solution.

(8)

Min $Z = 3X_1 + 2X_2$

S.t $5X_1 + X_2 \geq 10$

$X_1 + X_2 \geq 6$

$X_1 + 4X_2 \geq 12$

$X_1, X_2 \geq 0$

~~$5X_1 + X_2 = 10$~~

X_1	1	0.2
X_2	5	1

1, -5

$X_1 + X_2 = 6$

X_1	0	6
X_2	6	0

(0,6) (6,0)

$X_1 + 4X_2 = 12$

X_1	0	12
X_2	3	0

(0,3) (12,0)

X_1	0	2
X_2	10	0

(0,10), (2,0)

$5X_1 + X_2 = 10$

$5X_1 + 4X_2 = 12$

$5X_1 + X_2 = 10$

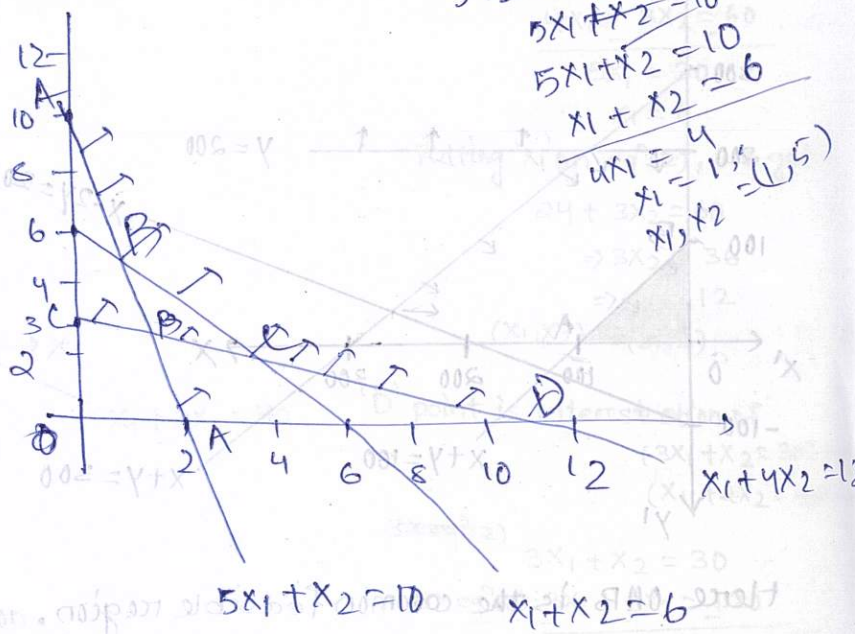
$5X_1 + X_2 = 10$

$X_1 + X_2 = 6$

$4X_1 = 4$

$X_1 = 1$; (1,5)

X_1, X_2



~~ETP~~	Max $Z = 3X_1 + 2X_2$
A(10,0)	$Z = 30$
B(1,5)	$Z = 3 + 10 = 13$ (Min)
C(4,2)	$Z = 16$
D(12,0)	$Z = 36$

In this case if the objective function is a maximization one, then the soln is unbounded because the feasible region has no boundary.

Q. Max $Z = x - 3y$

s.t $x + y \leq 300$

$x - 2y \leq 200$

$x + y \leq 100$

$y \geqslant 200$

$x, y \geqslant 0$

→ $x + y = 300$

x	0	300
y	300	0

(0,300) (300,0)

$x - 2y = 200$

x	0	200
y	-100	0

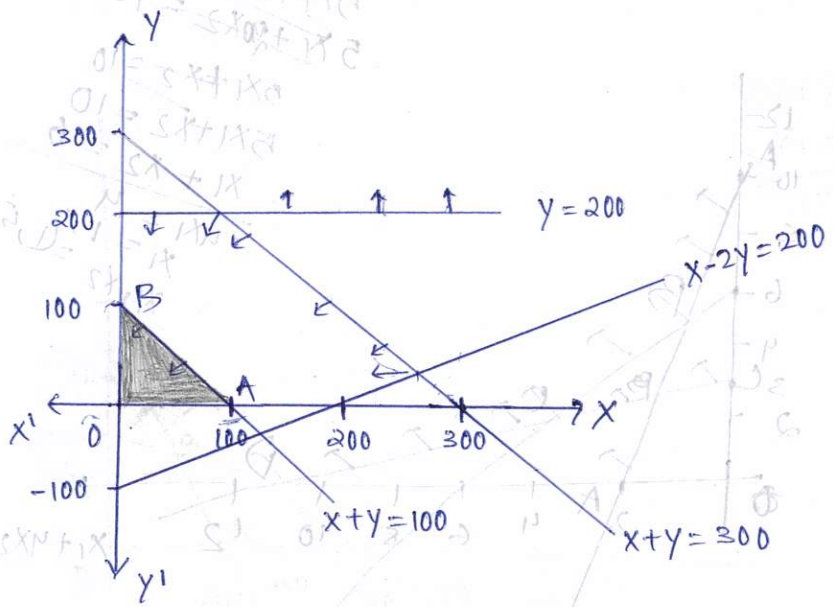
(0,-100) (200,0)

$x + y = 100$

x	0	100
y	100	0

(0,100) (100,0)

$y = 200$ line



Here OAB is the common feasible region.

Points	Max $Z = x - 3y$
O(0,0)	$Z = 0$
A(100,0)	$Z = 100$ (Maximum)
B(0,100)	$Z = -300$

∴ Z is maximum at point A(100,0). Optimal solution is $x = 100$, $y = 0$ & Max $Z = 100$.

Q.2

(10)

Max $Z = 20X_1 + 10X_2$

s.t. $X_1 + 2X_2 \leq 40$

$3X_1 + X_2 \geq 30$

$4X_1 + 3X_2 \geq 60$

$X_1, X_2 \geq 0$

→

$X_1 + 2X_2 = 40$

X_1	0	40
X_2	20	0

(0,20) (40,0)

$3X_1 + X_2 = 30$

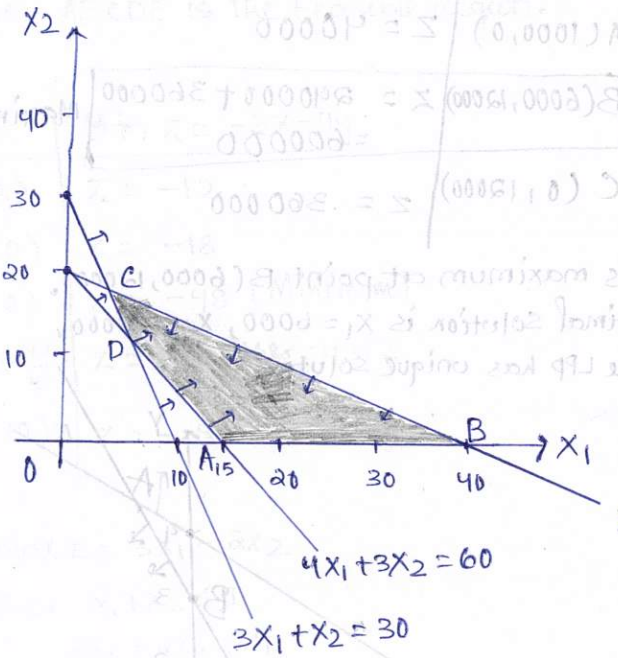
X_1	0	10
X_2	30	0

(0,30) (10,0)

$4X_1 + 3X_2 = 60$

X_1	0	15
X_2	20	0

(0,20) (15,0)



Here ABCD is the feasible region.

Points	Max $Z = 20X_1 + 10X_2$
A(15,0)	Z = 300
B(40,0)	Z = 800 (Maximum)
C(6,12)	Z = 120 + 120 = 240
D(4,18)	Z = 80 + 180 = 260

'C' point is intersection of $(3X_1 + X_2 = 30) - (1)$

$(4X_1 + 3X_2 = 60) - (2)$

3 × eq'(1) $9X_1 + 3X_2 = 90$

$4X_1 + 3X_2 = 60$

$5X_1 = 30$

$X_1 = 6$

Putting X_1 on eq'(2), we get -

$24 + 3X_2 = 60$

$\Rightarrow 3X_2 = 36$

$\Rightarrow X_2 = 12$

$(X_1, X_2) = (6, 12)$

'D' point is intersection of

$(3X_1 + X_2 = 30) - (1)$

$(X_1 + 2X_2 = 40) - (2)$

~~3 × eq'(2)~~ $3X_1 + X_2 = 30$

3 × eq'(2) $3X_1 + 6X_2 = 120$

$-5X_2 = -90$

$X_2 = 18$

~~$(X_1 + X_2) =$~~

Putting X_2 on eq'(1), we get -

$3X_1 + 18 = 30$

$\Rightarrow 3X_1 = 12$

$\Rightarrow X_1 = 4$

$(X_1, X_2) = (4, 18)$

∴ Z is maximum at point B(40,0). The LPP has a unique solution, whence optimal solution is $X_1 = 40$, $X_2 = 0$ & Max $Z = 800$.

Q.6

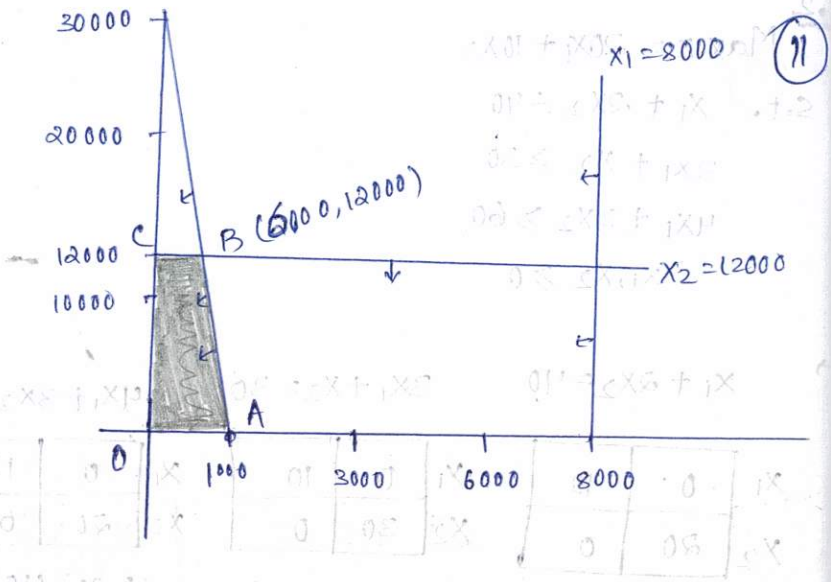
→ Max $Z = 40x_1 + 30x_2$

s.t. $3x_1 + x_2 \leq 30000$

$x_1 \leq 8000$

$x_2 \leq 12000$

$x_1, x_2 \geq 0$



$3x_1 + x_2 = 30000$

x_1	0	10000
x_2	30000	0

$x_1 = 8000$ and $x_2 = 12000$

Point 'B' is the intersection of $3x_1 + x_2 = 30000$ and $x_2 = 12000$

Thus $3x_1 + 12000 = 30000$

$\Rightarrow 3x_1 = 18000$

$\Rightarrow x_1 = 6000$

$(x_1, x_2) = (6000, 12000)$

Points	Max $Z = 40x_1 + 30x_2$	
A(1000, 0)	$Z = 40000$	
B(6000, 12000)	$Z = 240000 + 360000 = 600000$	Maximum
C(0, 12000)	$Z = 360000$	

∴ Z is maximum at point B(6000, 12000).
optimal solution is $x_1 = 6000$, $x_2 = 12000$.
The LPP has unique solution.

Q.8 Min $Z = 3x - 2y$

s.t. $-2x + 3y \leq 9$

$x - 5y \geq -20$

$x, y \geq 0$

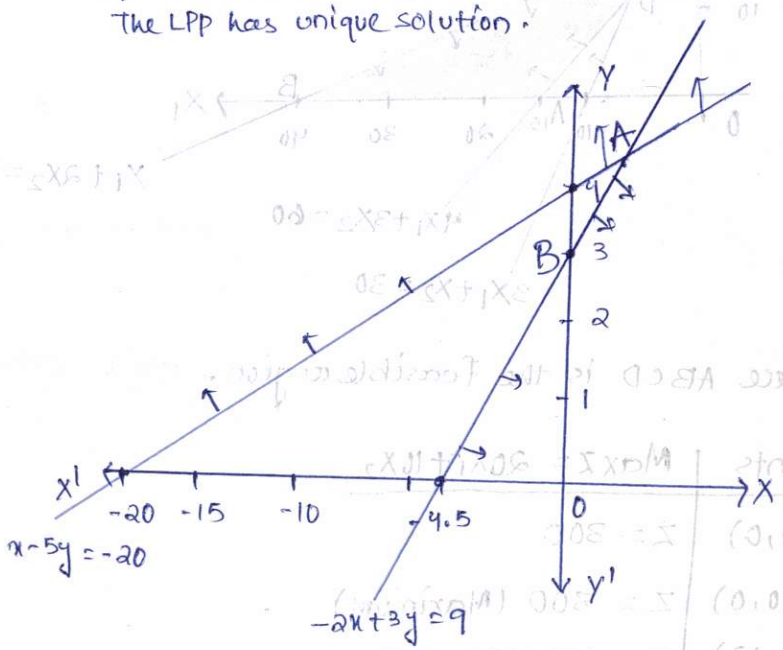
→ $-2x + 3y = 9$ $\quad x - 5y = -20$

x	0	-4.5
y	3	0

(0, 3) (-4.5, 0)

x	0	-20
y	4	0

(0, 4) (-20, 0)



A is the intersection of $x - 5y = -20$ and $-2x + 3y = 9$

2 × eq^n(1) $\quad 2x - 10y = -40$

$-2x + 3y = 9$

$-7y = -31$

$y = 4.42$

$x - 22.14 = -20$

$\Rightarrow x = 2.14$

$(x, y) = (2.14, 4.42)$

Points	Max $Z = 3x - 2y$
A(2.14, 4.42)	$Z = 6.42 - 8.84 = -2.42$
B(0, 3)	$Z = -6$

Q.8

Min $Z = -6x_1 - 4y$

s.t. $2x + 3y \geq 30$

$3x + 2y \leq 24$

$x + y \geq 3$

$x, y \geq 0$

→ $2x + 3y = 30$, $3x + 2y = 24$, $x + y = 3$

x	0	15
y	10	0

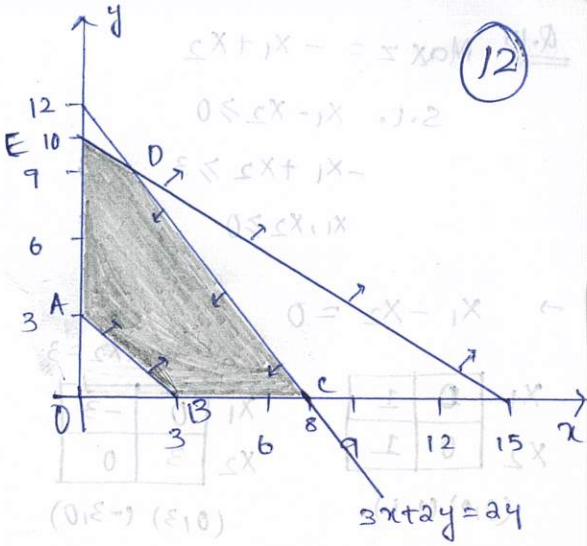
(0,10) (15,0)

x	0	8
y	12	0

(0,12) (8,0)

x	0	3
y	3	0

(0,3) (3,0)



Hence ABCDE is the feasible region.

Points	Min $Z = -6x - 4y$
A(0,3)	$Z = -12$
B(3,0)	$Z = -18$
C(8,0)	$Z = -48$ (Minimum)
D($\frac{12}{5}$, $\frac{42}{5}$)	$Z = \frac{-48}{5} - \frac{168}{5} = -43.2$
E(0,10)	$Z = -40$

Point D is the intersection point of $2x + 3y = 30$ and $3x + 2y = 24$,

3 × eq'n(1) = $6x + 9y = 90$

2 × eq'n(2) = $6x + 4y = 48$

$5y = 42$

$y = \frac{42}{5}$,

Putting 'y' value on eq'n(1), we get-

$2x + \frac{126}{5} = 30$, $\Rightarrow 2x = \frac{150 - 126}{5}$

$\Rightarrow 2x = \frac{24}{5} \Rightarrow x = \frac{12}{5}$

$\therefore (x, y) = (\frac{12}{5}, \frac{42}{5})$

Q.9 Max $Z = 3x_1 - 2x_2$

s.t. $x_1 + x_2 \leq 1$

$2x_1 + 2x_2 \geq 4$

$x_1, x_2 \geq 0$

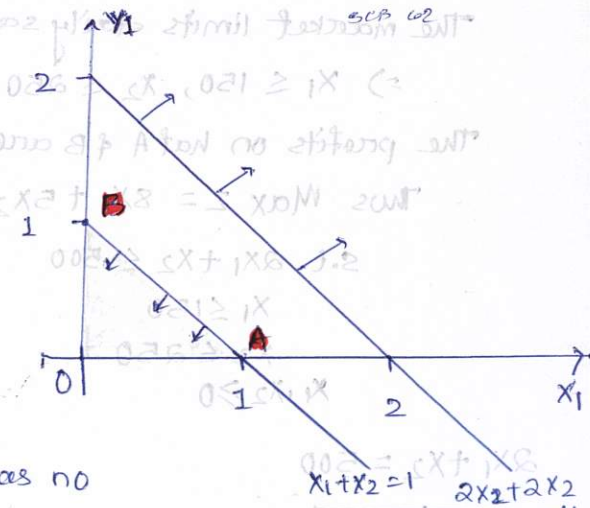
→ $x_1 + x_2 = 1$, $2x_1 + 2x_2 = 4$

x_1	0	1
x_2	1	0

(0,1) (1,0)

x_1	0	2
x_2	2	0

(0,2) (2,0)



~~Hence DAB is the feasible region.~~

This LPP has no feasible region.

Hence this an infeasible solution.

~~Points~~	~~Max $Z = 3x_1 - 2x_2$~~
~~O(0,0)~~	~~$Z = 0$~~
~~A(1,0)~~	~~$Z = 3$ (Maximum)~~
~~B(0,1)~~	~~$Z = -2$~~

~~The LPP has unique solution, Z is maximum at point A(1,0).~~

~~Optimal solution $x_1 = 1$, $x_2 = 0$.~~

Q.10 Max $z = -x_1 + x_2$

s.t. $x_1 - x_2 \geq 0$

$-x_1 + x_2 \geq 3$

$x_1, x_2 \geq 0$

→ $x_1 - x_2 = 0$

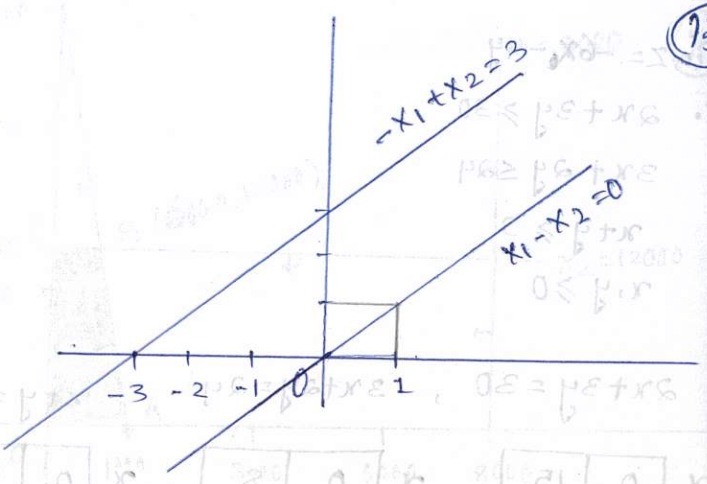
x_1	0	1
x_2	0	1

(0,0) (1,1)

$-x_1 + x_2 = 3$

x_1	0	-3
x_2	3	0

(0,3) (-3,0)



This LPP has no feasible region, hence no solution.

Q. → A company produces 2 types of hats A and B.

Let x_1 for total production for hat A

& x_2 " " " " " B.

Every hat A requires twice as much labour time as the second hat B.

Thus $2x_1 + x_2$.

If the company only hat B, then it can produce a total of 500 hats per day. that means in one day, company can also produce twice of hat A compared to hat B, and the total hat is equal to 500.

That implies $2x_1 + x_2 \leq 500$.

The market limits daily sales of hat A & B to 150 & 250 respectively.

$\Rightarrow x_1 \leq 150, \; x_2 \leq 250$

The profits on hat A & B are Rs.8 & Rs.5 respectively.

Thus Max $Z = 8x_1 + 5x_2$

s.t $2x_1 + x_2 \leq 500$

$x_1 \leq 150$

$x_2 \leq 250$

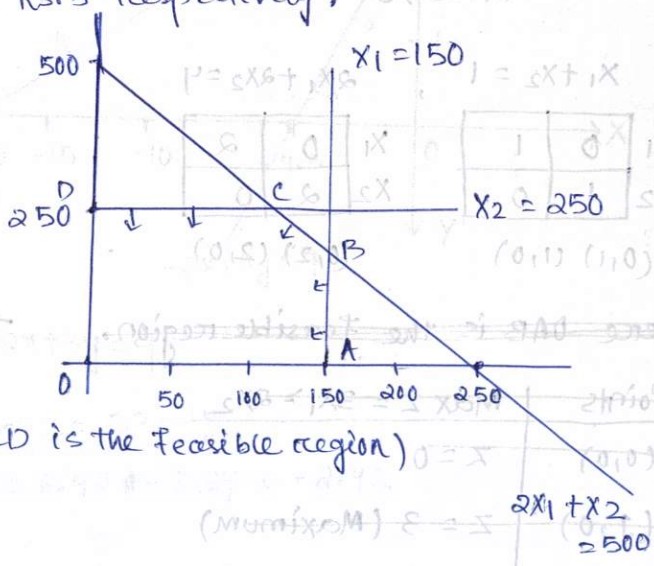
$x_1, x_2 \geq 0$

$2x_1 + x_2 = 500$

x_1	0	250
x_2	500	0

$x_1 = 150$

$x_2 = 250$



(OABCD is the feasible region)

Points	Max $Z = 8x_1 + 5x_2$
O (0,0)	$Z = 0$
A (150,0)	$Z = 1200$
B (150,200)	$Z = 2200$
C (125, 250)	$Z = 2250$ (Maximum)
D (0,250)	$Z = 1250$

Assignment

d.

Given AM of 5 observations, $\bar{x} = 6$

and $\sigma_x^2 = 8$

Also three observations are 2, 6 and 10

let others two are x_1 and x_2

Thus $\frac{2+6+10+x_1+x_2}{5} = 6$ (Given)

$\Rightarrow x_1 + x_2 + 18 = 30$

$\Rightarrow x_1 + x_2 = 12$ ——(1)

Now $\sigma_x^2 = 8$

$\Rightarrow \frac{\Sigma x^2}{n} - (\bar{x})^2 = 8$

$\Rightarrow \frac{x_1^2 + x_2^2 + 2^2 + 6^2 + 10^2}{5} - 6^2 = 8$

$\Rightarrow \frac{x_1^2 + x_2^2 + 140}{5} = 8 + 36$

$\Rightarrow x_1^2 + x_2^2 + 140 = 220$

$\Rightarrow x_1^2 + x_2^2 = 80$ ——(11)

Again from eq^n(1), we have $x_2 = (12 - x_1)$

Now putting x_2 value on eq^n(2), we get -

$x_1^2 + (12 - x_1)^2 = 80$

$\Rightarrow x_1^2 + 144 + x_1^2 - 24x_1 - 80 = 0$

$\Rightarrow 2x_1^2 - 24x_1 + 64 = 0$

$\Rightarrow x_1^2 - 12x_1 + 32 = 0$

$x_1 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{12 \pm \sqrt{144 - 128}}{2}$

$= \frac{12 \pm \sqrt{16}}{2}$

$= \frac{12 \pm 4}{2}$

$= \frac{12+4}{2}$ or $\frac{12-4}{2}$

$= 8$ or 4

Now putting x_1 value on eq^n(1), we get -

If $x = 8$, then eq^n(1) will be -

$8 + x_2 = 12$

$\Rightarrow x_2 = 4$

If $x = 4$, then eq^n(1) will be -

$4 + x_2 = 12$

$\Rightarrow x_2 = 8$

$\therefore (x_1, x_2) = (8, 4)$ or $(4, 8)$

(e)

→ Given $f(x,y) = 0.5$

$n = 10$

As $f(x,y) = 0.5$

$$\Rightarrow 1 - \frac{6\Sigma d^2}{n(n^2-1)} = 0.5$$

$$\Rightarrow -\frac{6\Sigma d^2}{10(10^2-1)} = 0.5 - 1$$

$$\Rightarrow \frac{6\Sigma d^2}{990} = 0.5$$

$$\Rightarrow 6\Sigma d^2 = 495$$

$$\Rightarrow \Sigma d^2 = \frac{495}{6} = 82.5$$

Later on it was found that the difference between the ranks (d) in the two subject was wrongly taken as 3 in place of 7.

Hence correct Σd^2 = Wrong Σd^2 - (wrongly taken observation)^2 + (right observation)^2

$$\text{correct } \Sigma d^2 = 82.5 - 3^2 + 7^2$$

$$\text{correct } \Sigma d^2 = 82.5 - 9 + 49 = 122.5$$

So correct $f(x,y) = 1 - \frac{6\Sigma d^2}{n(n^2-1)} = 1 - \frac{6 \times 122.5}{990}$

$$= 1 - \frac{735}{990}$$

$$= 1 - 0.74$$

$$= 0.25$$

(9)

	A	B	C	D	E
1	7	9	13	8	9
2	10	7	7	8	7
3	8	9	12	9	5
4	12	6	10	4	6
5	0	0	0	0	0

$\rightarrow$

0	2	6	1	2
3	0	0	1	0
3	4	7	4	0
8	2	6	0	2
0	0	0	0	0

(Row reduction)

$\rightarrow$

0	2	6	1	2
3	0	0	1	0
3	4	7	4	0
8	2	6	0	2
0	0	0	0	0

(Column reduction)

[0]	2	6	1	2
3	[0]	~~0~~	1	~~0~~
3	4	7	4	[0]
8	2	6	[0]	2
~~0~~	~~0~~	[0]	~~0~~	~~0~~

Assignment Process

$\rightarrow$

Machine		Job	cost
1	$\rightarrow$	A	7
2	$\rightarrow$	B	7
3	$\rightarrow$	E	5
4	$\rightarrow$	D	4
			23

SIMPLEX METHOD

A LPP having two variable or more than two variable can be solved by simplex method.

Step-1 Put the LPP into standard form

1. The objective function must be in maximize form If it is minimize, then convert to maximize by multiplying (-) ve sign to both the sign.
2. Convert all the inequality constraints into eq^n by adding slack variable and subtract surplus variables.
3. The right hand side constants of the constraints must be positive, if not then multiply a (-ve) sign to both the side of constraints, ~~then~~.

Slack Variable

When the inequality constraints having ($\leq$) then add a slack variable to make it equation and the coefficient of slack variable to the objective function is zero.

$C_B \rightarrow$ Coefficient of B.V in objective function
$Br \rightarrow$ Basic Variables
$X_B \rightarrow$ Right hand side of the constraint

Simplex Table

			C_j 3	2	0	0	
C_B	Br	X_B	x_1	x_2	s_1	s_2	leaving min+ve ($\frac{X_B}{Ec}$)
0	s_1	4	1	1	1	0	4/1 = 4
0	s_2	2	1	-1	0	1	2/1 = 2
			-3	-2	0	0	

The most (-ve) net evalution value corresponding to the variable is known as entering variable and min +ve ratio of $\frac{X_B}{EC}$ corresponding to the variable is known as leaving variable.

Max $Z = 3x_1 + 2x_2 + 0s_1 + 0s_2$

s.t. $x_1 + x_2 \leq 4$
$x_1 - x_2 \leq 2$
$x_1, x_2 \geq 0$

$\rightarrow$ $x_1 + x_2 + s_1 = 4$
$x_1 - x_2 + s_2 = 2$

$$\begin{matrix} x_1 & x_2 & s_1 & s_2 \end{matrix}$$
$$\begin{pmatrix} 1 & 1 & 1 & 0 \\ 1 & -1 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ s_1 \\ s_2 \end{pmatrix}$$

Pivot element is the intersection element of entering column and leaving row. Pivot element must be one (1), if not then make it one by dividing the same amount throughout the ~~row~~ pivot row.

		Cj	3	2	0	0	XB/EC
CB	BV	XB	x_1	x_2	s_1	s_2	
0	s_1	1	0	2	1	-1	1 ←
3	x_1	2	1	-1	0	1	-2
		Zj	3	-3	0	3	
		Zj-Cj	0	-5 ↑	0	3	

$2 + 2P_2$
$2 +$

		Cj	3	2	0	0
CB	BV	XB	x_1	x_2	s_1	s_2
2	x_2	1	0	1	$\frac{1}{2}$	$\frac{-1}{2}$
3	x_1	3	1	0	$\frac{1}{2}$	$\frac{1}{2}$
		Zj	3	2	$\frac{5}{2}$	$\frac{1}{2}$
		Zj-Cj	0	0	$\frac{5}{2}$	$\frac{1}{2}$

$1 + \frac{3}{2}$

$-1 + \frac{3}{2}$

$-2 + \frac{3}{2}$

the solution is optimal and $x_1 = 3$, $x_2 = 1$ and max $Z = 11$

Assignments

Q. Max $Z = X_1 + X_2 + 3X_3$

s.t. $3X_1 + 2X_2 + X_3 \leq 2$

$2X_1 + X_2 + 2X_3 \leq 2$

$X_1, X_2, X_3 \geq 0$

→ Max $Z = X_1 + X_2 + 3X_3 + 0S_1 + 0S_2$

s.t. $3X_1 + 2X_2 + X_3 + S_1 + 0S_2 = 2$

$2X_1 + X_2 + 2X_3 + 0S_1 + S_2 = 2$

Simplex Table

CB	BV	Cj / XB	1 / X_1	1 / X_2	3 / X_3	0 / S_1	0 / S_2	XB / EC
0	S_1	2	3	2	1	1	0	2
0	S_2	2	2	1	2	0	1	1 ←
	Z_j	0	0	0	0	0	0	
	~~$Z_j - C_j$~~ $Z_j - C_j$		-1	-1	-3 ↑	0	0	

Here there are three $Z_j - C_j$ are (-)ve and the most (-)ve is "-3".

Therefore X_3 is the entering variable.

and S_2 is the leaving variable.

Now we will convert the key element as unity and all other entries in that column to zero.

Now the revised simplex table as follows.

$R_2 \rightarrow \frac{R_2}{\text{Key element}}$

$R_1 \rightarrow R_1 - \frac{R_2}{2}$

CB	BV	Cj / XB	1 / X_1	1 / X_2	3 / X_3	0 / S_1	0 / S_2
0	S_1	1	2	$\frac{1}{2}$	0	1	$-\frac{1}{2}$
3	X_3	1	1	$\frac{1}{2}$	1	0	$\frac{1}{2}$
	Z_j	3	3	$\frac{3}{2}$	3	0	$\frac{3}{2}$
	$Z_j - C_j$		2	$\frac{1}{2}$	0	0	$\frac{3}{2}$

As all $Z_j - C_j \geq 0$, so the optimum solution to the LPP has been reached.

The solution is, $X_1 = 0$, $X_2 = 0$, $X_3 = 1$ and Max $Z = 3$

Q. Max $Z = 10x_1 + 6x_2$

s.t. $x_1 + x_2 \leq 2$

$2x_1 + x_2 \leq 4$

$3x_1 + 8x_2 \leq 12$

$x_1, x_2 \geq 0$

→ Max $Z = 10x_1 + 6x_2 + 0s_1 + 0s_2 + 0s_3$

$x_1 + x_2 + s_1 + 0s_2 + 0s_3 = 2$

$2x_1 + x_2 + 0s_1 + s_2 + 0s_3 = 4$

$3x_1 + 8x_2 + 0s_1 + 0s_2 + 8s_3 = 12$

		Cj	10	6	0	0	0	XB/EC
CB	BV	XB	x_1	x_2	s_1	s_2	s_3	
0	s_1	2	1	1	1	0	0	2 ←
0	s_2	4	2	1	0	1	0	2
0	s_3	12	3	8	0	0	1	4
	Z_j	0	0	0	0	0	0	
	$Z_j - C_j$		↑ −10	−6	0	0	0	

Here, there are two $Z_j - C_j$ are (-ve) and the most (-) ve is "-10".

Therefore x_1 is the entering variable

and the min +ve ratio of $\frac{XB}{EC} = \min\{2, 2, 4\} = 2$

So we choose s_1 as the leaving variable.

Now as the key element is equals to 1, we only have to convert all other entries in that column to zero.

$R_2 \rightarrow$	$R_1 - \frac{R_2}{2}$
$R_3 \rightarrow$	$R_1 - \frac{R_3}{3}$

Hence Revised simplex table as follows.

		Cj	10	6	0	0	0
CB	BV	XB	x_1	x_2	s_1	s_2	s_3
10	x_1	2	1	1	1	0	0
0	s_2	0	0	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	0
0	s_3	−2	0	$-\frac{5}{3}$	1	0	$-\frac{1}{3}$
	Z_j	20	10	10	10	0	0
	$Z_j - C_j$		0	4	10	0	0

Since all $Z_j - C_j \geq 0$, our optimal solution to the LPP has been reached.

The solution is $x_1 = 2, x_2 = 0$ and Max $Z = 20$

Q. Max $Z = 30x_1 + 23x_2 + 29x_3$

s.t. $6x_1 + 5x_2 + 3x_3 \leq 52$

$6x_1 + 2x_2 + 5x_3 \leq 14$

$x_1, x_2, x_3 \geq 0$

→ Max $Z = 30x_1 + 23x_2 + 29x_3 + 0s_1 + 0s_2$

$6x_1 + 5x_2 + 3x_3 + s_1 + 0s_2 \leq 52$

$6x_1 + 2x_2 + 5x_3 + 0s_1 + s_2 = 14$

Simplex Table

CB	BV	Cj XB	30 x_1	23 x_2	29 x_3	0 s_1	0 s_2	XB / EC
0	s_1	52	6	5	3	1	0	$\frac{52}{6} = 8.66$
0	s_2	14	6	2	5	0	1	$\frac{14}{6} = 2.33$ ←
	Z_j	0	0	0	0	0	0	
	$Z_j - C_j$		-30↑	-23	-29	0	0	

Here, there are three $Z_j - C_j$ are (−)ve and the most (−)ve is "−30".

Therefore x_1 is the entering variables.

and the min = +ve $\{8.66, 2.33\} = 2.33$

So s_2 is the leaving variable.

Now we will convert the key element as unity and all other entries in that column to zero.

Now the revised simplex table as follows.

CB	BV	Cj XB	30 x_1	23 x_2	29 x_3	0 s_1	0 s_2	XB / EC
0	s_1	38	0	3	-2	1	-1	$38/3 = 12.66$
30	x_1	$\frac{14}{6}$	1	$\frac{1}{3}$	$\frac{5}{6}$	0	$\frac{1}{6}$	7 ←
	Z_j	70	30	10	25	0	5	
	$Z_j - C_j$		0	-13↑	-4	0	5	

$R_2 \rightarrow \frac{R_2}{\text{Key element}}$

$R_1 \rightarrow R_1 - R_2$

$R_2 \rightarrow 3R_2$

$R_1 \rightarrow R_1 - 9R_2$

CB	BV	Cj XB	30 x_1	23 x_2	29 x_3	0 s_1	0 s_2
0	s_1	17	-9	0	$\frac{-19}{2}$	1	$\frac{-5}{2}$
23	x_2	7	3	1	$\frac{5}{2}$	0	$\frac{1}{2}$
	Z_j	161	69	23	57.5	0	11.5
	$Z_j - C_j$		39	0	28.5	0	11.5

Since all $Z_j - C_j \geq 0$, the optimal solution to the LPP has been reached.

The solution is $x_1 = 0$, $x_2 = 7$, $x_3 = 0$ and Max $Z = 161$.

Q. Max $Z = x_1 + 2x_2 + x_3$

s.t. $2x_1 + x_2 - x_3 \geq -2$

$-2x_1 + x_2 - 5x_3 \leq 6$

$4x_1 + x_2 + x_3 \leq 6$

$x_1, x_2, x_3 \geq 0$

→ Max $Z = x_1 + 2x_2 + x_3 + 0s_1 + 0s_2 + 0s_3$

Multiplying (-) into first constraints, which is $-2x_1 - x_2 + x_3 \leq 2$

~~$-2x_1 - x_2 + x_3 \leq 2$~~

$-2x_1 - x_2 + x_3 + s_1 + 0s_2 + 0s_3 = 2$

~~$4x_1 + x_2 - 5x_3 + 0s_1 + s_2 + 0s_3$~~

$-2x_1 + x_2 - 5x_3 + 0s_1 + s_2 + 0s_3 = 6$

$4x_1 + x_2 + x_3 + 0s_1 + 0s_2 + s_3 = 6$

CB	BV	c_j / XB	1 / x_1	2 / x_2	1 / x_3	0 / s_1	0 / s_2	0 / s_3	XB / EC
0	s_1	2	-2	-1	1	1	0	0	-2
0	s_2	6	-2	1	-5	0	1	0	6 ←
0	s_3	6	4	1	1	0	0	1	6
	Z_j	0	0	0	0	0	0	0	
	$z_j - c_j$	-1	-2↑	-1	0	0	0		

$R_1 \rightarrow R_2 + R_1$

$R_3 \rightarrow R_2 - R_3$

$R_1 \rightarrow R_1 + R_2$

Here, there are three $z_j - c_j$ are (-)ve and the most (-)ve is "-2".

Therefore x_2 is the entering variable.

also as $\min\{\frac{XB}{EC}\} = 6$, we choose s_3 as leaving variable.

Now as the key element is equal to 1, we only have to convert all other entries in that column to zero. Hence revised simplex table as follows.

CB	BV	c_j / XB	1 / x_1	2 / x_2	1 / x_3	0 / s_1	0 / s_2	0 / s_3	XB / EC
0	s_1	8	-4	0	-4	1	1	0	
2	x_2	6	-2	1	-5	0	1	0	
0	s_3	0	-6	0	-6	0	1	-1	
	Z_j	12	-4	2	-10	0	2	0	
	$z_j - c_j$		-5	0	-11↑	0	2	0	

unbounde

		Cj	1	2	1	0	0	0
CB	BV	XB	x_1	x_2	x_3	s_1	s_2	s_3
0	s_1	8	2	0	2	1	0	1
0	s_2	0	6	0	6	0	-1	1
2	x_2	6	4	1	1	0	0	1
	Zj	12	8	2	2	0	0	2
	Zj-Cj		7	0	1	0	0	2

$R_1 \rightarrow R_1 + R_3$

$R_2 \rightarrow$

Since all $Z_j - C_j \geq 0$, the optimum solution to this LPP has been reached.

The solution is $x_1 = 0$, $x_2 = 6$, $x_3 = 0$ and Max $Z = 12$.

		Cj	1	2	1	0	0	0	
CB	BV	XB	x_1	x_2	x_3	s_1	s_2	s_3	$\frac{XB}{EC}$
0	s_1	8	-4	0	-4	1	1	0	-2
2	x_2	6	-2	1	-5	0	1	0	$-\frac{6}{5}$
0	s_3	0	6	0	6	0	-1	1	
	Zj	12	-4	2	-10	0	2	0	
	Zj-Cj		-5	0	-11 ↑	0	2	0	

Q. Max $Z = x_1 + 2x_2 + x_3$

s.t. $2x_1 + x_2 - x_3 \geq -2$

$-2x_1 + x_2 - 5x_3 \leq 6$

$4x_1 + x_2 + x_3 \leq 6$

$x_1, x_2, x_3 \geq 6$

→ Multiplying (-) into first constraints, which will be $-2x_1 - x_2 + x_3 \leq 2$

Max $Z = x_1 + 2x_2 + x_3 + 0s_1 + 0s_2 + 0s_3$

$-2x_1 - x_2 + x_3 + s_1 + 0s_2 + 0s_3 = 2$

$-2x_1 + x_2 - 5x_3 + 0s_1 + s_2 + 0s_3 = 6$

$4x_1 + x_2 + x_3 + 0s_1 + 0s_2 + s_3 = 6$

		Cj	1	2	1	0	0	0	XB/EC
CB	BV	XB	x_1	x_2	x_3	s_1	s_2	s_3	
0	s_1	2	-2	-1	1	1	0	0	-2
0	s_2	6	-2	1	-5	0	1	0	6
0	s_3	6	4	1	1	0	0	1	6 ←
		Zj	0	0	0	0	0	0	
		Zj-Cj	-1	-2↑	-1	0	0	0	

Here, there are three Zj-Cj are (-)ve and the most (-)ve is "-2". Therefore x_2 is the entering variable.

also as min +ve $\left\{\frac{XB}{EC}\right\} = 6$, we choose s_3 as leaving variable.

Now as the key element is equal to 1, we only have to convert all other entries in that column to zero. Hence revised simplex table as follows.

		Cj	1	2	1	0	0	0
CB	BV	XB	x_1	x_2	x_3	s_1	s_2	s_3
0	s_1	8	2	0	2	1	0	1
0	s_2	0	-6	0	-6	0	1	-1
2	x_2	6	4	1	1	0	0	1
	Zj	12	8	2	2	0	0	2
	Zj-Cj		7	0	1	0	0	2

CB.	BV	c_j / XB	1 / x_1	2 / x_2	1 / x_3	0 / s_1	0 / s_2	0 / s_3	XB/EC
0	s_1	8	-4	0	-4	1	1	0	-2
2	x_2	6	-2	1	-5	0	1	0	$-\frac{6}{5}$
0	s_3	0	6	0	6	0	-1	1	0 ←
	z_j	12	-4	2	-10	0	2	0	
	$z_j - c_j$		-5	0	-11↑	0	2	0	

CB	BV	c_j / XB	1 / x_1	2 / x_2	1 / x_3	0 / s_1	0 / s_2	0 / s_3	XB/EC
0	s_1	8	0	0	0	1	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{8}{3}$
2	x_2	6	3	1	0	0	$\frac{1}{6}$	$\frac{5}{6}$	1
1	x_3	0	1	0	1	0	-1	0	0 ←
	z_j	12	7	2	1	0	$-\frac{2}{3}$	$\frac{5}{3}$	
	$z_j - c_j$		6	0	0	0	$-\frac{2}{3}$↑	$\frac{5}{3}$	

		c_j / XB	1 / x_1	2 / x_2	1 / x_3	0 / s_1	0 / s_2	0 / s_3
CB	BV							
0	s_1							
2	x_2							

* When all the elements of entering are (-)ve, the sol^n of LPP possess unbounded solution.
* when there is tie occures in min +ve ratio column, this is known as degeneracy.
* Alternative optimal sol^n
When the net evaluation of a non basic variable is zero in that case alternative optimal solution arises.

Q. Max $Z = 3x_1 + 5x_2$

s.t. $x_1 \le 4$

$2x_2 \le 12$

$3x_1 + 2x_2 \le 18$

$x_1, x_2 \ge 0$

→ Max $Z = 3x_1 + 5x_2 + 0s_1 + 0s_2 + 0s_3$

$x_1 + 0x_2 + s_1 + 0s_2 + 0s_3 = 4$

$0x_1 + 2x_2 + 0s_1 + s_2 + 0s_3 = 12$

$3x_1 + 2x_2 + 0s_1 + 0s_2 + s_3 = 18$

		Cj	3	5	0	0	0	XB/EC
CB	BV	XB	x_1	x_2	s_1	s_2	s_3	
0	s_1	4	1	0	1	0	0	4/0 ∴
0	s_2	12	0	2	0	1	0	6 ←
0	s_3	18	3	2	0	0	1	9
	Zj	0	0	0	0	0	0	
	Zj-Cj		-3	-5↑	0	0	0	

		Cj	3	5	0	0	0	XB/EC
CB	BV	XB	x_1	x_2	s_1	s_2	s_3	
0	s_1	4	1	0	1	0	0	4
5	x_2	6	0	1	0	1/2	0	6/0
0	s_3	6	3	0	0	-1	1	2 ←
	Zj	30	0	5	0	5/2	0	
	Zj-Cj		-3↑	0	0	5/2	0	

		Cj	3	5	0	0	0
CB	BV	XB	x_1	x_2	s_1	s_2	s_3
0	s_1	2	0	0	1	1/3	-1/3
5	x_2	6	0	1	0	1/2	0
3	x_1	2	1	0	0	-1/3	1/3
	Zj	36	3	5	0	3/2	1
	Zj-Cj		0	0	0	0	0

→ $x_1 = 2$, $x_2 = 6$, Max $Z = 36$

Q. Max $Z = 12x_1 + 16x_2$

s.t. $10x_1 + 20x_2 \leq 120$

$8x_1 + 8x_2 \leq 80$

x_1 and $x_2 \geq 0$

→ Max $Z = 12x_1 + 16x_2 + 0S_1 + 0S_2$

$10x_1 + 20x_2 + S_1 + 0S_2 = 120$

$8x_1 + 8x_2 + 0S_1 + S_2 = 80$

→

		Cj	12	16	0	0	
CB	BV	XB	x_1	x_2	S_1	S_2	$\frac{XB}{EC}$
0	S_1	120	10	20	1	0	6
0	S_2	80	8	8	0	1	10
		Zj-Cj	-12	-16↑	0	0	

		Cj	12	16	0	0	
CB	BV	XB	x_1	x_2	S_1	S_2	$\frac{XB}{EC}$
16	x_2	6	1/2	1	1/20	0	3 ←
0	S_2	32	4	0	$-\frac{8}{20}$	1	8
		Zj	96	8	16	$\frac{16}{20}$	0
		Zj-Cj	-4↑	0	$\frac{16}{20}$	0	

		Cj	12	16	0	0
CB	BV	XB	x_1	x_2	S_1	S_2
12	x_1	12	1	2	$\frac{1}{10}$	0
0	S_2	-4	0	-2	$-\frac{2}{5}$	1
	Zj	144	12	24	0	0
	Zj-Cj	0	8	0	0	

The LPP has been reached optimum solution,

solution is $x_1 = 12$, $x_2 = 0$ and Max $Z = 144$

Q. Max $Z = 3x_1 + 2x_2 + 5x_3$

s.t. $x_1 + 2x_2 + x_3 \le 430$

$3x_1 + 2x_3 \le 460$

$x_1 + 4x_2 \le 420$

$x_1, x_2, x_3 \ge 0$

→ Max $Z = 3x_1 + 2x_2 + 5x_3 + 0S_1 + 0S_2 + 0S_3$

$x_1 + 2x_2 + x_3 + S_1 + 0S_2 + 0S_3 = 430$

$3x_1 + 2x_2 + 2x_3 + 0S_1 + S_2 + 0S_3 = 460$

$x_1 + 4x_2 + 0x_3 + 0S_1 + 0S_2 + S_3 = 420$

		Cj	3	2	5	0	0	0	
CB	BV	XB	x_1	x_2	x_3	S_1	S_2	S_3	XB/EC
0	S_1	430	1	2	1	1	0	0	430
0	S_2	460	3	0	2	0	1	0	230 ←
0	S_3	420	1	4	0	0	0	1	(420/0)
		Zj	0	0	0	0	0	0	
		Zj-Cj	-3	-2	-5↑	0	0	0	

		Cj	3	2	5	0	0	0	
CB	BV	XB	x_1	x_2	x_3	S_1	S_2	S_3	XB/EC
0	S_1	200	-1/2	1	0	1	-1/2	0	200 ←
5	x_3	230	3/2	0	1	0	1/2	0	(230/0)
0	S_3	1	4	0	0	0	0	1	(1/0)
		Zj	1150	15/2	0	5	0	5/2	0
		Zj-Cj	9/2	-2↑	0	0	5/2	0	

		Cj	3	2	5	0	0	0	
CB	BV	XB	x_1	x_2	x_3	S_1	S_2	S_3	
2	x_2	200	-1/2	1	0	1	-1/2	0	
5	x_3	230	3/2	0	1	0	1/2	0	
0	S_3	1	4	0	0	0	0	1	
		Zj	1550	13/2	2	5	2	3/2	0
		Zj-Cj	7/2	0	0	2	3/2	0	

→ $x_1 = 0$

$x_2 = 200$

$x_3 = 230$

Max $Z = 1550$

Big-M Method

When a LPP having "$\geq$" inequality constraints and equal to "$=$" constraints, in that case to get intial basis matrix, we have to use artificial variable to get the basis matrix.

So when an artificial variable is used in a LPP then that LPP can be solved by two methods,

(1) Big-M Method

(2) Two Phase Method

Q. Max $Z = x_1 + 5x_2$

s.t. $4x_1 + 4x_2 \leq 6$

$x_1 + 3x_2 \geq 2$

$x_1, x_2 \geq 0$

$x_1 + 5x_2 + 0S_1 + 0S_2 - MA_1$

$4x_1 + 4x_2 + S_1 + 0S_2 + 0A_1 = 6$

$x_1 + 3x_2 + 0S_1 - S_2 + A_1 = 2$

x_1	x_2	S_1	S_2	A_1
4	4	1	0	0
1	3	0	-1	1

		c_j	1	5	0	0	-M	
CB	BV	XB	x_1	x_2	S_1	S_2	A_1	$\frac{XB}{EC}$
0	S_1	6	4	4	1	0	0	3/2 ←
-M	A_1	2	1	3	0	-1	1	2/3 ←
		Z_j	-2M	-M -3M	0	M	-M	
		$Z_j - c_j$	-M-1	-3M-5+0	M	0		

When an artificial variable left the basis, removed that artificial variable column from next table onwards.

		c_j	1	5	0	0	-M	
CB	BV	XB	x_1	x_2	s_1	s_2	A_1	$\frac{XB}{EC}$
0	s_1	10/3	8/3	0	1	4/3	-4/3	$\frac{10}{4}$ ←
5	x_2	2/3	1/3	1	0	-1/3	1/3	-2
	z_j	10/3	5/3	5	0	-5/3	5/3	
	$z_j - c_j$		2/3	0	0	-5/3 ↑	5/3+M	

		c_j	1	5	0	0	-M
CB	BV	XB	x_1	x_2	s_1	s_2	~~A~~
0	s_2	$\frac{10}{4}$	2	0	$\frac{3}{4}$	1	
5	x_2	$\frac{3}{2}$	1	1	1/4	0	
	z_j	$\frac{15}{2}$	5	5	5/4	0	
	$z_j - c_j$		4	0	5/4	0	

~~A~~ → $x_1 = 0$, $x_2 = \frac{3}{2}$ Max $z = \frac{15}{2}$

Assignment

Q. what do you mean by artificial variable?
Q. what is unbounded sol^n in simplex method?
Q. what is degeneracy in simplex method?
Solve the following LPP using Big M method

Max $z = 3x_1 + 2x_2 + 6x_3$
s.t. $4x_1 + x_2 + x_3 \leq 100$
$x_1 + x_2 \geq 40$
$x_2 + x_3 \leq 30$
$x_1, x_2, x_3 \geq 0$

Max $z = 3x_1 + 2x_2 + 6x_3 + 0s_1 + 0s_2 + 0s_3 - MA_1$
$4x_1 + x_2 + x_3 + s_1 + 0s_2 + 0s_3 + 0A_1$
$0x_1 + x_2 + 0x_3 + 0s_1 - s_2 + 0s_3 + A_1$
$0x_1 + x_2 + x_3 + 0s_1 + 0s_2 + 0s_3 + 0A_1$

		Cj	3	2	6	0	0	0	-M	
CB	BV	XB	x_1	x_2	x_3	S_1	S_2	S_3	A_1	XB/EC
0	S_1	100	4	1	1	1	0	0	0	100
0	S_3	40	1	1	0	0	0	0	1	40/0
-M	A_1	30	0	1	1	0	0	1	0	30 ←
		Zj	0	-M	-M	0	0	-M	0	
		Zj-Cj	-3	-M-2	-M-6	0	0	-M	M	
					↑					

		Cj	3	2	6	0	0	0	
CB	BV	XB	x_1	x_2	x_3	S_1	S_2	S_3	XB/EC
0	S_1	70	4	0	1	0	-1	0	70/4 ←
0	S_3	40	1	1	0	0	0	0	40
6	x_3	30	0	1	1	0	0	1	30/0
	Zj	180	0	6	6	0	0	6	
	Zj-Cj	-3 ↑	4	0	0	0	0	6	

		Cj	3	2	6	0	0	0	
CB	BV	XB	x_1	x_2	x_3	S_1	S_2	S_3	XB/EC
3	x_1	70/4	1	0	1/4	0	-1/4	0	(←)
0	S_3	90/4	0	1	-1/4	0	1/4	0	90
6	x_3	30	0	1	1	0	0	1	30 ←
	Zj	930/4	3	6	27/4	0	-3/4	6	
	zj-cj	0	4	3/4	0	-3/4 ↑	6		

CB	BV	Cj / XB	3 x_1	2 x_2	6 x_3	0 s_1	0 s_2	0 s_3
3	x_1						0	
0	s_3						0	
0	s_2	31	1	2	2	0	1	2

$4x_1 + x_2 + x_3 + s_1 = 100$
$x_1 + x_2 - s_2 + A_1 = 40$
$x_2 + x_3 + s_3 = 30$

	x_1	x_2	x_3	s_1	s_2	s_3	A_1
s_1	4	1	1	1	0	0	0
A_1	1	1	0	0	−1	0	1
s_3	0	1	1	0	0	1	0

Q. $3x_1 + 2x_2 + 6x_3 + 0s_1 + 0s_2 + 0s_3 - MA_1$

s.t. $4x_1 + x_2 + x_3 + 0s_1 + 0s_2 + 0s_3 + 0A_1 = 100$
$x_1 + x_2 + 0x_3 + 0s_1 - s_2 + 0s_3 + A_1 = 40$
$0x_1 + x_2 + x_3 + 0s_1 + 0s_2 + 0s_3 + 0A_1 = 30$

CB	BV	Cj / XB	3 x_1	2 x_2	6 x_3	0 s_1	0 s_2	0 s_3	−M A_1	XB/EC
0	s_1	100	4	1	1	1	0	0	0	25 ←
−M	A_1	40	1	1	0	0	−1	0	1	40
0	s_3	30	0	1	1	0	0	1	0	(−)
		Z_j	−M	−M	0	0	M	0	−M	
		$Z_j - C_j$	−M−3	−M−2	−6	0	M	0	0	

↑

CB	BV	Cj / XB	3 x_1	2 x_2	6 x_3	0 s_1	0 s_2	0 s_3	−M A_1	XB/EC
3	x_1	25	1	1/4	1/4	1/4	0	0	0	100
−M	A_1	15	0	3/4	−1/4	−1/4	−1	0	1	20
0	s_3	30	0	1	1	0	0	1	0	30
		Z_j	3	$\frac{3}{4}-\frac{3M}{4}$	$\frac{3}{4}+\frac{M}{4}$	$\frac{3}{4}+\frac{3M}{4}$	M	0	−M	
		$Z_j - C_j$	0	$-\frac{3M}{4}-\frac{5}{2}$						

↑

CB	BV	XB	3 x_1	2 x_2	6 x_3	0 s_1	0 s_2	0 s_3	XB/EC
3	x_1	20	1	0	1/3	1/3	−1/3	0	60
2	x_2	20	0	1	−1/3	−1/3	4/3	0	−60
0	s_3	10	0	0	4/3	1/3	4/3	1	30/4 ←
		Z_j	100	3	2	7/3	7/3	−5/3	0
		$Z_j - C_j$	0	0	−11/3 ↑	7/3	−5/3	0	

CB	BV	C_j XB	3 x_1	2 x_2	6 x_3	0 s_1	0 s_2	0 s_3
3	x_1	70/4	1	0	0	1/4	0	-1/4
2	x_2	90/4	0	1	0	-3/4	-1	1/4
6	x_3	30/4	0	0	1	1/4	1	3/4
	Z_j	570/4	3	2	6	7/4	4	17/4
	$Z_j - C_j$		0	0	0	7/4	4	17/4

$$\text{Max } Z = \frac{570}{4}$$

$$\left[x_1 = \frac{70}{4},\ x_2 = \frac{90}{4},\ x_3 = \frac{30}{4}\right]$$

① $4x_1 + x_2 + x_3 \le 100$

$$4 \times \frac{70}{4} + \frac{90}{4} + \frac{30}{4}$$

$$= 70 + \frac{90+30}{4}$$

$$= \frac{280 + 90 + 30}{4}$$

$$= \frac{400}{4}$$

$$= 100$$

② $x_1 + x_2 \ge 40$

$$\Rightarrow \frac{70}{4} + \frac{90}{4}$$

$$= \frac{70+90}{4} = \frac{160}{4} = 40$$

③ $x_2 + x_3 \le 30$

$$= \frac{90}{4} + \frac{30}{4}$$

$$= \frac{90+30}{4} = \frac{120}{4} = 30$$

maximize

$Z = X_1 + 5X_2$

Subject to

$4X_1 + 4X_2 \leq 6$

$X_1 + 3X_2 \geq 2$

$X_1, X_2 \geq 0$

$4X_1 + 4X_2 + S_1 = 6$

$X_1 + 3X_2 - S_2 + A_1 = 2$

$Z = X_1 + 5X_2 + 0S_1 - 0S_2 - MA_1$

	Cj		1	5	0	0	−M	
Cont^n	variable	const	X_1	X_2	S_1	S_2	A_1	
0	S_1	6	4	4	1	0	0	6/4 = 1.5
−M	A_1	2	1	3	0	−1	1	2/3 = 0.6 ← KR
	Zj	−2M	−M	−3M	0	M	−M	
	Cj − Zj		1+M	5+3M	0	−M	0	

↓ KC

value for main row :−

$\frac{2}{3}, \frac{1}{3}, \frac{3}{3}, \frac{0}{3}, -\frac{1}{3}, \frac{1}{3}$

Q.4.

$$\text{Max } z = 4x_1 + x_2 + 3x_3 - x_4$$

s.t. $x_1 + 2x_2 + 3x_3 = 15$

$2x_1 + x_2 + 5x_3 = 20$

$x_1 + 2x_2 + x_3 + x_4 = 10$

$x_1, x_2, x_3, x_4 \geq 0$

→

$$\text{Max } z = 4x_1 + x_2 + 3x_3 - x_4 - MA_1 - MA_2 - MA_3$$

s.t. $x_1 + 2x_2 + 3x_3 + A_1 = 15$

$2x_1 + x_2 + 5x_3 + A_2 = 20$

$x_1 + 2x_2 + x_3 + x_4 + A_3 = 10$

	x_1	x_2	x_3	x_4	A_1	A_2	A_3	
	1	2	3	0	1	0	0	15
	2	1	5	0	0	1	0	20
	1	2	1	1	0	0	1	10

	cj	4	1	3	-1	-M	-M	-M	XB/EC
CB	BV	XB	x_1	x_2	x_3	x_4	A_1	A_2	A_3	
-M	A_1	15	1	2	3	0	1	0	0	5
-M	A_2	20	2	1	5	0	0	1	0	4 ←
-M	A_3	10	1	2	1	1	0	0	1	10
	z_j		-4M	-5M	-9M	-M	-M	-M	-M	
	$z_j - c_j$		(-4M-4)	(-5M-1)	(-9M-3) ↑	(-M+1)	0	0	0	

	cj	4	1	3	-1	-M	-M	-M	XB/EC
CB	BV	XB	x_1	x_2	x_3	x_4	A_1	A_2	A_3	
-M	A_1	3	-1/5	7/5	0	0	1	-3/5	0	15/7 ←
3	x_3	4	2/5	1/5	1	0	0	1/5	0	20
-M	A_3	6	3/5	9/5	0	1	0	-1/5	1	30/9
	z_j		$\frac{-2M+6}{5}$	$\frac{-16M+3}{5}$	0	-M	-M	$\frac{4M+3}{5}$	-M	
	$z_j - c_j$		$\frac{-2M-14}{5}$	$\frac{-16M-2}{5}$ ↑	-3	-M+1	0		0	

CB	BV	Cj / XB	4 / x_1	1 / x_2	3 / x_3	-1 / x_4	-M / A_3	XB/EC
1	x_2	15/7	-1/7	1	0	0	0	(-)
3	x_3	25/7	3/7	0	1	0	0	25/3
-M	A_3	15/7	6/7	0	0	1	1	15/6 ←
		z_j	$\frac{8-6M}{7}$	1	3	-M	-M	
		$z_j - c_j$	$\frac{-6M-20}{7}$ ↑	0	0	-M+1	0	

CB	BV	Cj / XB	4 / x_1	1 / x_2	3 / x_3	-1 / x_4
1	x_2	95/6	0	1	0	-1/6
3	x_3	105/6	0	0	1	-1/2
4	x_1	15/6	1	0	0	7/6
		z_j	4	1	3	$\frac{20}{6}$
		$z_j - c_j$	0	0	0	$\frac{14}{6}$

All $z_j - c_j \geq 0$, the sol^n is optimal,

and sol^n is $x_1 = \frac{15}{6}$, $x_2 = \frac{95}{6}$, $x_3 = \frac{105}{6}$

and Max $z = \frac{95}{6} + 3 \times \frac{105}{6} + 4 \times \frac{15}{6}$

$= \frac{470}{6}$

Q. $4x_1 + x_2 + 3x_3 - x_4 = \text{Max } Z$

s.t. $x_1 + 2x_2 + 3x_3 = 15$

$2x_1 + x_2 + 5x_3 = 20$

$x_1 + 2x_2 + x_3 + x_4 = 10$

$x_1, x_2, x_3 \geq 0$

$\rightarrow$

$\text{Max } Z = 4x_1 + x_2 + 3x_3 - x_4 - MA_1 - MA_2$

$x_1 + 2x_2 + 3x_3 + A_1 = 15$

$2x_1 + x_2 + 5x_3 + A_2 = 20$

$x_1 + 2x_2 + x_3 + x_4 = 10$

		Cj	4	1	3	-1	-M	-M	XB/EC
CB	BV	XB	x_1	x_2	x_3	x_4	A_1	A_2	
-M	A_1	15	1	2	3	0	1	0	$\frac{15}{3}=5$
-M	A_2	20	2	1	5	0	0	1	$\frac{20}{5}=4$ ←
-1	x_4	10	1	2	1	1	0	0	10
		Zj	-3M-1	-3M-2	-8M-1	-1	-M	-M	
		Zj-Cj	-3M-5	-3M-3	-8M-4 ↑	0	0	0	

		Cj	4	1	3	-1	-M	XB/EC
CB	BV	XB	x_1	x_2	x_3	x_4	A_1	
-M	A_1	3	-1/5	7/5	0	0	1	$\frac{15}{7}$ ←
3	x_3	4	2/5	1/5	1	0	0	20
-1	x_4	6	3/5	9/5	0	1	0	$\frac{30}{9}$
		Zj	$\frac{M+3}{5}$	$\frac{-7M-6}{5}$	3	-1	-M	
		Zj-Cj	$\frac{M-17}{5}$	$\frac{-7M-11}{5}$ ↑	0	0	0	

		Cj	4	1	3	-1	XB/EC
CB	BV	XB	x_1	x_2	x_3	x_4	
1	x_2	$\frac{15}{7}$	$\frac{-1}{7}$	1	0	0	(-)
3	x_3	$\frac{25}{7}$	$\frac{15}{7}$	0	1	0	5/3
-1	x_4	$\frac{15}{7}$	$\frac{30}{7}$	0	0	1	1/2 ←
		Zj	2	-1	3	-1	
		Zj-Cj	-2 ↑	-2	0	0	

C_j			4	1	3	-1
C_B	BV	X_B	x_1	x_2	x_3	x_4
1	x_2	95/6	0	1	0	1/30
3	x_3	35/6	0	0	2	-1
4	x_1	1/2	1	0	0	7/30
	Z_j		4	1	6	

$\text{Max } Z = 4x_1 + x_2 + 3x_3 - x_4$

s.t. $x_1 + 2x_2 + 3x_3 = 15$ →

$2x_1 + x_2 + 5x_3 = 20$

$x_1 + 2x_2 + x_3 + x_4 = 10$

$x_1, x_2, x_3, x_4 \geq 0$

→ $\text{Max } Z = 4x_1 + x_2 + 3x_3 - x_4 - MA_1 - MA_2 - MA_3$

s.t. $x_1 + 2x_2 + 3x_3 + A_1 = 15$

$2x_1 + x_2 + 5x_3 + A_2 = 20$

$x_1 + 2x_2 + x_3 + x_4 + A_3 = 10$

	cj	4	1	3	-1	-M	-M	-M	
CB	BV	XB	x_1	x_2	x_3	x_4	A_1	A_2	A_3	XB/EC
-M	A_1	15	1	2	3	0	1	0	0	5
-M	A_2	20	2	1	5	0	0	1	0	4 ←
-M	A_3	10	1	2	1	1	0	0	1	10
	zj	-4M	-5M	-9M	-M	-M	-M	-M		
	zj-cj	(-4M-4)	(-5M-1)	(-9M-3) ↑	(-M+1)	0	0	0		

$R_1 \to R_1 - \frac{3}{5} R_2$

$R_3 \to R_3 - \frac{1}{5} R_2$

	cj	4	1	3	-1	-M	-M	
CB	BV	XB	x_1	x_2	x_3	x_4	A_1	A_3	XB/EC
-M	A_1	3	-1/5	7/5	0	0	1	0	15/7 ←
3	x_3	4	2/5	1/5	1	0	0	0	20
-M	A_3	6	3/5	9/5	0	1	0	1	10/3
	zj	$\frac{-2M+6}{5}$	$\frac{-16M+3}{5}$	3	-M	-M	-M		
	zj-cj	$\frac{-2M+4}{5}$	$\frac{-16M-2}{5}$ ↑	0	-M+1	0	0		

$R_2 \to R_2 - \frac{1}{7} R_1$

$R_3 \to R_3 - \frac{9}{7} R_1$

	cj	4	1	3	-1	-M	
CB	BV	XB	x_1	x_2	x_3	x_4	A_3	XB/EC
1	x_2	15/7	-1/7	1	0	0	0	(-)
3	x_3	25/7	3/7	0	1	0	0	25/3
-M	A_3	3/5	6/7	0	0	1	1	7/10 ←
	zj	$\frac{-6M+8}{7}$	1	3	-M	-M		
	zj-cj	$\frac{-6M-20}{7}$ ↑	0	0	-M+1	0		

CB	BV	c_j / x_B	4 / x_1	1 / x_2	3 / x_3	-1 / x_4
2	x_2	$\frac{157}{70}$	0	1	0	1/6
3	x_3	$\frac{229}{70}$	0	0	1	-1/2
4	x_1	$\frac{7}{10}$	1	0	0	7/6
		Z_j	4	2	3	$\frac{105}{18}$
		$Z_j - c_j$	0	1	0	$\frac{87}{18}$

$R_1 \to R_1 + \frac{1}{6} R_3$

$R_2 \to R_2 - \frac{1}{2} R_3$

Q. What do you mean by Game?

→ Game is defined ~~by~~ as an activity between two or more persons according to a set of rules at the end of which each person receives some benefit or suffers loss.

Q.2 What do you mean by pure strategy & Mixed?

→ A force of action taken by a player is called strategy.

Pure strategy :- If a player select the same strategy each time or if one knows in advance of the play that is certain to be adopted

Mixed strategy :- When a player use combination of strategies then it is called mixed strategy.

Q3 What do you mean by pay off?

→ Each game has an outcome i.e. gain or loss or draw that is called pay-off. And when the player select their strategies it can be ~~termed~~ represent in form of matrix, that is called matrix.

Q.4

Module-III — GAME THEORY — (Quantitative Techniques)

(1)

A Game is a physical activity which involves two or more than two players, with certain set of rules, during the play of game, players add different types of strategies, there are two types strategies (1) Pure strategy and (2) Mixed strategies.

Pure strategy – Are the strategy which is known to both the players, during the play of game.

Mixed strategy – Are the strategies which is unknown to both the players during the game, they only can predict the chances of different strategies during the game.

Pay off Matrix is represented as in a ~~rect~~ react ~~ungular~~ ways with rows and columns and the entries of pay-off matrix are the outcomes of different strategies adopted by the players during the game.

Game theory divided into two parts one ~~is~~ based on (1) pure strategy (2) mixed strategy

Pure Strategy

- Maximin and Minmax method.
- Saddle point
- value of the Game

Mixed strategy

→ 2x2 game
→ 2xn game, mx2 game
→ mxn game

Q. solve the following game, whose pay-off matrix is

Player A \ Player B	B_1	B_2	B_3
A_1	1	3	1
A_2	0	-4	-3
A_3	1	5	-1

Player A \ Player B	B_1	B_2	B_3	Row minimum
A_1	1 (*) (+)	3	1 (+) (*)	(1)
A_2	0	-4 (*)	-3	(-4)
A_3	1 (+)	5 (+)	-1 (*)	(-1)
Column max	(1)	(5)	(1)	

maximin — (Player A) — minimax

Max (Row minimum) = max {1, -4, -1} = 1

Min (column maximum) = Min {1, 5, 1} = 1

value of the game = 1

Since max (min) = min (max) = 1, and optimal strategy is $(A_1 B_1)$ & $(A_1 B_3)$

Saddle point is a position in the pay off matrix where maximum of row minimum intersects with minimum of column maximum, and the saddle point is called as the value of game and max min is denoted by $\underline{v}$ and

The game having a saddle point is known as pure

Q.

	B_1	B_2	B_3	B_4	B_5	Row minimum
A_1	-2	0	0	5	3	(-2)
A_2	3	2	1	2	2	(1)
A_3	-4	-3	0	-2	6	(-4)
A_4	5	3	-4	2	-6	(-6)
Column max	(5)	(2)	(1)	(5)	(6)	

Max (Row min) = Max $\{-2, 1, -4, -6\} = 1$

Min (Column max) = Min $\{5, 2, 1, 5, 6\} = 1$

$\underline{v} = 1, \quad \bar{v} = 1$

the value of the game or saddle point = 1 and this is corresponding to $(A_2 B_3)$

Q. Determine the optimal

	B_1	B_2	B_3	B_4	(row column)
A_1	-5	2	0	7	(-5)
A_2	5	6	4	8	(4)
A_3	4	0	2	-3	(-3)
(Column max)	(5)	(6)	(4)	(8)	

Max (row min) = max $\{-5, 4, -3\} = 4$

Min (column max) = Min $\{5, 6, 4, 8\} = 4$

Ignoring the value of "λ"

Q.

	B_1	B_2	B_3	R-min
A_1	λ	6 (†)	2 (†)*	(2) ⟶ $\underline{v}$ (maximin)
A_2	-1 (†)	λ	-1 *	(-7)
A_3	-2 *	4	λ	(-2)
c-Max	(-1)	(6)	(2) ⟶ $\overline{v}$ (minimax)	

Here the value of the game = 2 and this is corresponding to $(A_1 B_3)$.

Q.

(a)

	B_1	B_2	R-min
A_1	5 (†)	0 *	(0)
A_2	0 *	2 (†)	(0)
c-Max	(5)	(2)	

$\overline{v}$ (minimax)

The ~~saddle~~ game has no saddle point as $\underline{v} \neq \overline{v}$.

(b)

	B_1	B_2	R-min
A_1	0 (†)(*)	2	(0) ⟶ $\underline{v}$ (maximin)
A_2	-1 (*)	4 (†)	(-1)
c-Max	(0)	(4)	

$\overline{v}$ (minimax)

The saddle point is zero. So the game is called fair.

Q.

	~~B_1~~	B_2	B_3	R-min
A_1	-1	2	-2 †*	(-2) ⟶ $\underline{v}$ (maximin)
A_2	6 †	4 †	-6 *	(-6)
c-Max	(6)	(4)	(-2)	

$\overline{v}$ (minimax)

The value of the game = (-2) and game is strictly determinable corresponding to (A_1, B_3)

Q.

	B_1	B_2	B_3	B_4	R-min
A_1	-5^*	2	0	7	(-5)
A_2	$5^\dagger$	$6^\dagger$	$4^{*\dagger}$	$8^\dagger$	(4) — $\underline{v}$ (maximin)
A_3	4	0	2	-3^*	(-3)
C-Max	(5)	(6)	(4)	(8)	

$\bar{v}$ (minimax)

The saddle point = 4, and it is corresponding to (A_2, B_3)

Q. A game having 2 players is known as 2 person game.

A game having more than 2 players is known as 'n' person game.

A game is said to be zero sum game, when the gain of one player is equal to the loss of another player.

Mixed Strategy (Game without saddle point)

→ 2x2 game (Algebric Method)
→ 2xn or mx2 game (Graphical & Algebric)
→ mxn game (Dominance principle & Algebric)

$$P_1 = \frac{a_{22} - a_{21}}{(a_{11} + a_{22}) - (a_{12} + a_{21})}, \quad q_1 = \frac{a_{22} - a_{12}}{(a_{11} + a_{22}) - (a_{12} + a_{21})}$$

$$v = \frac{a_{11}a_{22} - a_{12}a_{21}}{(a_{22} + a_{11}) - (a_{12} + a_{21})}$$ (Value of the game)

$$\begin{bmatrix} 5 & 0 \\ 0 & 2 \end{bmatrix}$$

$$P_1 = \frac{2}{7}$$

$$P_2 = \frac{5}{7}$$

$$q_1 = \frac{2}{7}, \quad q_2 = \frac{5}{7}$$

$$v = \frac{10 - 0}{7 - 0} = \frac{10}{7}$$

$$\begin{bmatrix} 5 & 1 \\ 3 & 4 \end{bmatrix}$$

(1) (3) (5) (4)

$a_{11}=5,\ a_{12}=1$

$a_{21}=3,\ a_{22}=4$

$$P_1 = \frac{4-3}{(5+4)-(1+3)} = \frac{1}{9-4} = \frac{1}{5}$$

$$P_2 = \frac{4}{5}$$

$$q_1 = \frac{4-1}{(5+4)-(1+3)} \qquad q_2 = \frac{2}{5}$$

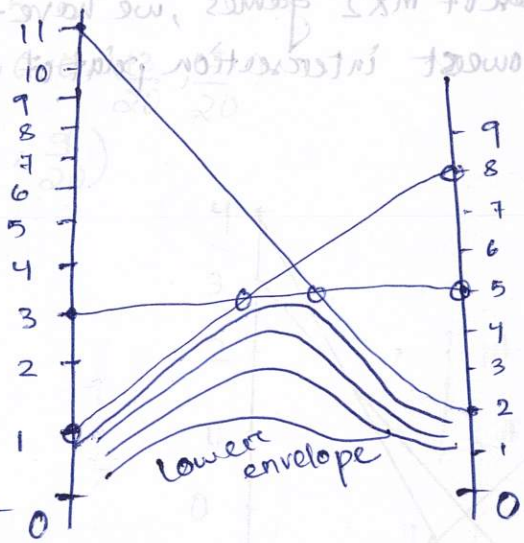
$$= \frac{3}{9-4}$$

$$= \frac{3}{5}$$

$$v = \frac{20-3}{(5+4)-(1+3)} = \frac{17}{5}$$

2xn or mx2 (Graphical & Algebraic)

$$\begin{bmatrix} 1 & 3 & 11 \\ 8 & 5 & 2 \end{bmatrix}$$



When a game is a (2xn) game, we have to use graphical technique to reduce the matrix into a (2x2) game.

In a 2xn game, we have to consider the highest intersection points of the lower envelope from the graph.

$a_{11}=3,\ a_{12}=11$

$a_{21}=5,\ a_{22}=2$

$$\begin{bmatrix} 3 & 11 \\ 5 & 2 \end{bmatrix} \quad \begin{matrix} 3 \\ 8 \end{matrix}$$

$$P_1 = \frac{2-5}{(2+3)-(11+5)} = \frac{-3}{5-16} = \frac{3}{11}$$

$$P_2 = \frac{8}{11}$$

$$q_1 = \frac{a_{22}-a_{12}}{(a_{11}+a_{22})-(a_{12}+a_{21})}$$

$$= \frac{2-11}{(3+2)-(11+5)}$$

$$= \frac{-9}{-11} = \frac{9}{11}$$

$$q_2 = \frac{2}{11}$$

$$v = \frac{9}{11} + \frac{40}{11} = \frac{49}{11}$$

$$S_A = \begin{pmatrix} SA_1 & SA_2 \\ 3/11 & 8/11 \end{pmatrix}$$

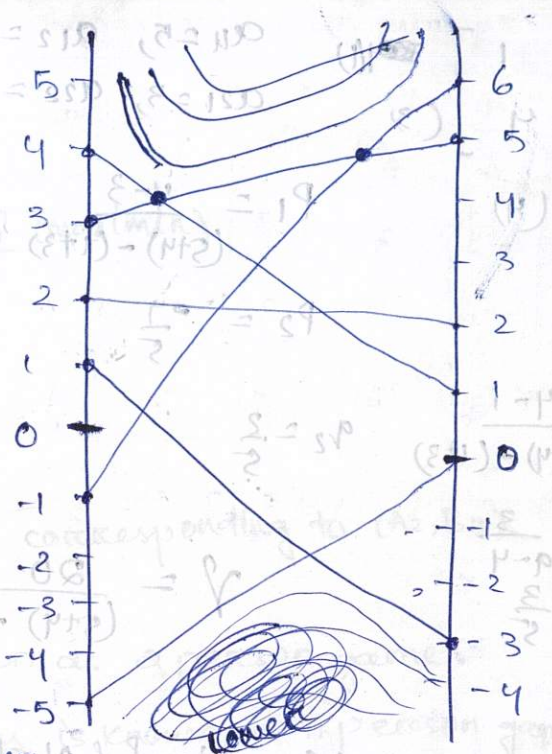
$$S_B = \quad SB_1 \quad SB_2 \quad SP_3$$

$$\begin{bmatrix} 1 & -3 \\ 3 & 5 \\ -1 & 6 \\ 4 & 1 \\ 2 & 2 \\ -5 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 4 & 1 \\ -1 & 6 \end{bmatrix}$$

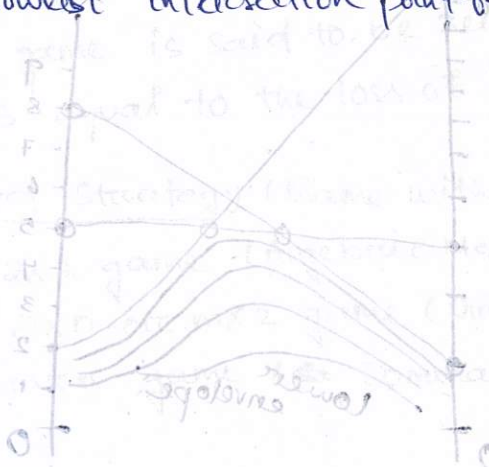
$$\begin{bmatrix} 3 & 5 \\ 4 & 1 \end{bmatrix} \quad \begin{matrix} 3 & \frac{3}{13} \\ 2 & \frac{2}{13} \end{matrix}$$

$$\begin{matrix} \frac{4}{13} & \frac{1}{13} \end{matrix}$$

$$v = \frac{9}{13} + \frac{8}{13} = \frac{17}{13}$$



In case of mx2 games, we have to consider the lowest intersection point of upper envelope from the graph.

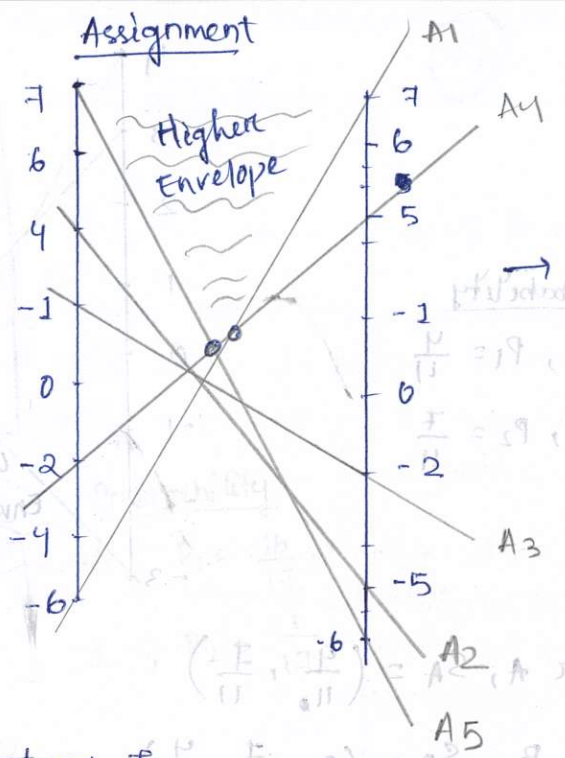


Assignment

(7)

Q.

	B_1	B_2
A_1	-6	7
A_2	4	-5
A_3	-1	-2
A_4	-2	5
A_5	7	-6



	B_1	B_2	P_1
A_4	-2	5	$\frac{13}{20}$
A_5	7	-6	$\frac{7}{20}$ (P_2)
	11	9	

$q_1 = \frac{11}{20}$ $\quad \frac{9}{20} = q_2$

$v = \frac{-13}{20}(-2) + \frac{7}{20}(7)$

$= \frac{-26+49}{20} = \frac{23}{20}$

∴ optimal strategy for A, $S_A = (0, 0, 0, \frac{13}{20}, \frac{7}{20})$

optimal strategy for B, $S_B = (\frac{11}{20}, \frac{9}{20})$

Q.

	B_1	B_2	B_3
A_1	4	-3	3
A_2	-3	1	-1

→

	B_1	B_2
A_1	4	-3
A_2	-3	1

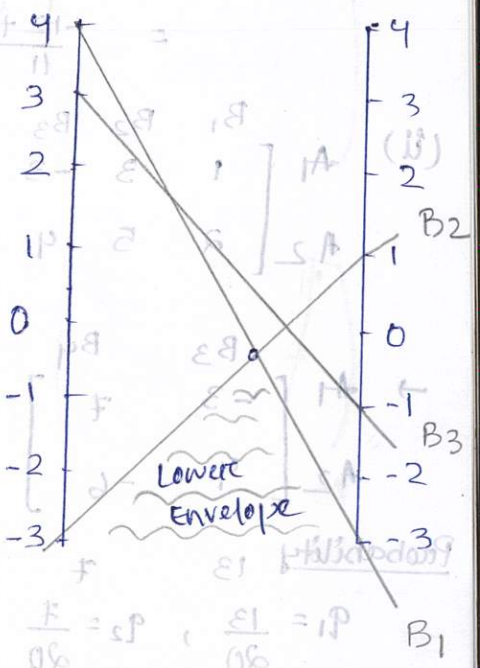
Probability

4 , $P_1 = \frac{4}{11}$

7 , $P_2 = \frac{7}{11}$

Probability 4 7

$q_1 = \frac{4}{11}$, $q_2 = \frac{7}{11}$



optimal strategy for A, $S_A = (\frac{4}{11}, \frac{7}{11})$

optimal strategy for B, $S_B = (\frac{4}{11}, \frac{7}{11}, 0)$

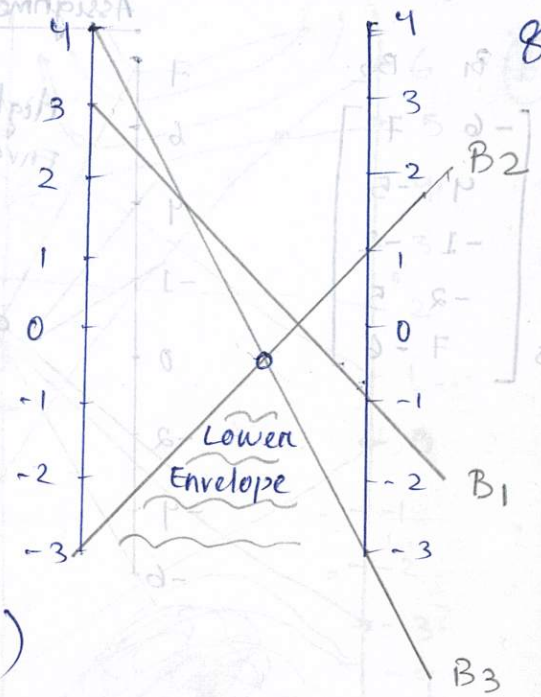
and the saddle point, $v = \frac{4}{11} \times 4 + \frac{7}{11} \times (-3)$

$= \frac{16}{11} - \frac{21}{11}$

$= -\frac{5}{11}$

Q.1

	B_1	B_2	B_3
A_1	3	-3	4
A_2	-1	1	-3



→

	B_2	B_3	Probability
A_1	-3	4	4 , $P_1 = \frac{4}{11}$
A_2	1	-3	7 , $P_2 = \frac{7}{11}$

Probability $\quad \frac{7}{11}, \quad \frac{4}{11}$

$q_1 = \frac{7}{11}, \quad q_2 = \frac{4}{11}$

∴ optimal strategy for A, $S_A = \left(\frac{4}{11}, \frac{7}{11}\right)$

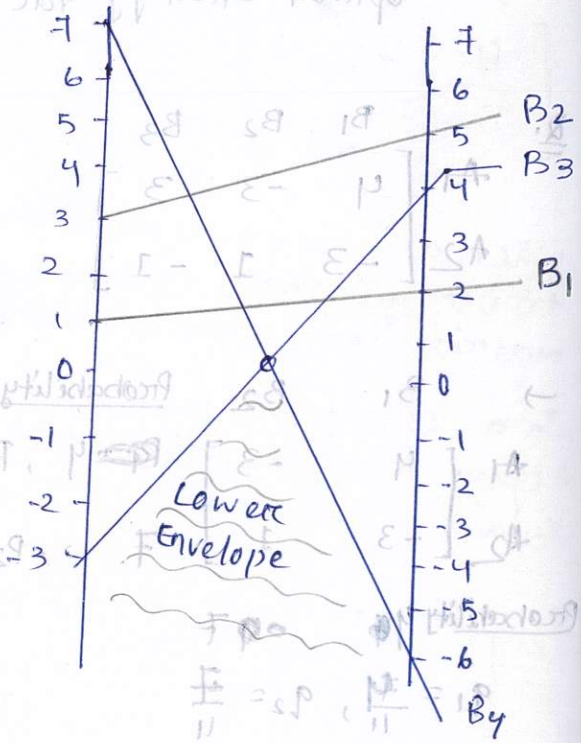
" " " B, $S_B = \left(0, \frac{7}{11}, \frac{4}{11}\right)$

$$v = \frac{4}{11}(-3) + \frac{7}{11}(1)$$

$$= \frac{-12+7}{11} = \frac{-5}{11}$$

(ii)

	B_1	B_2	B_3	B_4
A_1	1	3	-3	7
A_2	2	5	4	-6



→

	B_3	B_4	Probability
A_1	-3	7	10 , $P_1 = \frac{10}{20} = \frac{1}{2}$
A_2	4	-6	10 , $P_2 = \frac{10}{20} = \frac{1}{2}$

Probability $\quad 13 \quad 7$

$q_1 = \frac{13}{20}, \quad q_2 = \frac{7}{20}$

optimal strategy for, A, $S_A = \left(\frac{1}{2}, \frac{1}{2}\right)$

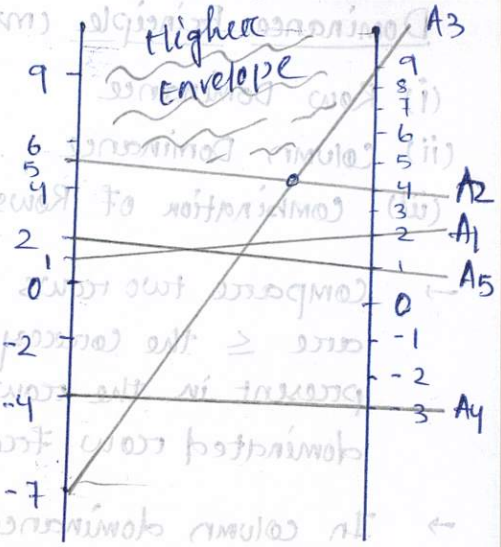
" " " B, $S_B = \left(0, 0, \frac{13}{20}, \frac{7}{20}\right)$

And $v = \frac{1}{2} \times (-3) + \frac{1}{2} \times (4)$

$$= \frac{-3+4}{2} = \frac{1}{2}$$

(iii)

	B_1	B_2
A_1	1	2
A_2	5	4
A_3	-7	9
A_4	-4	-3
A_5	2	1



→

	B_1	B_2	Probability
A_2	5	4	$16, \; P_1 = \frac{16}{17}$
A_3	-7	9	$1, \; P_2 = \frac{1}{17}$

Probability 5 12

$q_1 = \frac{5}{17}, \; q_2 = \frac{12}{17}$

Optimal strategy for A, $S_A = \left(0, \frac{16}{17}, \frac{1}{17}, 0, 0\right)$

" " " B, $S_B = \left(\frac{5}{17}, \frac{12}{17}\right)$

and $v = \frac{16}{17}(5) + \frac{1}{17}(-7)$

$= \frac{80}{17} - \frac{7}{17}$

$= \frac{73}{17}$

Dominance Principle (m×n matrix)

(i) Row Dominance

(ii) Column Dominance

(iii) Combination of Rows and Column Dominance

→ Compare two rows from the pay-off matrix if all the elements of a row are ≤ the corresponding element of another row then the smallest element present in the row dominated by largest element, Then remove the dominated row from matrix.

→ In column dominance, all the elements in a column ≤ the corresponding elements of another column then the largest element present in the column is dominated by the smallest element in the column then remove the dominated column from matrix.

Q.

$$\begin{pmatrix} 3 & 2 & 4 & 0 \\ 2 & 4 & 2 & 4 \\ 4 & 2 & 4 & 0 \\ 0 & 4 & 0 & 8 \end{pmatrix}$$

Row Dominance

$$\rightarrow \begin{pmatrix} \cancel{3} & \cancel{2} & \cancel{4} & \cancel{0} \\ 2 & 4 & 2 & 4 \\ 4 & 2 & 4 & 0 \\ 0 & 4 & 0 & 8 \end{pmatrix}$$

As $r_4 < r_3$

Column Dominance

$$\begin{pmatrix} \cancel{2} & 4 & 2 & 4 \\ 4 & 2 & 4 & 0 \\ 0 & 4 & 0 & 8 \end{pmatrix}$$

As $c_4 = c_3$

$$\begin{pmatrix} \cancel{4} & 2 & 4 \\ \cancel{2} & 4 & 0 \\ \cancel{4} & 0 & 8 \end{pmatrix} \begin{matrix} 3 \\ 2 \\ 4 \end{matrix}$$

As $c_1 > \frac{c_2 + c_3}{2}$

$$\rightarrow \begin{pmatrix} \cancel{2} & \cancel{4} \\ 4 & 0 \\ 0 & 8 \end{pmatrix} \rightarrow$$

As $r_1 = \frac{r_2 + r_3}{2}$

$$\underline{P} \quad \begin{matrix} 4 & 0 \\ 0 & 8 \end{matrix} \quad \begin{matrix} 8 \\ 4 \end{matrix}$$

$$\underline{Q} \quad \begin{matrix} 8 & 4 \end{matrix}$$

Probability

$P_1 = \frac{8}{12}$, $P_2 = \frac{4}{12}$

$q_1 = \frac{8}{12}$, $q_2 = \frac{4}{12}$

Saddle point, $v = 4 \times \frac{8}{12} + 0 \times \frac{4}{12} = \frac{8}{3}$

Optimal strategy for A, $S_A = \left(0, 0, \frac{8}{12}, \frac{4}{12}\right)$

$S_B = \left(0, 0, \frac{8}{12}, \frac{4}{12}\right)$

Q.1

(a) Quartiles divides the data set into 4 equal parts. By comparing Q_3, Q_2 & Q_1 we can measure the dispersion.

(b) The correlation coefficient ranges from -1 to 1.

(d) Coefficient of variation helps us to compare the consistency between two or more distributions.

(e) → P.E = 0.2 & n = 9

As $P.E = 0.6745\left(\frac{1-r^2}{\sqrt{n}}\right)$

$\Rightarrow 0.2 = 0.6745\left(\frac{1-r^2}{\sqrt{9}}\right)$

$\Rightarrow 0.2 \times 3 = (1-r^2)\,0.6745$

$\Rightarrow \frac{0.6}{0.6745} = 1-r^2$

$\Rightarrow r^2 = 1 - \frac{0.6}{0.6745}$

$\Rightarrow r^2 = 1 - 0.8895$

$\Rightarrow r = \sqrt{0.110}$

$\Rightarrow r = 0.331$

(f) Max $Z = 3x_1 - 2x_2$

s.t. $x_1 + x_2 \leq 1$

$2x_1 + 2x_2 \geq 4$

$\forall x \geq 0$

(g) → First we need to convert the maximization table/matrix to minimization matrix by subtracting all the elements of that matrix from the largest element of matrix, and then we can follow the hungarian technique.

(h) A T.P is said to be unbalanced, when sum of demand is not equal to sum of supply. In this situation we have to make it balance by adding a dummy row or column as per the requirement, with proper supply and demand.

Q.2

(a)

No. of Accidents	Frequency	fx
0	46	
1	f_1	
2	f_2	
3	25	
4	10	
5	5	
	f_1+f_2+86	

Given $\bar{X} = 1.46$

$N = 200$

$\Rightarrow f_1+f_2+86 = 200$

$\Rightarrow f_1+f_2 = 114$ —— (1)

(a)

X	f	fx
0	46	0
1	f_1	f_1
2	f_2	$2f_2$
3	25	75
4	10	40
5	5	25
	$140+f_1+2f_2$	

Given $\bar{X} = 1.46$

$N = 200$

$\Rightarrow f_1+f_2+86 = 200$

$\Rightarrow f_1+f_2 = 114$ —— (1)

Now $\bar{X} = 1.46$

$\Rightarrow \frac{\Sigma fx}{N} = 1.46$

$\Rightarrow \frac{140+f_1+f_2}{200} = 1.46$

$\Rightarrow f_1+2f_2+140 = 292$

$\Rightarrow f_1+2f_2 = 152$ —— (11)

Substracting eq^n(11) from eq^n(1), we get -

$$f_1 + 2f_2 = 152$$
$$-f_1 - f_2 = -114$$
$$f_2 = 38$$

Putting f_2 value on eq^n(1), we get -

$f_1 + 38 = 114$

$\Rightarrow f_1 = 76$

$\therefore (f_1, f_2) = (38, 76)$

(b) Given $\bar{x} = 43$

$\sigma x = 5$

and $n = 9$

→ So $\Sigma x = n \cdot \bar{x}$

$= 9 \times 43$

$= 387$

For addition of new item 63

$\Sigma x = 387 + 63$

$= 450$

Hence $\bar{x} = \frac{\Sigma x}{n} = \frac{450}{10} = 45$

Again $\Sigma x^2 = n(\sigma x^2 + (\bar{x})^2)$

$= 9(5^2 + 43^2)$

$= 9(25 + 1849)$

$= 9 \times 1874$

$= 16866$

For addition of new item 63,

$\Sigma x^2 = 16866 + 63^2 = 16866 + 3969$

$= 20835$

So $\sigma x = \sqrt{\frac{\Sigma x^2}{n} - (\bar{x})^2}$

$= \sqrt{\frac{20835}{10} - (45)^2}$

$= \sqrt{2083.5 - 2025}$

$= \sqrt{58.5} = 7.64$

∴ Mean = 45, & S.D = 7.64.

(c)

X	Y	Rx	Ry	D = Rx − Ry	D^2
48	13	3	5.5	−2.5	6.25
33	13	5	5.5	−0.5	0.25
40	24	4	1	3	9
9	6	10	8.5	1.5	2.25
16	15	8	4	4	16
16	4	8	10	−2	4
65	20	1	2	−1	1
24	9	6	7	−1	1
16	6	8	8.5	−0.5	0.25
57	19	2	3	−1	1
					$\Sigma = 41$

Here $m_1(16) = 3$, $m_2(13) = 2$,

& $m_3(6) = 2$

Hence $r = 1 - \frac{6\left[\Sigma d^2 + \frac{m_i(m_i^2 - 1)}{12}\right]}{n(n^2 - 1)}$

$= 1 - \frac{6\left[41 + \frac{3(3^2-1)}{12} + \frac{2(2^2-1)}{12} + \frac{2(2^2-1)}{12}\right]}{10(100 - 1)}$

$= 1 - \frac{6(41 + 2 + 0.5 + 0.5)}{990}$

$= 1 - \frac{264}{990} = 1 - 0.266 = 0.73$

(f) Max $Z = 5x_1 + 3x_2$

s.t. $3x_1 + 5x_2 \leq 15$

$5x_1 + 2x_2 \leq 10$

$\forall x \geq 0$

→ Max $Z = 5x_1 + 3x_2 + 0s_1 + 0s_2$

s.t. $3x_1 + 5x_2 + s_1 + 0s_2 = 15$

$5x_1 + 2x_2 + 0s_1 + s_2 = 10$

$x_1, x_2, s_1, s_2 \geq 0$

Simplex Table

		c_j	5	3	0	0	XB/EC
CB	BV	XB	x_1	x_2	s_1	s_2	
0	s_1	15	3	5	1	0	$\frac{15}{3} = 5$
0	s_2	10	5	2	0	1	$\frac{10}{5} = 2$ ←
		z_j	0	0	0	0	
		$z_j - c_j$	-5↑	-3	0	0	

		c_j	5	3	0	0	XB/EC
CB	BV	XB	x_1	x_2	s_1	s_2	
0	s_1	9	0	19/5	1	-3/5	$\frac{45}{19} = 0.78$ ←
5	x_1	2	1	2/5	0	1/5	$\frac{20}{2} = 10$
		z_j	5	2	0	1	
		$z_j - c_j$	0	-1↑	0	1	

		c_j	5	3	0	0	
CB	BV	XB	x_1	x_2	s_1	s_2	
3	x_2	$\frac{45}{19}$	0	1	$\frac{5}{19}$	$\frac{-3}{19}$	
5	x_1	20/19	1	0	$\frac{2}{19}$	5/19	
		z_j	5	3	[illegible]	[illegible]	5/19 16/19
		$z_j - c_j$	0	0	[illegible]	[illegible]	0 0

Since all $z_j - c_j \geq 0$, the soln has reached its optimal

~~and Max Z~~ = [illegible]

$x_1 = \frac{20}{19}$, $x_2 = \frac{45}{19}$, Max $Z = \frac{235}{19}$

(h) Machines — Row Reduction — Column Reduction

Jobs	A	B	C	D
1	18	24	28	32
2	8	13	17	18
3	10	15	19	22

$\rightarrow$

	0	6	10	14
	0	5	9	10
	0	5	9	12

$\rightarrow$

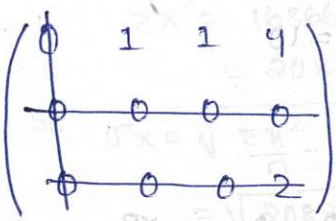
	0	1	1	4
	0	0	0	0
	0	0	0	2

$\rightarrow$

	[0]	1	1	4
	~~0~~	~~0~~	~~0~~	[0]
	~~0~~	[0]	~~0~~	2

(Assignment Process)

$\rightarrow$



(Drawing Lines)

$\rightarrow$

	1	0	0	3
	1	0	0	0
	1	0	0	2

(New Table)

	1	[0]	~~0~~	3
	1	~~0~~	~~0~~	[0]
	1	~~0~~	[0]	2

(h)

Jobs	A	B	C	D
1	18	24	28	32
2	8	13	17	18
3	10	15	19	22

$\rightarrow$

	18	24	28	32
	8	13	17	18
	10	15	19	22
	0	0	0	0

(Balanced Matrix)

$\rightarrow$

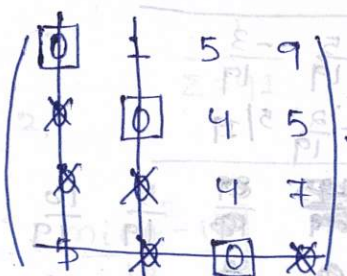
	0	6	10	14
	0	5	9	10
	0	5	9	12
	0	0	0	0

(Row Reduction & Column Reduction)

	[0]	6	10	14
	~~0~~	5	9	10
	~~0~~	5	9	12
	~~0~~	[0]	~~0~~	~~0~~

(Assignment process & Drawing lines)

$\rightarrow$



(New table, Assignment process & Drawing lines)

$\rightarrow$

	[0]	1	1	5
	~~0~~	[0]	~~0~~	1
	~~0~~	~~0~~	[0]	3
	9	4	~~0~~	[0]

Ans is

Jobs	Machines	Cost
1	A	18
2	B	13
3	C	19
		50

Q.4 A retired person's broker recommends him to invest his money in two bonds A & B

Let x_1 rupees invest in bond A

& x_2 rupees " " " B.

Bond A is yielding 7% and Bond B is yielding 10%.

Thus Max $Z = \frac{7}{100} x_1 + \frac{10}{100} x_2$

Also the retired person wants to invest up to an amount of Rs. 30,000

Hence $x_1 + x_2 \leq 30{,}000$

He decides to invest at most Rs. 12000 in Bond B

So $x_2 \leq 12000$

and at least Rs. 6000 in Bond A

So $x_1 \geq 6000$

He also wants the amount invested in bond A (x_1) to be at least equal to the amount invested in Bond B (x_2).

that means $x_1 \geq x_2$

$\Rightarrow x_1 - x_2 \geq 0$

Hence the LPP is Max $Z = \frac{7}{100} x_1 + \frac{10}{100} x_2$

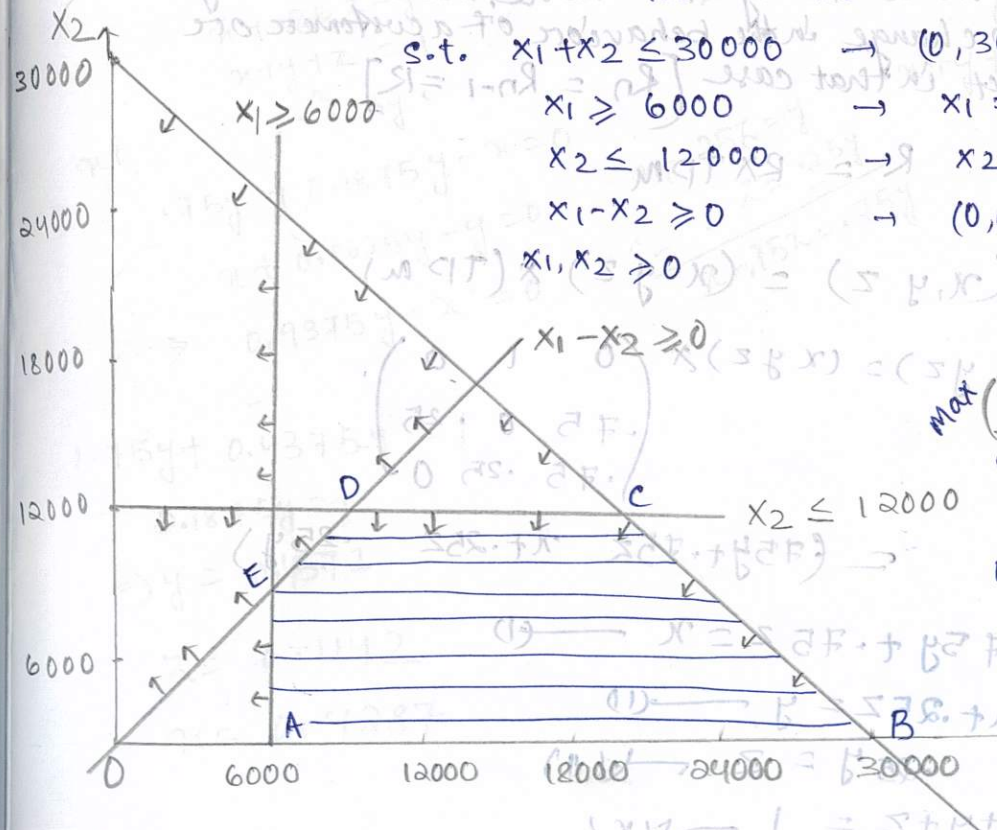
s.t. $x_1 + x_2 \leq 30000$ → (0, 30000) & (30000, 0)

$x_1 \geq 6000$ → $x_1 = 6000$

$x_2 \leq 12000$ → $x_2 = 12000$

$x_1 - x_2 \geq 0$ → (0,0)

$x_1, x_2 \geq 0$



Points	Max $Z = \frac{7}{100} x_1 + \frac{10}{100} x_2$
A (6000, 0)	$Z = \frac{42000}{100} = 420$
B (0, 30000)	$Z = 0 + 3000 = 3000$
C (18K, 12K)	$Z = 1260 + 1200 = 2460$
D (12K, 12K)	$Z = 840 + 1200 = 2040$
E (6K, 6K)	$Z = 420 + 600 = 1020$

Max

Here ABCDE is the feasible region

Markov process are analysis based on probability. In Markov process we will collect the

Q.

Today \ Next week	A	B	C
A	1	0	0
B	0	1	0
C	0	1	0

6 days

Next Week

	A	B	C
A	0	1	0
B	.75	0	.25
C	.75	.25	0



	A	B	C
A	90%	5%	5%
B	5%	85%	10%
C	10%	7%	8.3%

Row wise

In a TPM, the row wise movement depicted as retention & loss and more column wise it is described as retention & gain and the diagonal elements of a TPM described retention only.

In a long run the behavior of the customer on the market share of any product will be in a steady state mode, means in a long run there will be no change in the behavior of a customer or market share of product in that case $[R_n = R_{n-1} = R]$

$$R = R \times TPM$$

Q

	A	B	C
A	0	1	0
B	.75	0	.25
C	.75	.25	0

$$\Rightarrow (x, y, z) = (x\ y\ z) \times (TPM)$$

$$\Rightarrow (x\ y\ z) = (x\ y\ z) \times \begin{pmatrix} 0 & 1 & 0 \\ .75 & 0 & .25 \\ .75 & .25 & 0 \end{pmatrix}$$

$$= (.75y + .75z \quad x + .25z \quad .25y)$$

$$\Rightarrow .75y + .75z = x \quad \text{---(I)}$$

$$x + .25z = y \quad \text{---(II)}$$

$$.25y = z \quad \text{---(III)}$$

$$x + y + z = 1 \quad \text{---(IV)}$$

→ $.75y + .25y = x \quad (.25y = z)$

⇒ $x = y$

∴

$$.75y + .75z = x$$

Putting eq^n(3) in eq^n(1), we get

$$.75y + .75(.25y) = x$$

$$\Rightarrow .75y + 0.1875y = x$$

$$\Rightarrow x = 0.9375y \quad \text{—— (V)}$$

$$x + y + z = 1$$
$$-x + .75y + .75z = 0$$
$$1.75y + 1.75z = 1$$
$$-.25y + z = 0$$
$$1.5y + 2.75z = 1$$

$$.75z - x = -.75y$$
$$.25z + x = y$$
$$z = 0.25y$$

$$-.75y + .75z - x = 0$$
$$-y + .25z + x = 0$$
$$z - .25y = 1.75y$$

$$x + y + .25z = 0$$
$$2x + 1.25z = 1$$
$$z - .25$$

$$z - .25y = 0$$

$$.75y + .75z - x = 0$$
$$x + .25z - y = 0$$
$$x + y + z = 1$$
$$z = .25y$$

$$0.75y + 0.75z = x$$
$$x + 0.25z = y$$
$$0.25y = z$$
$$x + y + z = 1$$

$$.75y + 0.1875y - x = 0$$
$$x + 0.0625y - y = 0$$
$$\Rightarrow 0.9375y - x = 0$$

$$x + .25z = y$$
$$-z = .25y$$
$$x - .75z = .75y$$

$$.75y + .75z - x = 0$$
$$-y + .25z + x = 0$$
$$-0.25y + z = 0$$

$$1.75y + 0.4375y = 1$$
$$2.1875y = 1$$
$$\Rightarrow y = 0.4571$$
$$z = 0.1142$$
$$x = 0.4287$$

SIMULATION

Simulation is a process where experiment is carried out in an Artificial environment, where all the parameters are in control and through simulation we can gather the knowledge which helps us in real environment, for making decision.

The examples are :- Pilot training, car driving, Military training

Monte-Carlo simulation method / Technique

1) Objective
2) Data collection
3) Probability, cumulative probability
4) Tag number
5) Random number
6) Analysis
7) Interpretation of Result / Then Decision

Q. A manufacturing company keeps a stock of a special product, previous experience indicates the daily demand as follows

Daily Demand	Probability
5	~~0.01~~ .01
10	~~0.0101~~ 0.2
15	~~0.2~~ .15
20	~~0.2~~ .5
25	~~0.15~~ .12
30	~~0.5~~ .02

Simulate the demand for the next 10 days also find the daily average demand for that product on the basis of ~~simulate~~ simulated data.

→ Random Number

82
96
18
76
20
84
56
11
52

Daily Demand	P	cP	Tag number
5	0.01	0.01	0-0
10	0.2	0.21	1-20
15	0.15	0.36	21-35
20	0.5	0.86	36-85
25	0.12	0.98	86-97
30	0.02	1	98-99
	Σ=1		

R.N	Demand
82	20
96	25
18	10
96	25
20	10
84	20
56	20
11	10
52	20
03	10

Avg. Daily Demand = Total no. of unit / 10 = 170 / 10 = 17

Q.

Production	Probability	cP	Tag No
95	0.03	0.03	0-02
96	0.05	0.08	3-9
97	0.07	0.15	10-14
98	0.10	0.25	15-24
99	0.15	0.40	25-39
100	0.20	0.60	40-59
101	0.15	0.75	60-74
102	0.10	0.85	75-84
103	0.07	0.92	85-91
104	0.05	0.97	92-96
105	0.03	1	97-99

R.N	Tag No
97	0-2
2	3-7
80	8-14
66	15-24
	25-

R.N →	Production	Cars Waiting	Empty space
97	1050	105	95
02	95*	102	101
80	102°	104	100
66	101*	103	100
96	104*		99
55	100*	Σ=414	100
50	100*	Avg = 414/4	100
29	99*	=103.5	96
58	100*		98
51	100*		99
04	96*		100
86	103°		Σ=1088
24	98*		Avg = 98.90
39	99*		
47	100*		

~~RCA~~

Production	cores Waiting	Empty space
105	4	0
95	0	6
102	1	0
101	0	0
104	3	0
100	0	1
100	0	1
99	0	2
100	0	1
100	0	1
96	0	5
103	2	0
98	0	3
99	0	2
100	0	1
	$\Sigma = 10$	$\Sigma = 23$

~~Avg = $\frac{10}{4}$ = 2.5~~

Avg = $\frac{10}{15}$ = 0.66

~~Avg = $\frac{23}{10}$ = 2.3~~

Avg = $\frac{23}{15}$ = 1.53

Q.e

Time bet^n arrival (min)	Frequency	Probability	CP	Tag No.
1.0	18	18/100 = 0.18	0.18	0 - 17
1.5	15	15/100 = 0.15	0.33	18 - 32
2.0	36	36/100 = 0.36	0.69	33 - 68
2.5	19	19/100 = 0.19	0.88	69 - 87
3.0	12	12/100 = 0.12	1	88 - 99

Random Arrivals	Arrival (min)
25	1.5
39	2.0
65	2.0
76	2.5
12	1.0
05	1.0
73	2.5
89	3.0
19	1.5
49	2.0
	$\Sigma = 19$

Average Arrival Time = $\frac{19}{10} = 1.9$

Q.5

→ Let X_1 & X_2 be the number of ~~products~~ chemicals A and B.

The total production of the chemicals A and B should be at least 350kg.

$$X_1 + X_2 \geq 350$$

A major customer orders for 125kg of product A must also be supplied.

$$X_1 \geq 125$$

Product A requires 2 hrs of processing time/kg

and " B " 1 hrs " " " " .

But for the coming month, 600 hrs of processing time are available

$$2X_1 + X_2 \leq 600.$$

The company wants to meet the requirements at minimum total production cost, where A's ~~prodn~~ and B's production costs are ~~Rs~~

Rs. 2/kg & Rs. 3/kg respectively.

Q.

Time bet^n arrival (min)	frequency
0.5	2
1.0	6
1.5	10
2.0	25
2.5	20
3.0	14
3.5	10
4.0	7
4.5	4
5.0	2

Arrival R.N.
93
22
53
64
39
07
10
63
76
35

Time bet^n service	Frequency
0.5	12
1.0	21
1.5	36
2.0	19
2.5	7
3.0	5

Service R.N.
78
76
58
54
74
92
38
70
96
92

→

Inter Arrival	P	CP	Tag No
0.5	0.02	0.02	0–1
1.0	0.06	0.08	2–7
1.5	0.1	0.18	8–17
2.0	0.25	0.43	18–42
2.5	0.20	0.63	43–62
3.0	0.14	0.77	63–76
3.5	0.10	0.87	77–86
4.0	0.07	0.94	87–93
4.5	0.04	0.98	94–97
5.0	0.02	1	98–99

Inter Arrival (time)	Arrival time
4.0	4.0
2.0	6.0
2.5	8.5
3.0	11.5
2.0	13.5
1.0	14.5
1.5	16.0
3.0	19.0
3.0	22.0
2.0	24.0

Time bet^n service (min)	P	C.P	Tag No
0.5	0.12	0.12	0–11
1.0	0.21	0.33	12–32
1.5	0.36	0.69	33–68
2.0	0.19	0.88	69–87
2.5	0.07	0.95	88–94
3.0	0.05	1	95–99

Service time	Service start Intime	service end outtime	C.W	Server
2	4	6	—	4
2	6	8	—	—
1.5	8.5	10	—	0.5
1.5	11.5	13	—	1.5
2	13.5	15.5	—	0.5
2.5	15.5	18	1	0
1.5	18.0	19.5	2	—
2	19.5	21.5	0.5	—
3	22.0	25	0	0.5
2.5	25	27.5	1	—
			Σ = 4.5	Σ = 7

Customer waiting time = 4.5, Avg = $\frac{4.5}{10}$ = 0.45

Server idle time = 7, Avg = $\frac{7}{10}$ = 0.7

Q. A tourist car operator

1. What do you mean by simulation? What are the advantages and limitation of simulation.

2. What do you mean by Markov process? In what area of management markov process applied successfully.

3. A sample of 100 arrival of customer in a departmental store is acc. to the following. Simulate the for the next 8. R.N. 25, 39, 65, 76, 12, 5, 73, 89

Time bet^n arrival	Frequency
0.5	12
1	21
1.5	36
2	19
2.5	7
3	5

→

Time bet^n Arrival	Frequency	P	CP	Taq No	R.N	Arrival
0.5	12	0.12	0.12	0 – 11	25	1
1	21	0.21	0.33	12 – 32	39	1.5
1.5	36	0.36	0.69	33 – 68	65	1.5
2	19	0.19	0.88	69 – 87	76	2
2.5	7	0.07	0.95	88 – 94	12	1
3	5	0.05	1	95 – 99	5	0.5
					73	2
					89	2.5
						Σ = 12

Avg = $\frac{12}{8}$ = 1.5

Q.1 What do you mean by simulation? What are the Advantages & Limitations of simulation? (25)

→ Simulation is a process where experiment is carried out in an artificial environment, where all the parameter are in control and through simulation, we can gather the knowledge which helps us in real environment for making decision.

Simulation is one of the most widely used and popularly accepted tool in operation research. It helps us in deciding the best policy with prior assurances that it implementation will certainly prove to be beneficial to the organization.

Some examples where simulation is being used are :- Military Training, Car Training, Weather forecasting etc.

Advantages :-

(i) It can be used to study existing systems without disrupting the ongoing operations.

(ii) It allows us to control times.

(iii) It is best suited to analyze complex and large practical problems when it is not possible to solve them through a mathematical method.

(iv) It helps us to forecast or analyze the scenario for longer period.

Limitations :-

(i) It does not generate optimal solutions.

(ii) In certain cases simulation model can be very expensive.

(iii) It may take a long time to develope a good simulation model.

(iv) It needs all the information about the constraint for analyzing, as simulation does not give the answer by itself.

Q.2 What do you mean by Markov Process? In what area of management, Markov process applied successfully?

→ Markov process is a mathematical model that looks at an array of events and analyzes the tendencies of each event. It was named after the Russian mathematician Andrey Markov. Markov process have been used to analyze inventory problems, equipment replacement process, plant location problems, brand-switching problems and dynamic systems.

In the following area of management, Markov process can be applied successfully :-

1. Inventory Management :- It can be applied to model inventory systems where the availability of items fluctuates over time.

2. Quality control :- In Manufacturing, Markov process can model the state transitions of a system's quality. This is useful for improving product quality over time.

3. Financial Management :- Markov processes find applications in financial modelling, particularly in analyzing market states, portfolio management, and predicting changes in economic conditions.

4. Customer Relationship Management :- Markov processes are used to model customer behavior in areas such as retention and purchasing patterns. This helps businesses make informed decisions regarding marketing and customer engagement strategies.

Queueing Theory

A Queue is a waiting line where customers are waiting to get their service and the line sometime finite & sometime infinite and generally A queue arises when no. of customers are more than service facility.

Queueing System

There are 4 different system in queueing system

1) The Input
2) Queue Discipline
3) Service Mechanism
4) customer's Behavior

specific terms which describe the behavior's customer.

1) Managing
2) Balking
3) Jockeying

→ customer, who jumps from one Queue to another Queue by observing which queue is moving fast, those types of customer are known as Jockey type.

→ This type of customer left the queue due to impatience are Managing Balk

→ A customer, who do not join the queue because the queue is too long, are Balking type.

Queue Discipline

There are 4 type of Discipline

1) FIFO (First in first out)
2) LIFO (Last in first out)
3) SIRO (Service in Random order)
4) Priority (VIP, senior citizen)

→ FIFO is a usual discipline in a queueing system, also known as FCFS.

Service Mechanism / Service Pattern

1) Single Queue – single server
2) ,, ,, – Multiple server
3) Multiple queue – single server
4) Multiple queue – Multiple server

The most important parameter

λ is the parameter
μ (MU)

λ is defined as the arrival rate per unit time, per
μ is the parameter, which is defined as service rate/unit time.

Kendall's Notation (To represent Queueing Model)

Generally, the queueing models are described by the help of Kendall's notation, **a/b/c/:d/e**,

where a is the arrival pattern
b described as service pattern
c described as no. of channels / no. of service station
d ,, as queue length
e ,, as queue discipline.

There are 4 different types of models in queueing system

1) M/M/1 : ∞/FCFS
2) M/M/S : ∞/FCFS
3) M/M/1 : N/FCFS
4) M/M/S : N/FCFS

Important Measures in Queueing Theory

1) Traffic Intensity (Server Busy) (ρ) ($1-\rho$ = server idle)
2) Waiting Time of a Customer in a Queue (W_q)
3) Waiting Time of a ~~Customer~~ System in a ~~Queue~~ System (W_s)
4) ~~Max~~ Length of the Queue (L_q)
5) Length of the System (L_s)

What is Queue?
Define customer.
Describe Queueing System.
Kandall's Notation.

TP
Assignment Problem

possible 2 mark Q.

1) Define Decision variable
2) What do you mean by LPP
3) Define slack & surplus variable
4) What is feasible region.
5) What is unbalanced TP.
6) What is unbalanced Assignment problem
7) what is degeneracy is in TP.
8) what is unbounded solution in simplex method?
9) Define LOOP
10) What is Alternative optimal sol^n in Simplex method?
11) Define Artificial variable.

Q.1 What is Queue?

→ Queues are a part of everyday life. We all wait in queues to buy a movie ticket, to make bank deposit, pay for groceries, to have ride in an amusement park and have become adjustment to wait but still get annoyed by unusually long waits.

However, It can be defined as "A flow of customers from finite or infinite population towards the service facility forms a waiting line otherwise known as a Queue.

Q.2 Define customer

→ The one who requires service is called a customer. The customer may be persons, machines, vehicles, etc. The customer is otherwised known as "Arriving unit".

Q.3 Describe Queueing System.

→ A queueing system can be described as customers arriving for service, waiting for service if it is not immediate and leaving the service after being served.

There are 4 elements of a Queueing System.

(1) The Input
(2) Queue Discipline
(3) Service Mechanism
(4) Customer's Behavior

* The Input (Arrival Pattern)

It describes the way in which the customers arrive and join the system. Generally, customers arrive in a ~~poission~~ poisson fashion, which is not possible to predict. Hence, the arrival pattern can be described in terms of probabilities or probabilities for inter arrival times. The Mean arrival rate is denoted by λ.

* Queue Discipline

It can be defined as the rule for selecting the next customer from the queue for service. The most common disciplines are :-

1) FIFO
2) LIFO
3) SIRO
4) PRIORITY

* FIFO (First In First Out)

FIFO is a queueing discipline where the earliest item to join the queue is the first to be served.

ex:- checkout line at Grocery store. The first person to join the line is the first one to complete their purchase and leave.

* LIFO (Last In First Out)

LIFO is a queueing discipline, where the most recently added item or the last item that enters the queue is the first to be served.

ex:- Stack of Plates. The last plate added to the stack is the first plate you pick up to use.

* SIRO (Service In Random Order)

SIRO is a queueing discipline, where items are served in a random sequence without following a specific order.

ex:- Lottery ticket processing, where winners are randomly selected.

* PRIORITY

PRIORITY discipline is a queueing model, where items are assigned priority values, and the item with the highest priority is served first.

ex:- VIP's, Senior citizens, Emergency Patient.

SERVICE MECHANISM

This means, the arrangement of service facility to serve customers. A service station may have one or several servers arranged in parallel.

Basic elements of Service Mechanism/ Service Pattern

1) Single Queue - Single Server
   → A single cashier serving customers in a grocery store.
2) Single Queue - Multiple server
   → Airport security checkpoints with multiple screening lanes.
3) Multiple Queue - Single Server
   → Customer service desks at a large retail store.
4) Multiple Queue - Multiple Server
   → Fast-food restaurant with multiple order counters.

The most impotant parameter of service mechanism are "λ" and "μ"

λ is defined as the arrival rate per unit time.

μ is defined as the service rate per unit time.

CUSTOMER'S BEHAVIOR

It refers to how customers join the queue, their expectations, and their reaction to the queueing process. Understanding customer behavior is crucial for designing effective queueing systems and ensuring a positive experience for those waiting in line.

Customers generally behave in following ways.

1. Balking :- A customer who leaves the queue because the queue is too long, and he has no time to wait or does not have sufficient waiting space.

2. Reneging :- It refers to a situation, where a customer joins the queue but decides to leave before reaching the service point due to impatience or dissatisfaction with the wait.

3. Jockeying :- customer, who jumps from one queue to another queue by observing which queue is moving fast. This is most common in a supermarket.

8. In a supermarket, the avg. arrival rate of customer is 10 in every 30 min and the avg time taken by the server is 2.5m/customer. Find out server busy time and expected time spent by the customer in the system. (33)

Here λ is 10 in every 30 min $\lambda = \frac{10}{30}$ min

As avg time taken by the server $\mu = \frac{1}{2.5}$ min

$$\rho = \frac{\lambda}{\mu} = \frac{0.33}{0.4} = 0.825$$

$$W_s = \frac{1}{\mu - \lambda} \qquad \frac{1}{\mu - \lambda} = \frac{1}{0.4 - 0.33}$$

$$= \frac{1}{0.4} - 0.33 \qquad = \frac{1}{0.07}$$

$$= 2.5 - 0.33$$

$$= 2.17$$

People arrive at a cinema hall with a rate of 25 per hour and service time is per customer 2 mins. calculate the avg. no. of customer in the queue (L_q), W_s and utilization factor (ρ)

$$\lambda = \frac{25}{60} = 0.41$$

$$\mu = \frac{1}{2} = 0.5$$

$$\rho = \frac{\lambda}{\mu} = \frac{0.41}{0.5} = 0.82$$

$$L_q = \frac{\rho^2}{1 - \rho} = \frac{0.6724}{1 - 0.82} = \frac{0.6724}{0.18} = 3.735$$

$$W_s = \frac{1}{\mu - \lambda} = \frac{1}{0.5 - 0.41} = \frac{1}{0.09} = 11.11$$

$\lambda = 25$

$\mu = 30$

$\rho = \frac{25}{30} = 0.83$

$L_q = \frac{\rho^2}{1 - \rho} = \frac{0.6889}{1 - 0.83} = \frac{0.6889}{0.17}$

(1) Traffic Intensity / Server Busy / Utilization Factory (ρ) rho

$$\rho = \frac{\lambda}{\mu}$$

(2) Server Idle $= (1 - \rho)$

(3) Avg no. of customer in the system $L_s = \frac{\rho}{1-\rho}$

(4) Avg no. of customer in the queue $L_q = \frac{\rho^2}{1-\rho}$

(5) Waiting time of a customer in the system $W_s = \frac{1}{\mu - \lambda}$

(6) Waiting time of a customer in the queue $W_q = \frac{\lambda}{\mu(\mu-\lambda)}$

$$L_s = \frac{\rho}{1-\rho}$$

$$L_q = \frac{\rho^2}{1-\rho}$$

$$W_s = \frac{1}{\mu} - \lambda$$

$$W_q = \frac{\lambda}{\mu(\mu-\lambda)}$$

Q. A T.V repairemen repair a T.V with a mean rate of 30 mins per Tv. and the arrival of T.V to the repairman 10 per shift in a day. Find out the traffic intensity, Server idle, length of the system, length of the queue and W_s, W_q.

→ Here λ is 10

As 30min is the repair time per T.V,

Thus $\mu = 16$

Now Traffic intensity, $\rho = \frac{10}{16}$

Server Idle, $\rho = 1 - \frac{10}{16}$

Length of the System $L_s = \frac{\rho}{1-\rho} = \frac{0.625}{0.375} = 1.66$

Length of the queue, $L_q = \frac{\rho^2}{1-\rho} = \frac{0.390625}{0.375} = 1.041$

$$W_s = \frac{1}{\mu} - \lambda = \frac{1}{16} - 10 = 0.16$$

$$W_q = \frac{\lambda}{\mu(\mu-\lambda)} = \frac{10}{16 \times 6} = \frac{10}{96} = 0.104$$

Q. Cars arrive at a petrol pump with a mean rate of half min. The attendent take $\frac{1}{5}$ min to supply the petrol. calculate the avg. no waiting to be serve? The avg no. of cars in the queue and calculate the petrol pump attendent idle time. (35)

→ ~~$\frac{1}{2}$~~ Cars are

$\lambda = 2$

$\mu = \frac{1}{5}$ $\quad \mu = \frac{1}{1/5} = 5$

$\rho = \frac{1}{2} \times \frac{5}{1}$ $\quad \rho = 0.4$

$L_s = \frac{\rho}{1-\rho} = \frac{0.4}{1-0.4} = \frac{0.4}{0.6} = 0.66$

$L_q = \frac{\rho^2}{1-\rho} = \frac{0.16}{0.6} = 0.26$

idle time $1 - \rho = 0.6$

24 hr — 30 min

1 — $\frac{30}{24}$ min

1 min $\frac{30}{24 \times 60}$

$\frac{1}{5}$ hours $\quad \frac{60}{5} = 12$

Q.

→ Train arrive at a rate of 30 ~~mins~~ trains per day.

means 24 hr — 30 trains

1 hr → $\frac{30}{24}$ trains per hr

and 1 min → $\frac{\frac{30}{24}}{60} = \frac{30}{24 \times 60} = \frac{1}{48}$ trains/min

Also Service rate per unit, $\mu = \frac{1}{30}$ min

(i) Mean queue size, $\rho = \frac{\lambda}{\mu} = \frac{1}{48} \times 30 = 0.625$

(ii)

Q.

100	200	300
140	100	500
80	120	60

			Supply
100	200	300	100
140	100	500	110
Demand 80	120	60	

Supply

→ 100	80	200 20	300	~~100~~ 20 ~~0~~
140		100 100	500 10	~~110~~ 10
0		0	0 50	50
Demand	~~80~~ 0	~~120~~ ~~100~~ 0	60	

T.P cost = 100×80 + 200×20 + 100×100 + 500×10 + 0×50

= 8000 + 4000 + 10000 + 5000

= 27000

Using MODI

Allocated cell ($U_1 = 0$)

$C_{11} = U_1 + V_1 = 100$, $V_1 = 100$

$C_{12} = U_1 + V_2 = 200$, $V_2 = 200$

$C_{22} = U_2 + V_2 = 100$, $U_2 = -100$

$C_{23} = U_2 + V_3 = 500$, $V_3 = 600$

$C_{33} = U_3 + V_3 = 0$, $U_3 = -600$

Non Allocated

$\Delta_{13} = C_{13} - (U_1 + V_3) = 300 - (0 + 600) = -300 < 0$

$\Delta_{21} = C_{21} - (U_2 + V_1) = 140 - (-100 + 100) = 140 > 0$

$\Delta_{31} = C_{31} - (U_3 + V_1) = 0 - (-600 + 100) = 500 > 0$

$\Delta_{32} = C_{32} - (U_3 + V_2) = 0 - (-600 + 200) = 400 > 0$

As $\Delta_{13} < 0$, so we need to start a loop from C_{13}

100	80	200 20 (-)	300 (+)
140		100 (+) 100	500 10 (-)
0		0	0 50

$\theta = \min\{20, 10\} = 10$

→

100	80	200 10	300 10
140		100 110	500
0		0	0 50

It is a non-degeneracy.

Allocated cell ($U_1 = 0$)

$C_{11} = U_1 + V_1 = 100$, $V_1 = 100$

$C_{12} = U_1 + V_2 = 200$, $V_2 = 200$

$C_{13} = U_1 + V_3 = 300$, $V_3 = 300$

$C_{22} = U_2 + V_2 = 100$, $U_2 = -100$

$C_{33} = U_3 + V_3 = 0$, $U_3 = -300$

Net evalution

$\Delta_{21} = C_{21} - (U_2 + V_1) = 140 - (-100 + 100) = 140 > 0$

$\Delta_{23} = C_{23} - (U_2 + V_3) = 500 - (-100 + 300) = 300 > 0$

$\Delta_{31} = C_{31} - (U_3 + V_1) = 0 - (-300 + 100) = 200 > 0$

$\Delta_{32} = C_{32} - (U_3 + V_2) = 0 - (-300 + 200) = 100 > 0$

As all $\Delta_{ij} > 0$, so our Soln reached optimal.

And total cost = 100×80 + 200×10 + 300×10 + 100×110 + 0×50

= 8000 + 2000 + 3000 + 11000

= 24000

Q. Mean = 40

S.D = 5.1

n = 100

→ As Mean = 40

$\Rightarrow \frac{\Sigma X}{n} = 40$

$\Rightarrow \Sigma X = 400$

Thus correct sum = wrong sum − wrong obs^n + correct obs^n

$\Sigma X = 4000 - 50 + 40$

$= 3900$

Thus correct Mean $= \frac{3900}{100} = 39.9$

Now we know $\sigma_x^2 = \frac{\Sigma x^2}{n} - (\bar{x})^2$

$\Rightarrow (5.1)^2 = \frac{\Sigma x^2}{n} - (40)^2$

$\Rightarrow \frac{(26.01 + 1600)}{100} = \Sigma x^2$

$\Rightarrow \Sigma x^2 = (26.01 + 1600) \times 100$

$\Rightarrow \Sigma x^2 = 162601$

Hence correct sum square $= \Sigma x^2 - (50)^2 + (40)^2$

$\Sigma x^2 = 162601 - 2500 + 1600$

$\Rightarrow \Sigma x^2 = 161701$

$\therefore \sigma_x = \sqrt{\frac{\Sigma x^2}{n} - (\bar{x})^2}$

$= \sqrt{\frac{161701}{100} - (39.9)^2}$

$= \sqrt{1617.01 - 1592.01}$

$= 5$

Q.2

Price of A	Price of B	$(x-\bar{x})$	$(x-\bar{x})^2$	$(y-\bar{y})$	$(y-\bar{y})^2$
55	108	2	4	3	9
54	107	1	1	2	4
52	105	-1	1	0	0
53	105	0	0	0	0
56	106	3	9	1	1
58	107	5	25	2	4
52	104	-1	1	-1	1
50	103	-3	9	-2	4
51	104	-2	4	-1	1
49	101	-4	16	-4	16
530	1050		70		40

$\bar{x} = \frac{530}{10} = 53$ $\quad \bar{y} = \frac{1050}{10} = 105$

Now standard deviation of x,

$$\sigma_x = \sqrt{\frac{(x-\bar{x})^2}{N}} = \sqrt{\frac{70}{10}} = 2.64$$

and $$\sigma_y = \sqrt{\frac{(y-\bar{y})^2}{N}} = \sqrt{\frac{40}{10}} = 2$$

Q.8 Find out mean, median & mode

Marks (x)	Frequency f	M	fM	C.F
10-20	15	15	225	15
20-30	20	25	500	35
30-40	45	35	1575	80
40-50	15	45	675	95
50-60	5	55	275	100
	100		3250	

Mean $= \frac{\Sigma fM}{N} = \frac{3250}{100} = 32.5$

Median class $\frac{N}{2} = \frac{100}{2} = 50$,

So it lies betⁿ class (30-40) having frequency 45

So Median $= L_1 + \frac{L_2 - L_1}{f_1}(M - c)$

$= 30 + \frac{40-30}{45}(50-25)$

$= 30 + \frac{10}{45} \times 25$

$= 30 + 5.55 = 35.55$

and Modal class is 30-40 45

Hence Mode $= L_1 + \frac{f_1 - f_0}{2f_1 - f_2 - f_0}(L_2 - L_1)$

$= 30 + \frac{45-20}{90-15-20}(40-30)$

$= 30 + \frac{25}{55} \times 10$

$= 30 + 4.54$

$= 34.54$

Mode = 3Median - 2Mean

Emperical relation betⁿ Mean, Me, Mo

Q.3

Test scores	weekly sales (in 000)	$dx = x - \bar{x}$	$dy = y - \bar{y}$	dx^2	dy^2	$dx\,dy$
50	30	-10	-13	100	169	130
60	60	0	17	0	289	0
50	40	-10	-3	100	9	30
60	50	0	7	0	49	0
80	60	20	17	400	289	340
50	30	-10	-13	100	169	130
80	7	20	-36	400	1296	-720
40	50	-20	7	400	49	-140
70	60	10	17	100	289	170
540	387	$\Sigma = 0$	0	1600	2608	-60

$b_{yx} = \frac{\Sigma dx\,dy}{\Sigma dx^2} = \frac{-60}{1600} = -0.0375$

(i) regression eq yonx $\Rightarrow (y - \bar{y}) = b_{yx}(x - \bar{x})$

$(y - 43) = -0.0375(x - 54)$

$\Rightarrow y + 0.0375x + 45.025 = 0$

(ii) If $x = 65$

$\rightarrow y + 0.0375(65) + 45.025 = 0$

$\Rightarrow y + 2.4375 - 40.975$

$\Rightarrow y = 38.5375$

Q.7 Derive regression lines from the following data. (40)

$\Sigma X = 30$, $\Sigma X^2 = 190$, $\Sigma XY = 192$

$\Sigma Y = 30$, $\Sigma Y^2 = 190$, $n = 5$

$$b_{yx} = \frac{n\Sigma XY - \Sigma X \cdot \Sigma Y}{n\Sigma X^2 - (\Sigma X)^2}$$

$$b_{xy} = \frac{n\Sigma XY - \Sigma X \cdot \Sigma Y}{n\Sigma Y^2 - (\Sigma Y)^2}$$

→ According to direct method,

$$b_{xy} = \frac{n\Sigma XY - \Sigma X \cdot \Sigma Y}{n\Sigma Y^2 - (\Sigma Y)^2}$$

$$= \frac{5 \times 192 - (30 \times 30)}{5 \times 190 - (30)^2} = \frac{960 - 900}{950 - 900} = \frac{60}{50} = 1.2$$

And $b_{yx} = \frac{n\Sigma XY - \Sigma X \cdot \Sigma Y}{n\Sigma X^2 - (\Sigma X)^2} = \frac{5 \times 192 - (30 \times 30)}{5 \times 190 - (30)^2} = 1.2$

We know the regression eq^n X on Y is

~~$(x - \bar{x}) = b_{yx}(y - \bar{y})$~~

$$(X - \bar{X}) = b_{xy}(Y - \bar{Y})$$

$$\Rightarrow (X - 6) = 1.2(Y - 6) \quad \left[\because \text{As } \bar{X} = \frac{\Sigma X}{n} = \frac{30}{5} = 6, \ \bar{Y} = \frac{\Sigma Y}{n} = \frac{30}{5} = 6\right]$$

$$\Rightarrow X - 6 = 1.2Y - 7.2$$

$$\Rightarrow X - 1.2Y + 1.2 = 0$$

and Y on X is

$$(Y - \bar{Y}) = b_{yx}(X - \bar{X})$$

$$\Rightarrow (Y - 6) = 1.2(X - 6)$$

$$\Rightarrow Y - 6 = 1.2X - 7.2$$

$$\Rightarrow Y - 1.2X + 1.2 = 0$$

(F)

	B_1	B_2	B_3	B_4
A_1	3	2	4	0
A_2	3	4	2	4
A_3	4	2	4	0
A_4	0	4	0	8

Using Dominance

~~3 2 4 0~~

3 4 2 4

4 2 4 0

0 4 0 8

(As $\pi_1 < \pi_3$)

Using Dominance

3 4 2 4

4 2 4 0

0 4 0 8

(As $c_3 < c_4$)

Using Dominance

~~3 4 4~~

4 2 0

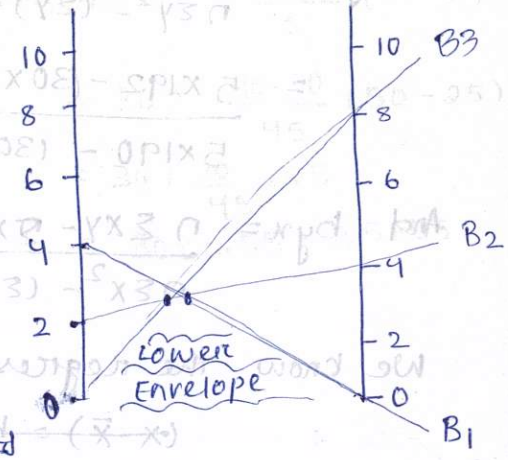
0 4 8

Avg of (π_2, π_3) (2) (3) (4)

(As $\pi_4 < \frac{\pi_2 + \pi_3}{2}$)

4 2 0

0 4 8



~~The new 2×2 game is~~

~~Probability~~

~~2 0~~ ~~4~~, $P_1 = \frac{4}{6}$

~~4 8~~ ~~2~~, $P_2 = \frac{2}{6}$

~~Probability~~ ~~2~~

The new 2×2 ~~order~~ game is

Probability

4 2 4, $P_1 = \frac{4}{6}$

0 4 2, $P_2 = \frac{2}{6}$

Probability 2 4

$q_1 = \frac{2}{6}$, $q_2 = \frac{4}{6}$

$\therefore$ Saddle point, $v = 4 \times \frac{4}{6} + 0 \times \frac{2}{6}$

$= \frac{16}{6} = 2.66$

And strategy for player A, $S_A = (0, 0, \frac{4}{6}, \frac{2}{6})$

" " " " B, $S_B = (\frac{2}{6}, \frac{4}{6}, 0, 0)$

Q6 Calculate co-relation co-efficient.

X	Y	U = X − A	V = Y − A	U^2	V^2	UV
1	25	−4	13	16	169	−52
2	23	−3	11	9	121	−33
3	(20)	−2	8	4	64	−16
4	16	−1	4	1	16	−4
(5)	12	0	0	0	0	0
6	(12)	1	0	0	0	0
7	10	2	−2	4	4	−4
8	8	3	−4	9	16	−12
9	5	4	−7	16	49	−28
10	4	5	−8	25	64	−40
55	135	5	15	85	439	189

$$r = \frac{n\Sigma UV - (\Sigma U)(\Sigma V)}{\sqrt{\left[n\Sigma U^2 - (\Sigma U)^2\right]\left[n\Sigma V^2 - (\Sigma V)^2\right]}}$$

$$= \frac{10 \times 189 - 5 \times 15}{\sqrt{\left[10 \times 85 - (5)^2\right]\left[10 \times 439 - (15)^2\right]}}$$

$$= \frac{1890 - 75}{\sqrt{(850 - 25)(4390 - 225)}}$$

$$= \frac{1815}{\sqrt{(825 \times 4165)}}$$

$$= \frac{1815}{28.72 \times 64.53}$$

$$= \frac{1815}{1853.49}$$

$$= 0.979$$

Q.5 (a) The mean and S.D of a set of 100 obs^n are 40 & 5 respectively. Later it was found, the computers by mistake took value 50 in place of 40 for one observation. Find Correct mean and S.D.

→ Given $n=100$,

$\bar{X} = 40$

and $\sigma_x = 5$

From mean we have,

$\frac{\Sigma X}{n} = 40$

$\Rightarrow \frac{\Sigma X}{100} = 40 \Rightarrow \Sigma X = 4000$

Again, we know $\sigma_x^2 = \frac{\Sigma X^2}{n} - (\bar{X})^2$

$\Rightarrow (5)^2 = \frac{\Sigma X^2}{100} - (40)^2 \Rightarrow \frac{\Sigma X^2}{100} = 25 + 1600$

$\Rightarrow \Sigma X^2 = 162500$ (Wrong sum square)

Correct sum square = (Wrong sum)^2 − (Wrong obs^n)^2 + (Correct sum)^2

$\Rightarrow \Sigma X^2 = 162500 - (50)^2 + (40)^2$

$\Sigma X^2 = (162500 - 2500 + 1600) = 161600$

Now Correct sum = Wrong Sum − Wrong obs^n + Correct obs^n

$\Rightarrow \Sigma X = 4000 - 50 + 40$

$= 3990$

Hence correct mean, $\bar{X} = \frac{\Sigma X}{n} = \frac{3990}{100} = 39.90$

Hence Correct S.D

$\sigma_x = \sqrt{\frac{\Sigma X^2}{n} - (\bar{X})^2}$

$= \sqrt{\frac{161600}{100} - (39.90)^2}$

$= \sqrt{1616 - 1592.01}$

$= \sqrt{23.99}$

$= 4.89$

(b)

No. of Goals (X)	f_1 Team (A)	f_2 Team (B)	(For Team A) f_1X	f_1X^2	(For Team B) f_2X	f_2X^2
0	27	17	0	0	0	0
1	9	9	9	9	9	9
2	8	6	16	36	12	24
3	5	5	15	45	15	45
4	4	3	16	64	12	48
5	1	2	5	25	10	50
	54	42	61	179	58	176

So $\bar{X}_A = \frac{\Sigma f_1 X}{N} = \frac{61}{54} = 1.13$

Now $\sigma_A = \sqrt{\frac{\Sigma f_1 X^2}{N} - (\bar{X}_A)^2}$

$= \sqrt{\frac{179}{54} - (1.13)^2}$

$= \sqrt{(3.31 - 1.27)}$

$= \sqrt{(2.04)}$

$= 1.42$

$\bar{X}_B = \frac{\Sigma f_2 X}{N} = \frac{58}{42} = 1.38$

$\sigma_B = \sqrt{\frac{\Sigma f_2 X^2}{N} - (\bar{X}_B)^2}$

$\sigma_B = \sqrt{\frac{176}{42} - (1.38)^2}$

$= \sqrt{4.2 - 1.90}$

$= \sqrt{2.3}$

$= 1.51$

Hence $CV_A = \frac{\sigma_A}{\bar{X}_A} \times 100$ and $CV_B = \frac{\sigma_B}{\bar{X}_B} \times 100$

$= \frac{1.42}{1.13} \times 100$ $= \frac{1.51}{1.38} \times 100$

$= 160.46$ $= 109.42$

$\therefore CV_B < CV_A$.

So Team B is more consistent.

$AM \geq GM \geq HM$

$G^2 = A \times H$

$(GM^2 = AM \times HM)$ For obs^n

Qy If the AM of two obs^n is 127.5 and their GM is 60. Find HM? and two obs^n?

→ Let the two observation are x and y

So $\frac{x+y}{2} = 127.5 \quad (AM = 127.5)$

$\Rightarrow x+y = 255$ —— (1)

Again $GM = 60$

So $\sqrt{xy} = 60$

On squaring both side, we get –

$xy = 3600$ —— (2)

and $HM = \frac{n}{\sum(\frac{1}{x})} = \frac{2}{(\frac{1}{x}+\frac{1}{y})}$

$= \frac{2xy}{x+y}$

$= \frac{2(3600)}{255}$ [$\because$ From eq^n (1) & (2)]

$= 28.23$

Now we know

$GM^2 = AM \times HM$

~~Now from eq^n (1), we have $y = (127.5 - x)$~~

~~Again, we know $(a+b)^2 = a^2 + b^2 + 2ab$ / $(x+y)^2 = x^2 + y^2 + 2xy$~~

~~$\Rightarrow (255)^2 = x^2 + y^2 + 2(3600)$~~

~~$\Rightarrow 65025 = x^2 + (127.5 - x)^2 + 7200$~~

~~$\Rightarrow x^2$~~

we know $(a-b)^2 = (a+b)^2 - 4ab$

$= (x-y)^2 = (x+y)^2 - 4xy$

$\Rightarrow (x-y)^2 = (255)^2 - 4 \times 3600$

$\Rightarrow x-y = \sqrt{65025 - 14400}$

$\Rightarrow x-y = 225$ —— (3)

From eq^n (2) and (3), we have

$x+y = 255$

$x-y = 225$

$2x = 480$

$\Rightarrow x = 240$

Hence $y = 15$

$\therefore x = 240$

$y = 15$

Q. (Q_1, Q_2, Q_3)

Heights in CM	No. of Person	Cummilative Frequency	
145–150	2	2	→
150–155	4	6	
155–160	12	18	
160–165	22	40	Q_1
165–170	30	70	Q_2
170–175	25	95	Q_3
175–180	10	105	

Q_1 class $= \frac{N}{4} = \frac{105}{4} = 26.25$

So $Q_1 = L_1 + \frac{L_2 - L_1}{f_1}\left(\frac{N}{4} - C\right)$

$= 160 + \frac{165-160}{22} + (26.25 - 18)$

$= 160 + \frac{5}{22} \times 8.25$

$= 160 + \frac{41.25}{22}$

$= 160 + 1.875$

$= 161.875$ (Lower Quartile)

Now Q_2 class $= \frac{N}{2} = \frac{105}{2} = 52.5$

Thus $Q_2 = L_1 + \frac{L_2 - L_1}{f_1}\left(\frac{N}{2} - C\right)$

$= 165 + \frac{170-165}{30}(52.5 - 40)$

$= 165 + \frac{5}{30}(12.5)$

$= 165 + \frac{62.5}{30}$

$= 165 + 2.083$

$= 167.083$ (Median)

Again Q_3 class $= \frac{3N}{4} = 3 \times \frac{105}{4} = 78.75$

Hence $Q_3 = L_1 + \frac{L_2 - L_1}{f_1}\left(\frac{3N}{4} - C\right)$

$= 170 + \frac{175-170}{25}(78.75 - 70)$

$= 170 + \frac{5}{25} \times 8.75$

$= 170 + \frac{8.75}{5}$

$= 170 + 1.75$

$= 171.75$ (Upper Quartile)